

# On the real-time joint determination of GNSS clocks and biases in UDUC PPP-RTK

**Dimitrios Psychas**

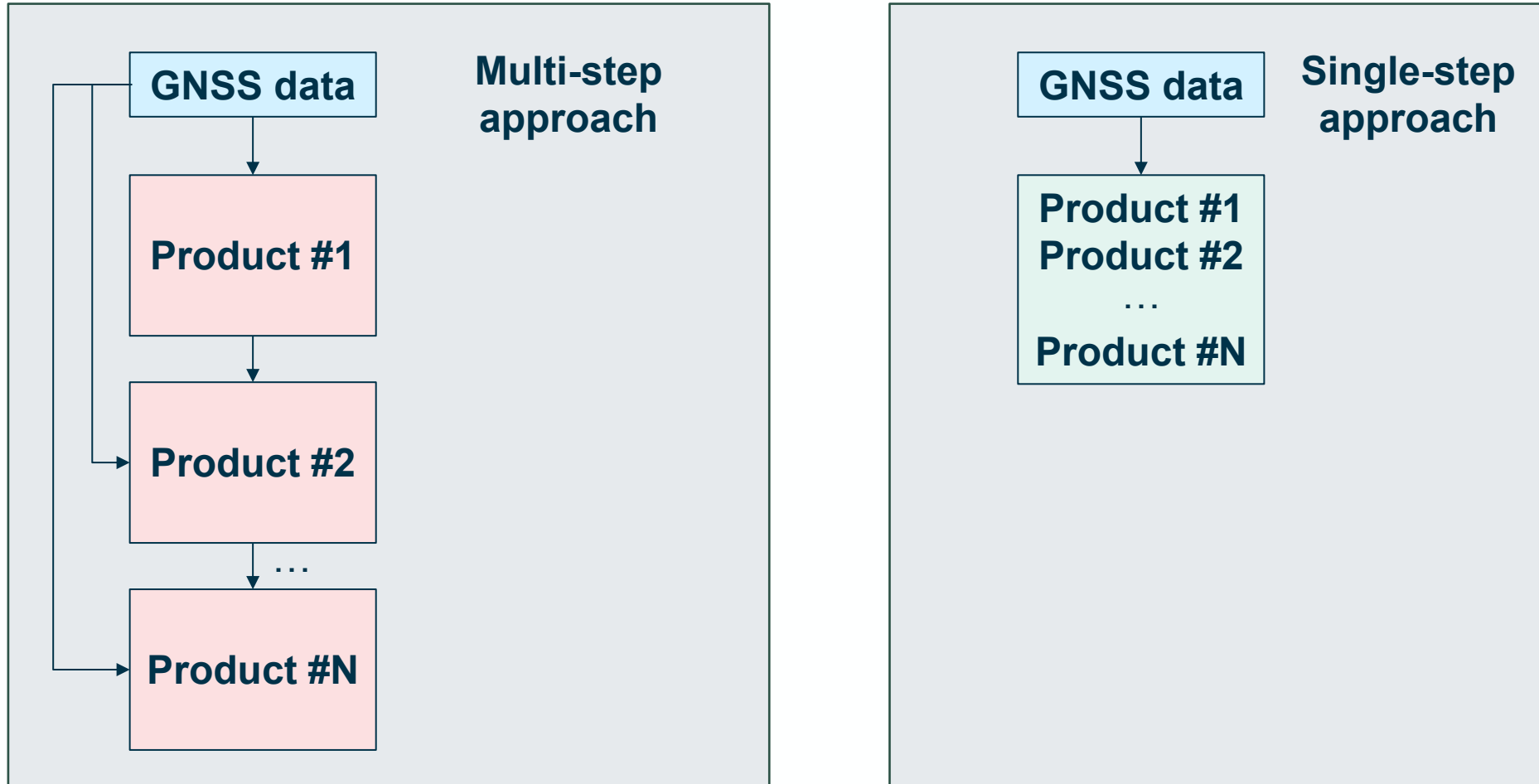
European Space Agency (ESA/ESTEC)

IGS BAR & WCC Splinter Meeting (SPM64)

EGU2026, 07/05/2026, Vienna, Austria

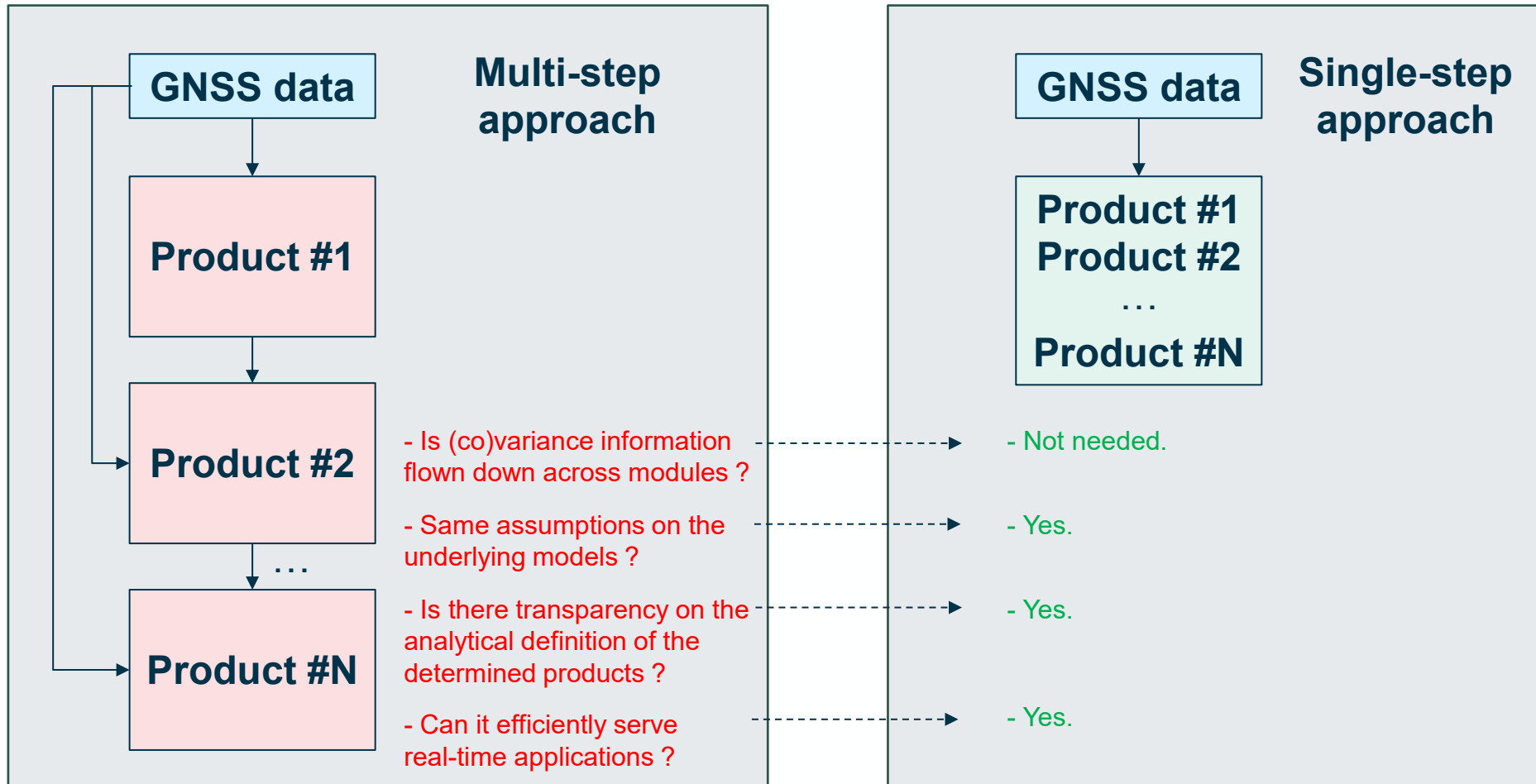
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# 1. Joint real-time estimation of GNSS corrections



Consistency in SSR correction generation is **key** for high-accuracy positioning.

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UDUC PPP-RTK framework with common-clocks pivot-receiver S-basis:

$$E(p_{r,j}^s) = d\tilde{t}_r - d\tilde{t}^s + m_r^s \tilde{T}_r + \mu_j \tilde{t}_r^s + \tilde{d}_{r,j>2} - \tilde{d}_{,j>2}^s$$
$$E(\phi_{r,j}^s) = d\tilde{t}_r - d\tilde{t}^s + m_r^s \tilde{T}_r - \mu_j \tilde{t}_r^s + \lambda_j (\tilde{\delta}_{r,j} - \tilde{\delta}_{,j}^s + \tilde{a}_{r,j}^s)$$

**Full-rank UDUC measurement + dynamic models**

$r = 1, \dots, n$  → receiver  
 $j = 1, \dots, f$  → frequency  
 $s = 1, \dots, m$  → satellite  
 $(\tilde{\cdot})$  → estimable parameter

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multi-GNSS multi-frequency code and phase data

**LSE at 1<sup>st</sup> epoch + KF from 2<sup>nd</sup> epoch onwards**

simultaneous estimation of products

**Satellite clocks:**  $\hat{d\tilde{t}}^s$

**Products**

**Satellite phase biases (phase OSBs):**  $\hat{\tilde{\delta}}_{,j}^s$  (for every frequency)

**Satellite code biases (code OSBs):**  $\hat{\tilde{d}}_{,j>2}^s$  (from 3<sup>rd</sup> freq. onwards)

- No code OSBs are estimable for the first two (pivot) frequencies, neither they are needed for user IAR.
- S-basis (datum):  $\tilde{d}_{,j=1,2}^s := 0$



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**Products**

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→ No a priori code OSBs are required for phase OSB generation

# 1. Joint real-time estimation of GNSS corrections

## Application of corrections for arbitrary-frequency PPP-AR

Phase combined corrections:  $\hat{C}_{\phi,j}^s = d\hat{t}^s + \lambda_j \hat{\delta}_{,j}^s \quad \forall j$

Code combined corrections:  $\hat{C}_{p,j}^s = \begin{cases} d\hat{t}^s & j = 1, 2 \\ d\hat{t}^s + \hat{d}_{,j>2}^s & j > 2 \end{cases}$

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### Full-rank UDUC user model

$$E(p_{u,j}^s + \hat{c}_{p,j}^s) = e_u^{sT} \Delta x_u + d\tilde{t}_u + m_u^s \tau_u + \mu_j \tilde{l}_u^s + \tilde{d}_{u,j>2}$$

$$E(\phi_{u,j}^s + \hat{c}_{\phi,j}^s) = e_u^{sT} \Delta x_u + d\tilde{t}_u + m_u^s \tau_u - \mu_j \tilde{l}_u^s + \lambda_j (\tilde{\delta}_{u,j} + \tilde{a}_{u,j}^s)$$

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$$(\cdot)_{,GF} = -\frac{1}{\mu_2 - \mu_1} [(\cdot)_{,1} - (\cdot)_{,2}]$$

### Full-rank UDUC user model

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➔ User integer-estimable ambiguities (always) in DD form

- The user-estimable ionospheric delays 'absorb' the (scaled) sat. DCBs
- In case of PPP-RTK, the (scaled) DCBs or code OSBs at pivot frequencies need to be consistent with the precise iono corrections.

$$\tilde{l}_u^s = l_u^s + d_{r,GF}^s - d_{,GF}^s$$

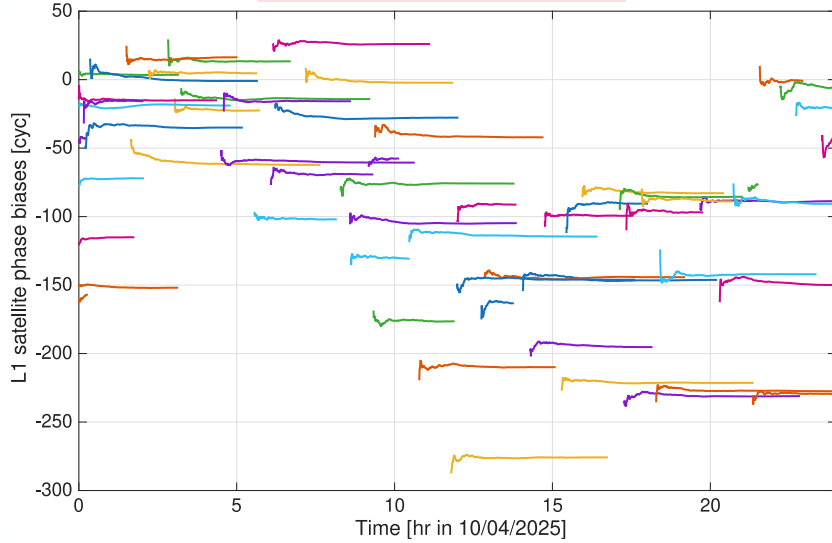
## Processing setup:

- Regional network in Europe
- GPS L1/L2, Galileo E1/E5a/E6, BeiDou-3 B1I/B3I/B2a code and phase data
- **Uncombined and undifferenced observation model**
- 10/04/2025 (30s sampling period)
- **Integrated network data processing** using the Generalized Kalman filter
  - Receiver clocks: unlinked in time
  - Satellite clocks: unlinked in time
  - Receiver/satellite biases: constant-state random process
  - Ion. slant delays: constant-state random process
  - Phase ambiguities: time-constant
- Integer ambiguity resolution: LAMBDA-based ILS
- PPP-AR processing using zero-latency products (post-processing)

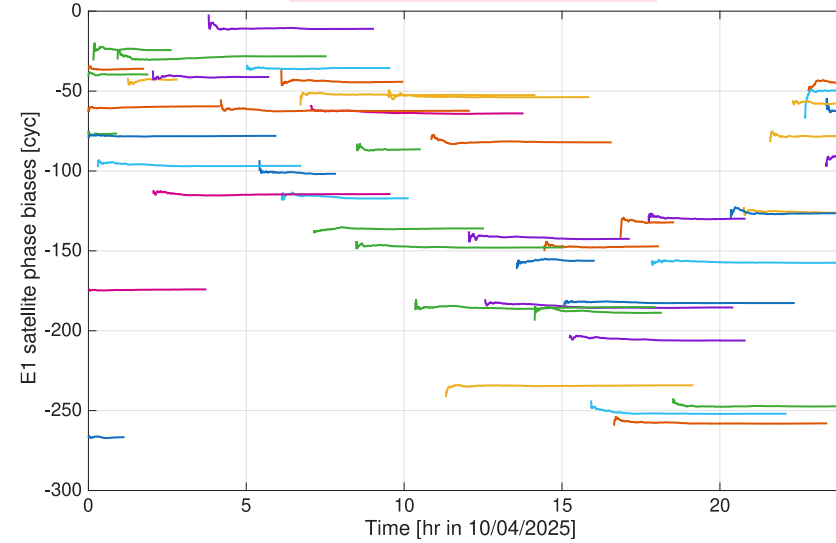
# 1. Joint real-time estimation of GNSS corrections



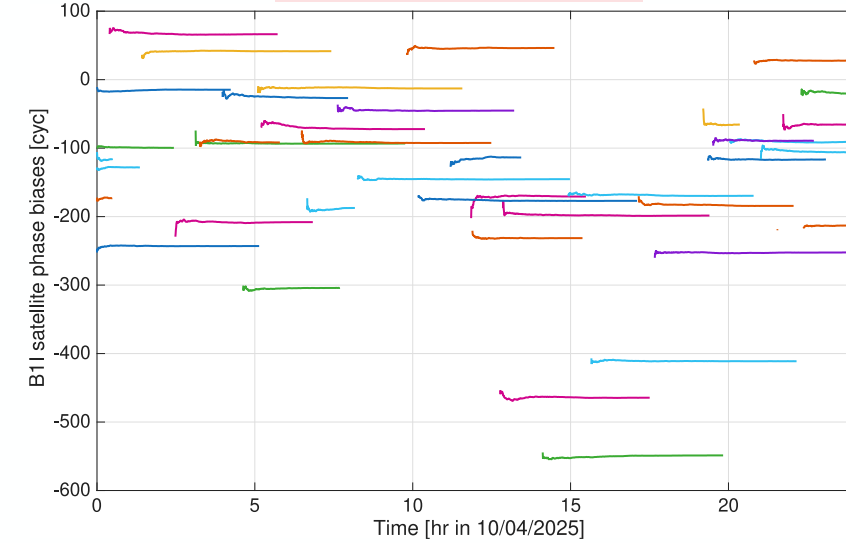
SPB L1



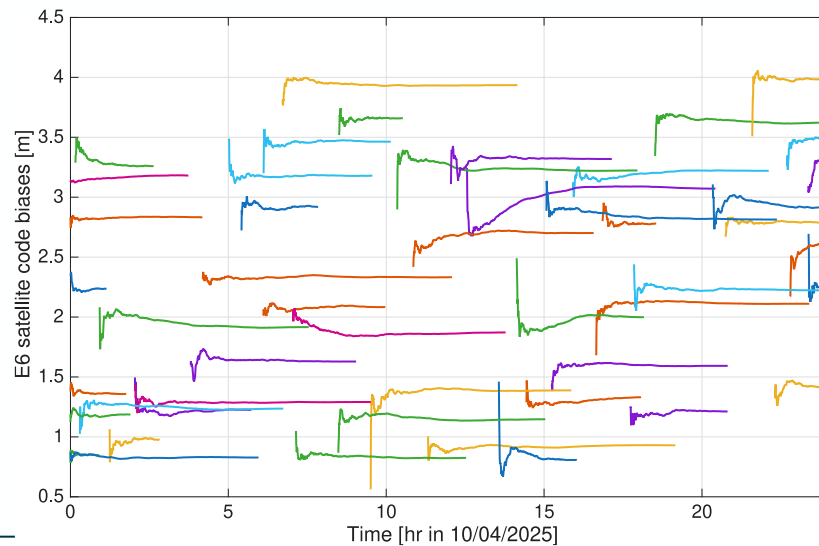
SPB E1



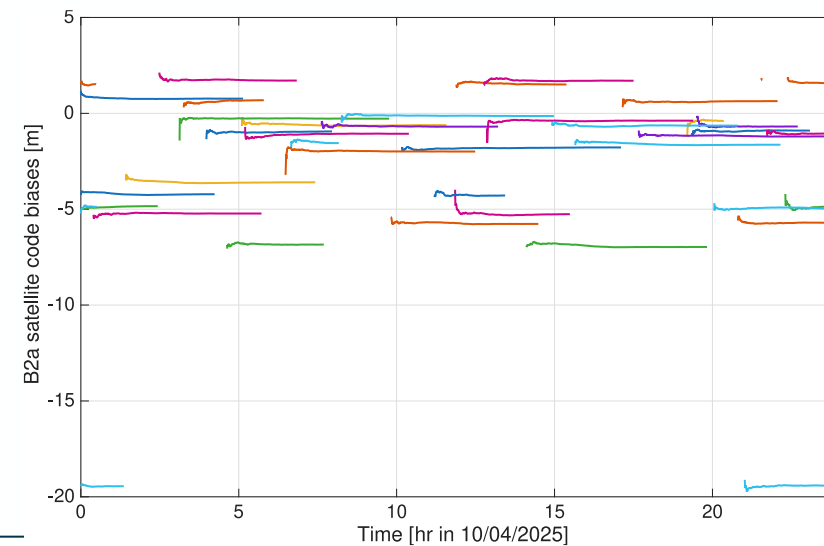
SPB B1I



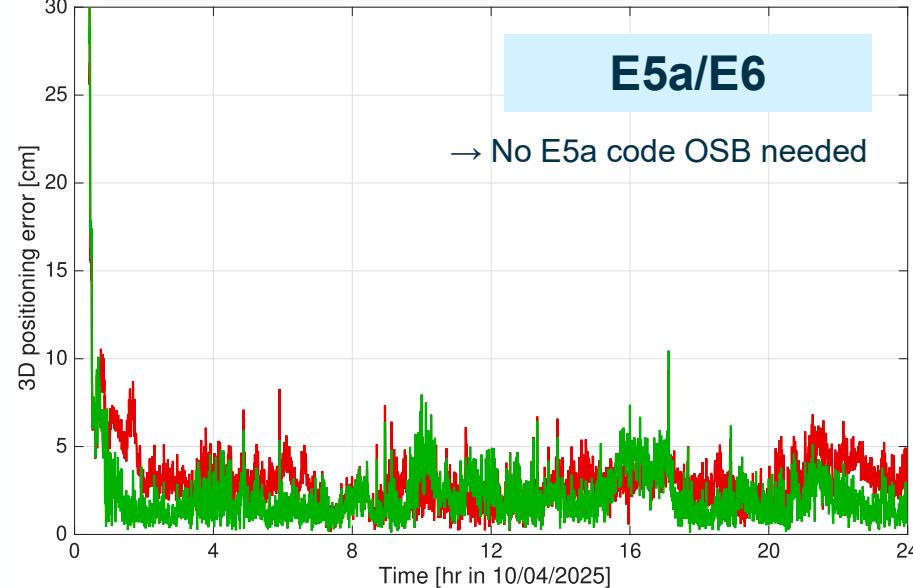
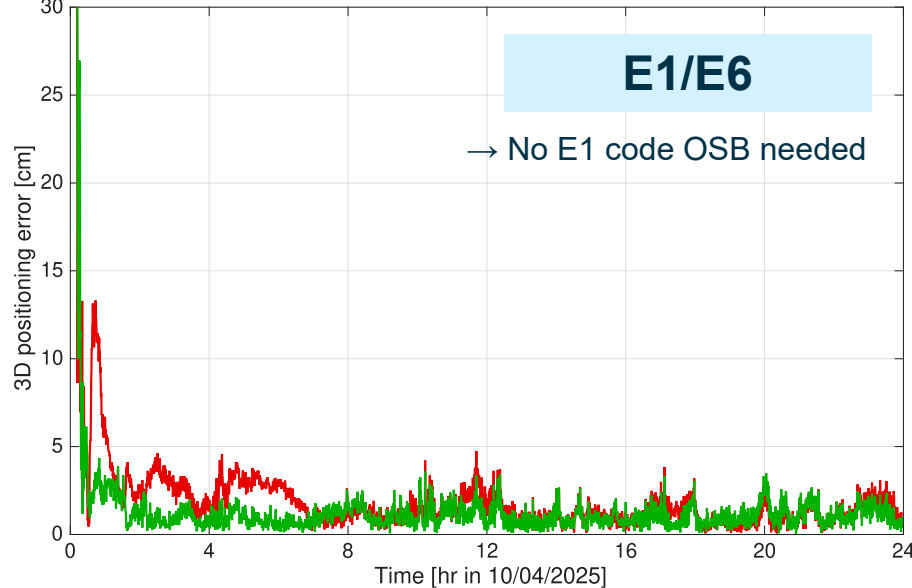
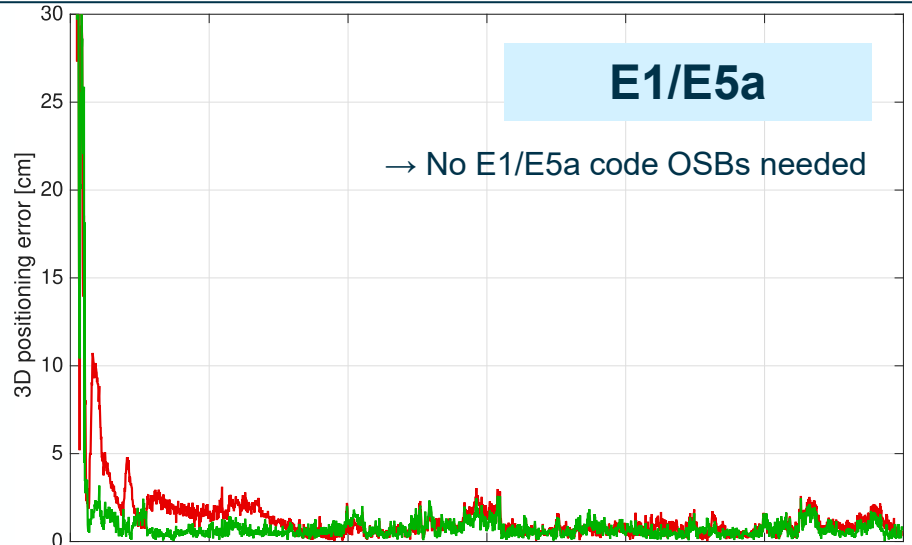
SCB E6



SCB B2a



# 1. Joint real-time estimation of GNSS corrections

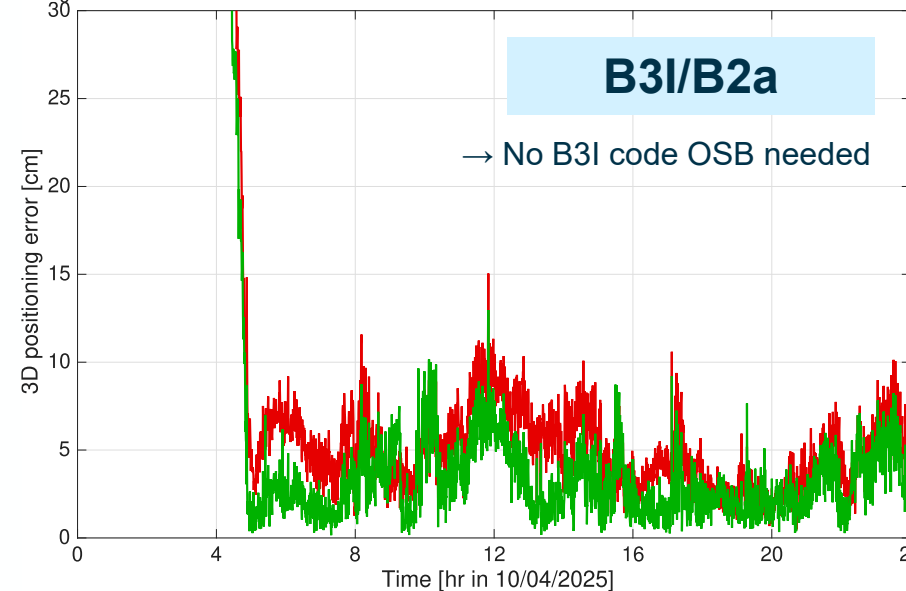
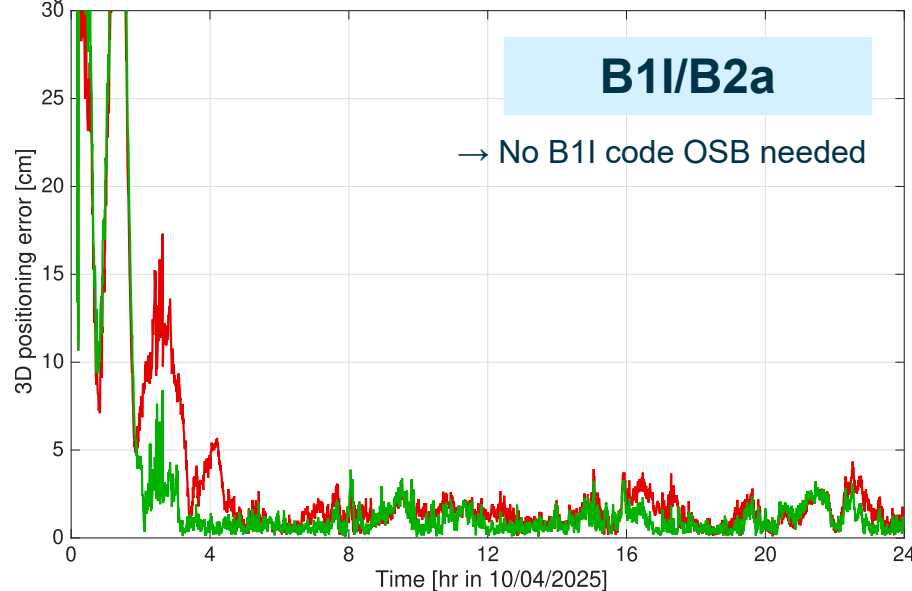
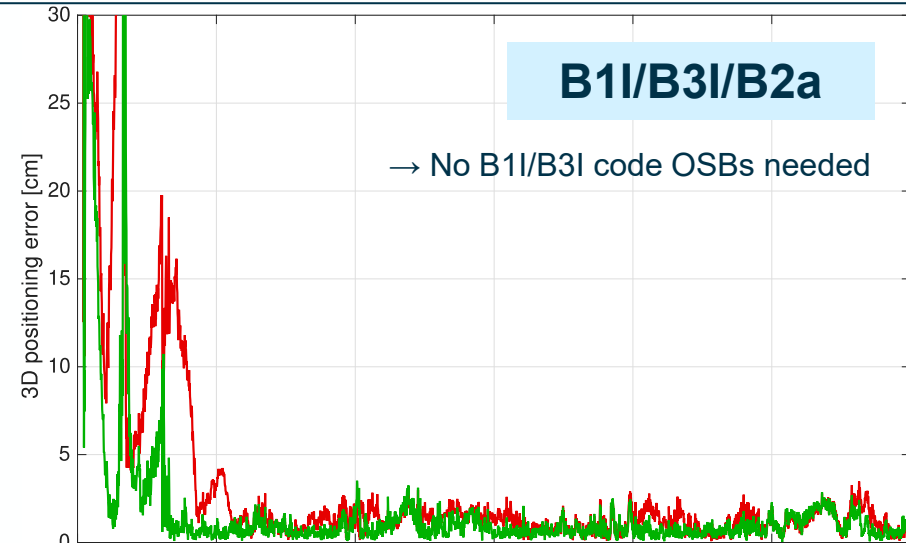
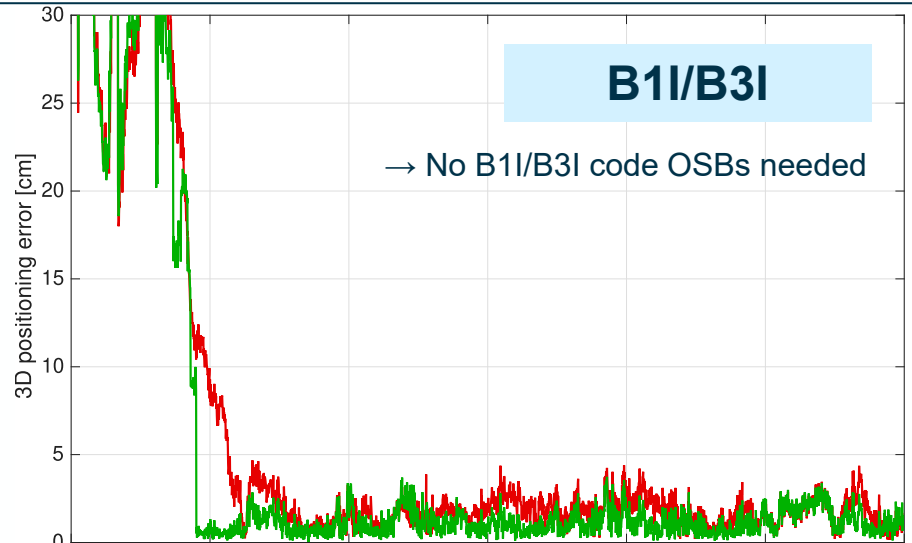


## Arbitrary-frequency PPP-AR with Galileo data

The impact of frequency spacing on IAR has been studied in:

Psychas, Teunissen, & Verhagen (2021)  
<https://doi.org/10.3390/rs13163077>

# 1. Joint real-time estimation of GNSS corrections

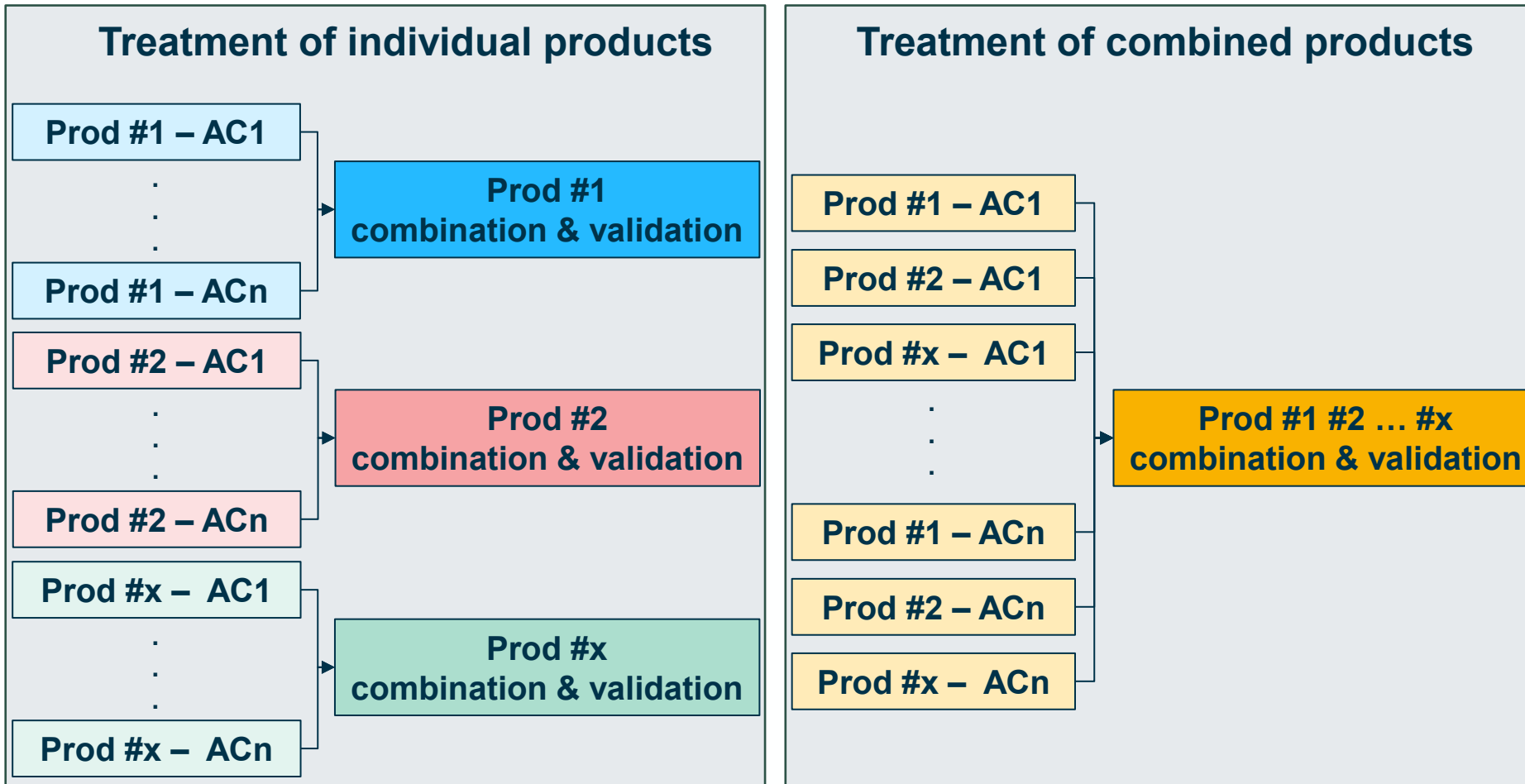


Arbitrary-frequency PPP-AR with BeiDou-3 data

More info on the unified user models in:

Hou, Psychas, & Zhang (2025)  
<https://doi.org/10.1007/s10291-025-01931-8>

# 2. Product quality assessment at combined level



The high correlation between SSR products has been analytically shown in:

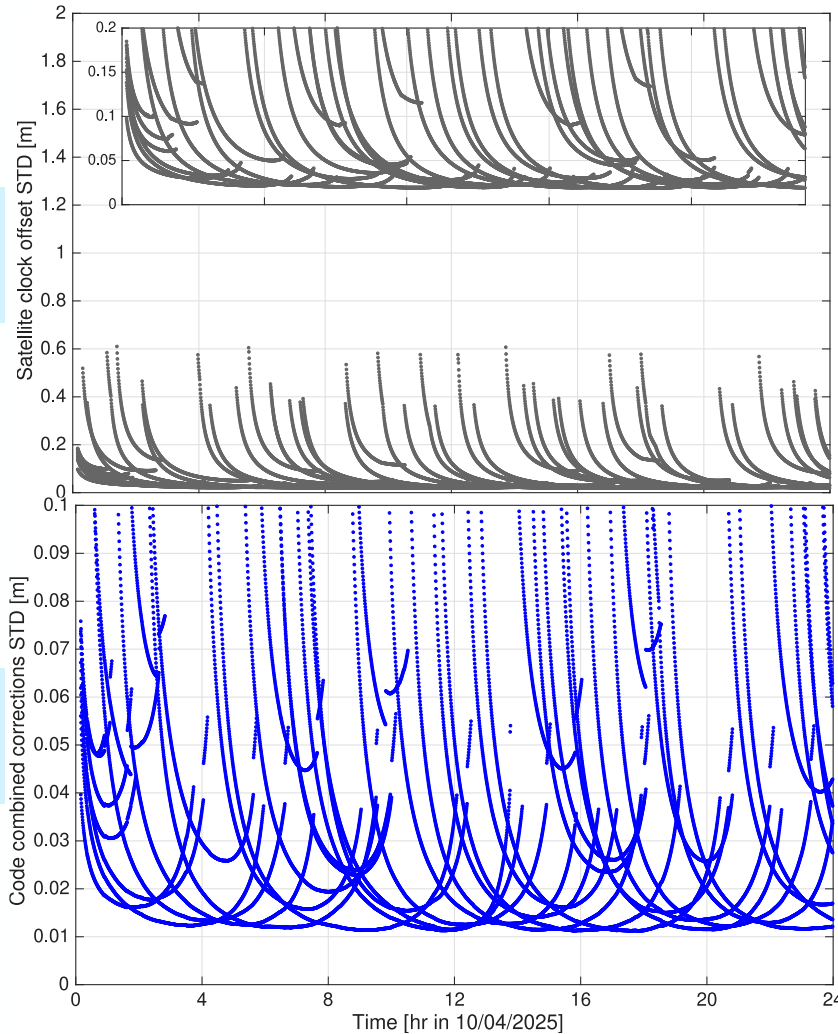
Khodabandeh & Teunissen (2015)  
<https://doi.org/10.1007/s00190-015-0838-9>

It is the **combined corrections' quality** that matters for IAR-enabled positioning users.

# 2. Product quality assessment at combined level

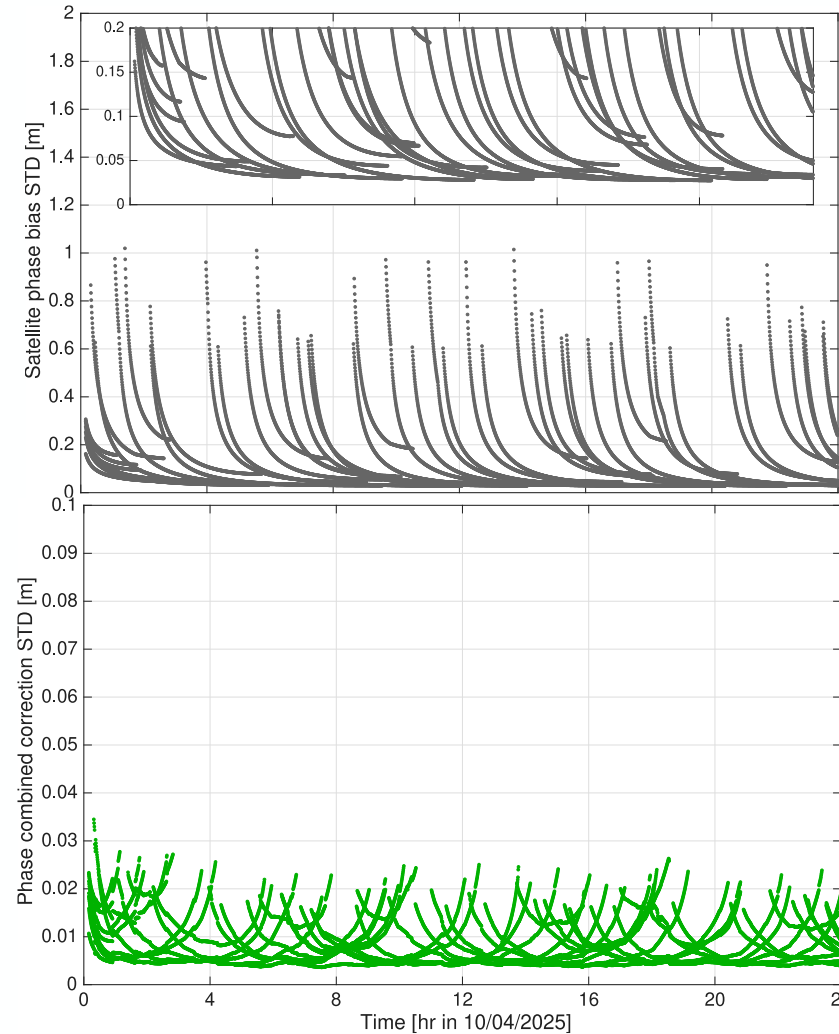
## Formal STDs of individual and combined network-derived products

Sat. clock offset STD



Code comb. correction STD

Sat. phase bias STD



Phase comb. correction STD

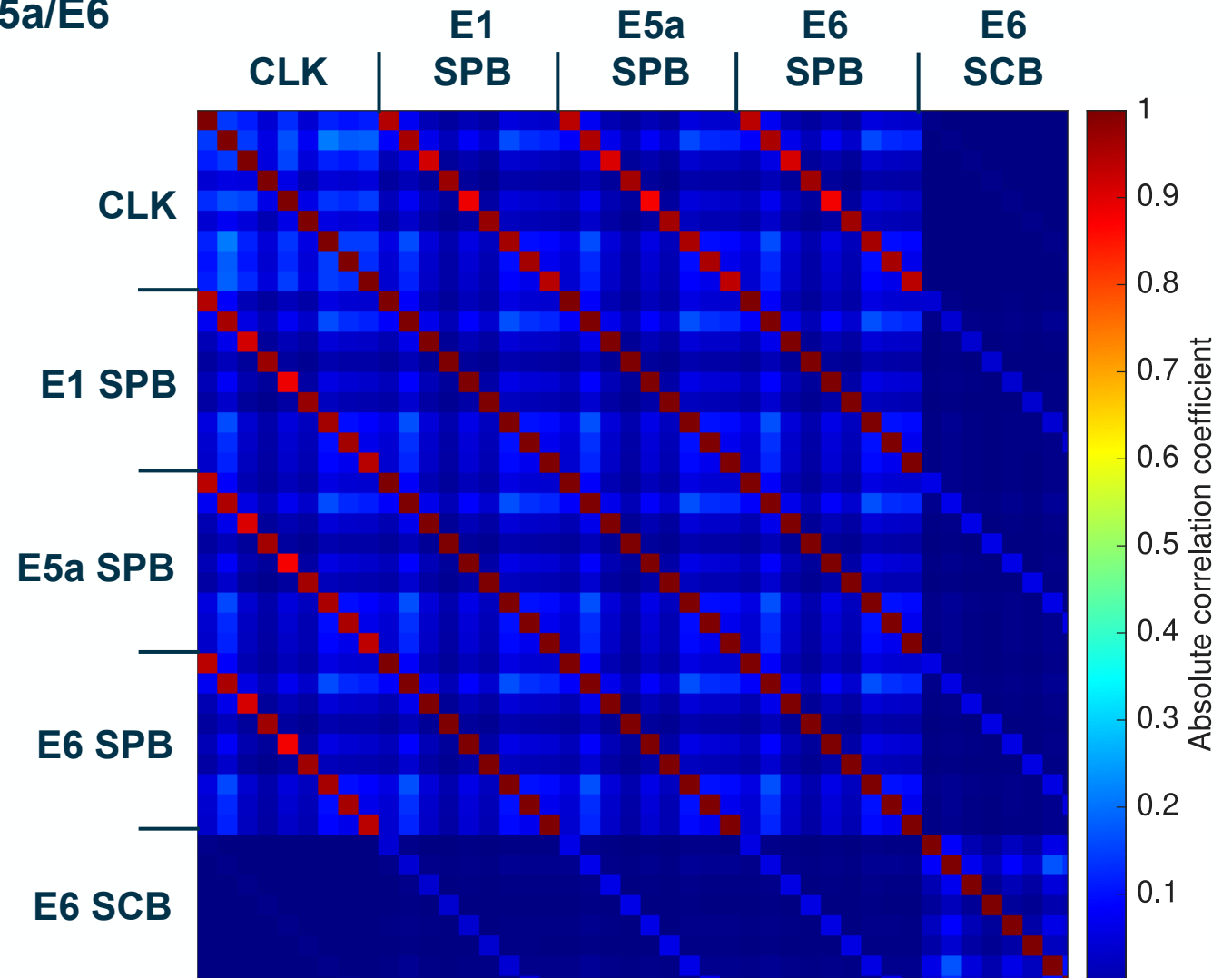
# 2. Product quality assessment at combined level

## Corrections' correlation matrix for GAL E1/E5a/E6

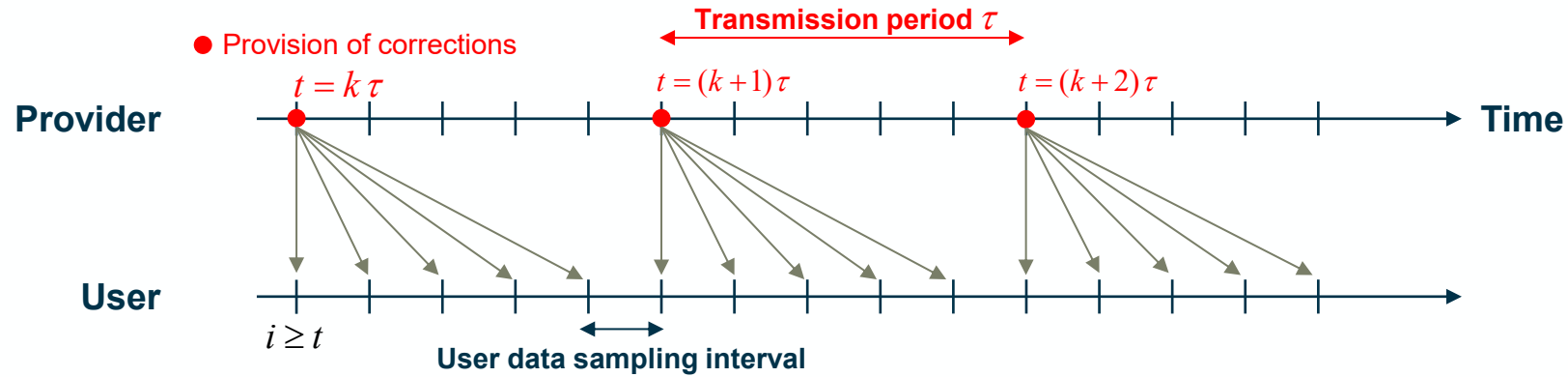
- Significant correlations between **clocks and phase biases**
- Significant correlations in **phase biases among frequencies**

A recent proposal for the assessment of the correction ensemble:

Duan, Hugentobler, & Montenbruck, O. (2024)  
<https://doi.org/10.1007/s10291-024-01634-6>



# 3. Uncertainty of SSR corrections for real-time PPP-AR



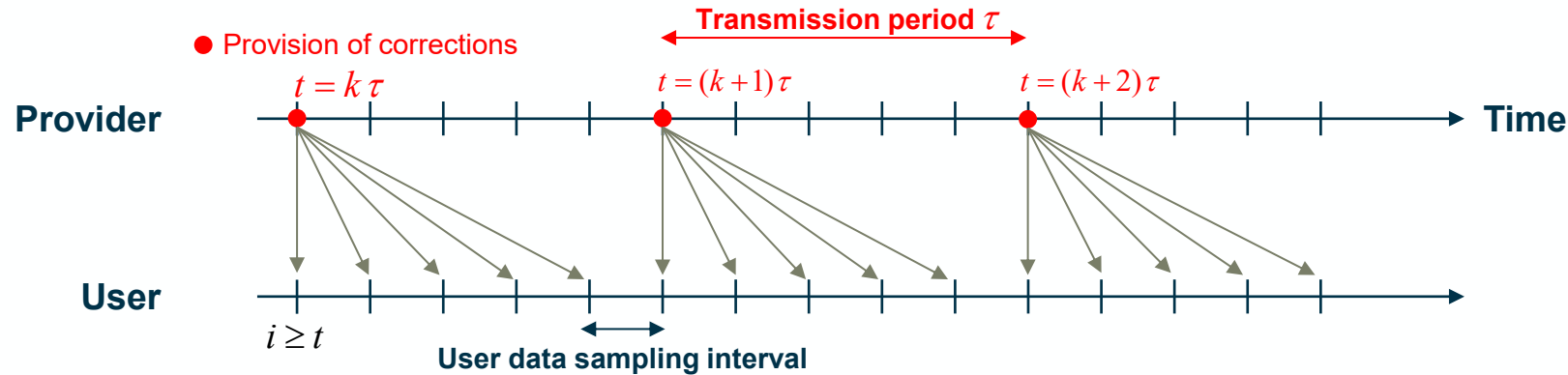
Real-Time

❑ For considerable latencies, the **corrections' prediction error cannot be ignored !**

$$y_i - C_i \hat{c}_{i|k\tau} = A_i x_i + n_i + C_i \underbrace{(c_i - \hat{c}_{i|k\tau})}_{\text{zero-mean, of random nature, but non-negligible for nonzero latency}}$$

zero-mean, of random nature, but  
**non-negligible for nonzero latency**

# 3. Uncertainty of SSR corrections for real-time PPP-AR



Real-Time

True user's stochastic model:

$$D \{y_i - C_i \hat{c}_i|_{k\tau}\} = Q_{y_i} + C_i (\Phi_{i,k\tau} Q_{\hat{c}_{k\tau|k\tau}} \Phi_{i,k\tau}^T + S_{i,k\tau}) C_i^T$$

→ cumulative system-noise variance  
→ can be neglected for long filter-initialization

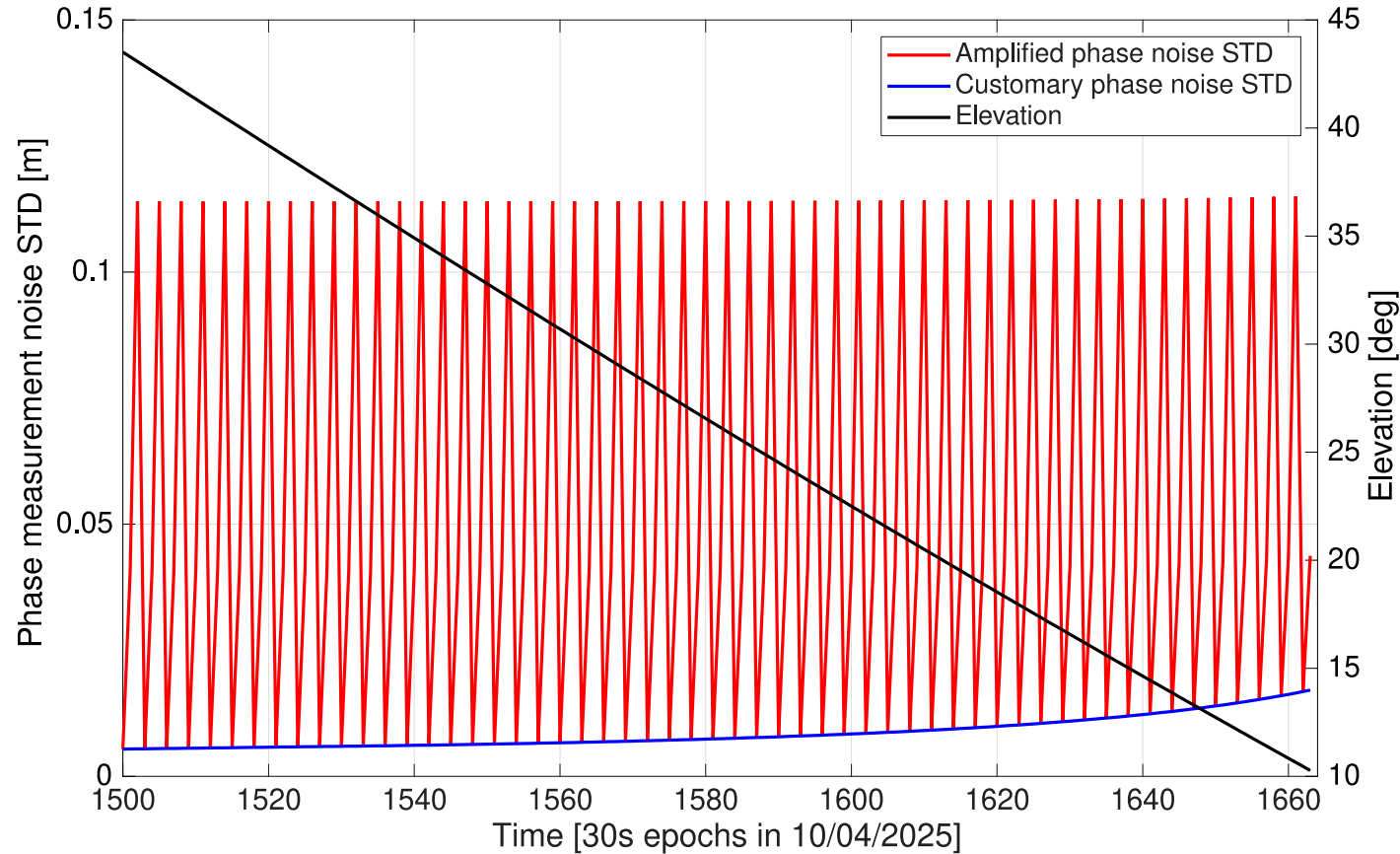
Close-to-optimal user performance can be achieved with the strategy below:

Psychas, Khodabandeh, & Teunissen (2022)  
<https://doi.org/10.1007/s10291-021-01214-y>

$$D \{y_i - C_i \hat{c}_i|_{k\tau}\} \approx Q_{y_i} + C_i S_{i,k\tau} C_i^T \quad \text{where: } S_{i,k\tau} = \sum_{j=k\tau+1}^i \Phi_{i,j} S \Phi_{i,j}^T$$

# 3. Uncertainty of SSR corrections for real-time PPP-AR

## What happens in the presence of nonzero latency



- Case where the products are provided every 90s (latency up to 60s)

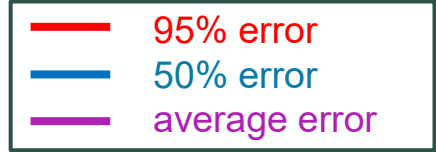
Phase measurement noise STD can exceed 10 cm

if not taken into account

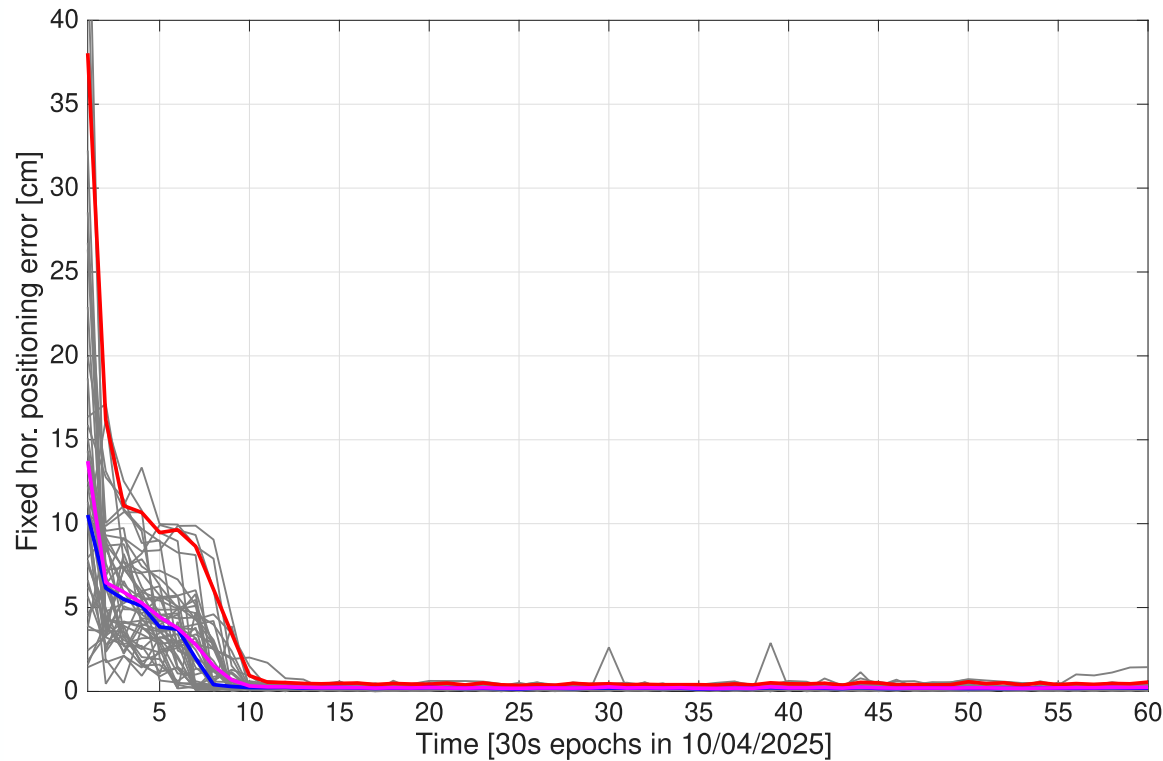
Misspecified user-filter !

# 3. Uncertainty of SSR corrections for real-time PPP-AR

## GPS+GAL triple-frequency PPP-AR | Positioning convergence

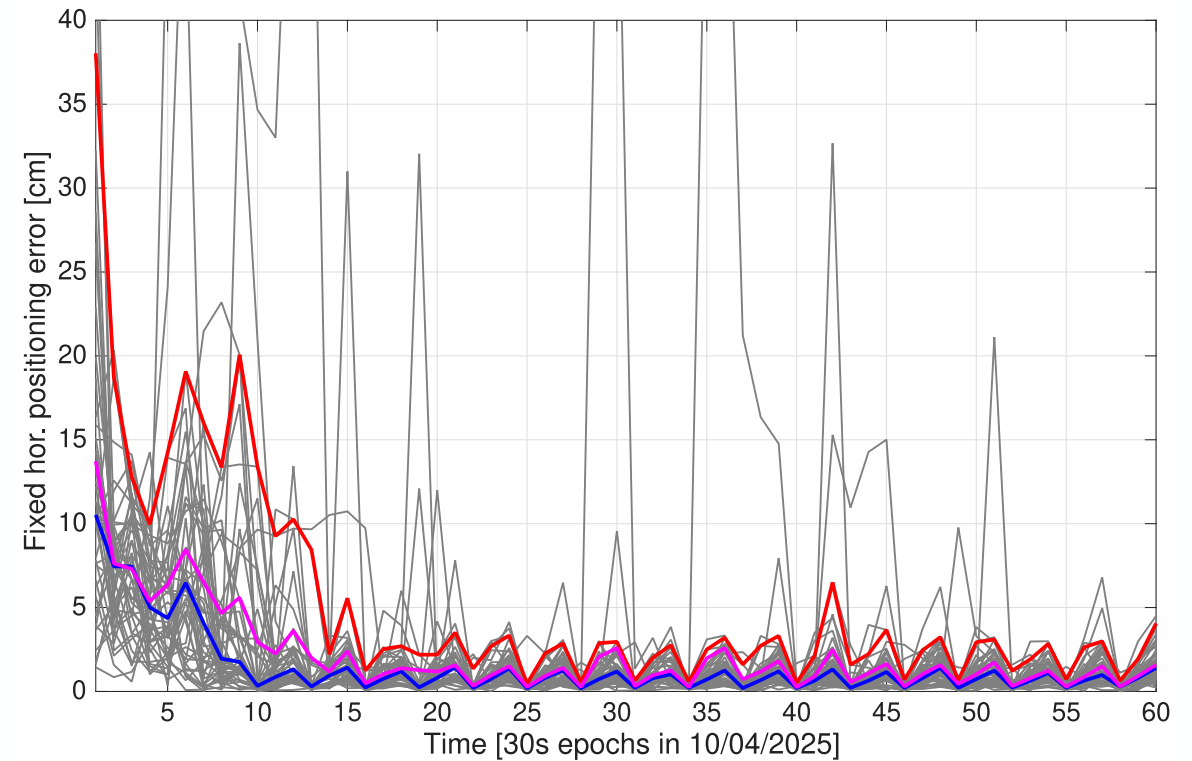


### zero latency | ignoring uncertainty



95% convergence → 2 min (5 epochs)

### nonzero latency | ignoring uncertainty



95% convergence → 6 min (13 epochs)









# Thank you for your attention



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