IGS 2026 WORKSHOP
SANTIAGO DE CHILE
33°26′ S 70°39′ O

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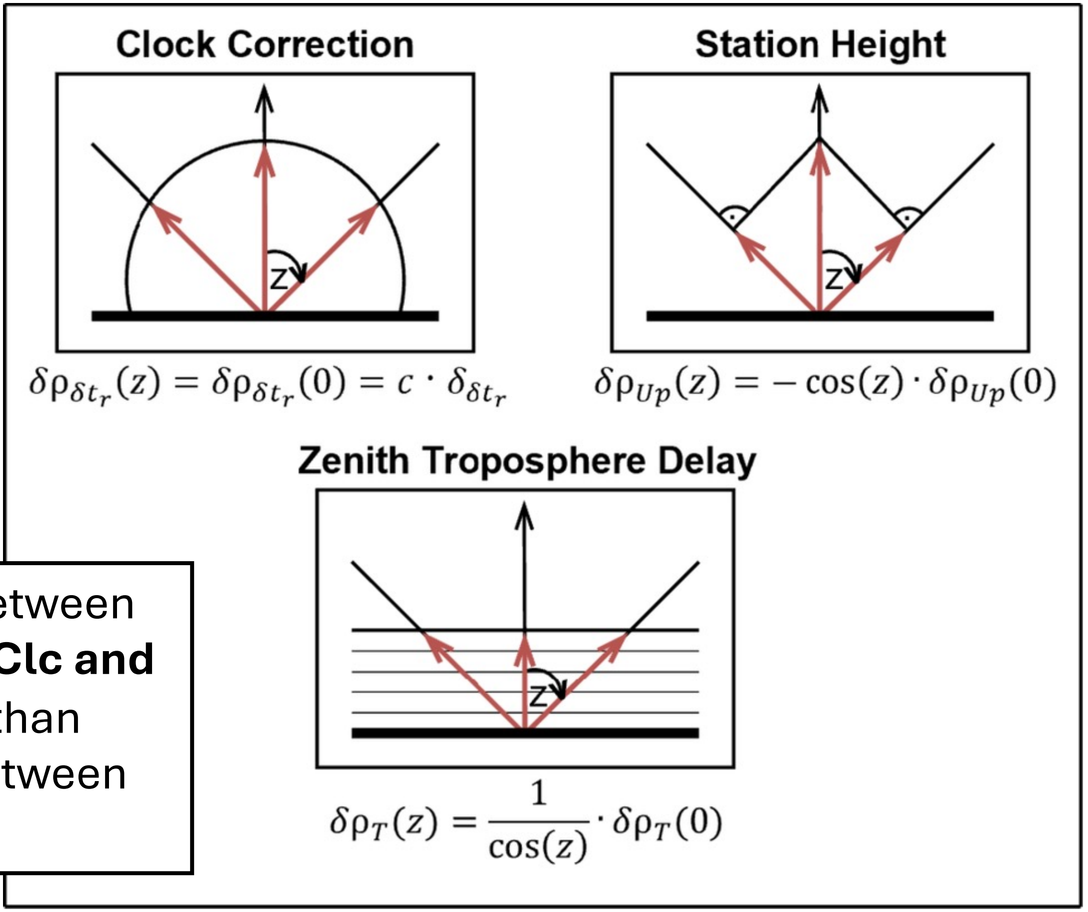
Improving GNSS positioning by clock constraints

ORGANIZING INSTITUTIONS

Correlations in GNSS solutions



Correlations between **Clc and Up** or **Clc and ZTD** are larger than correlations between Up and ZTD

Table 1 Correlation coefficients between Up, ZTD, and Clc parameter estimates assuming uniform satellite distribution between zenith angles z_{min} and z_{max}

z_{min} [°]	z_{max} [°]	corr.(Up, ZTD)	corr.(Up, Clc)	corr.(ZTD, Clc)
Station at the equator				
0	70	−0.964	0.990	−0.990
0	75	−0.943	0.985	−0.983
0	80	−0.907	0.975	−0.968
0	83	−0.869	0.966	−0.949
0	**85**	**−0.830**	**0.956**	**−0.926**
0	86	−0.802	0.949	−0.907
0	87	−0.764	0.940	−0.880
0	88	−0.709	0.928	−0.836
0	89	−0.614	0.909	−0.746
Station at the pole				
45	70	−0.983	0.996	−0.995
45	75	−0.967	0.992	−0.990
45	80	−0.937	0.984	−0.980
45	83	−0.904	0.976	−0.965
45	**85**	**−0.868**	**0.967**	**−0.946**
45	86	−0.841	0.961	−0.930
45	87	−0.805	0.953	−0.906
45	88	−0.752	0.941	−0.866
45	89	−0.656	0.922	−0.782

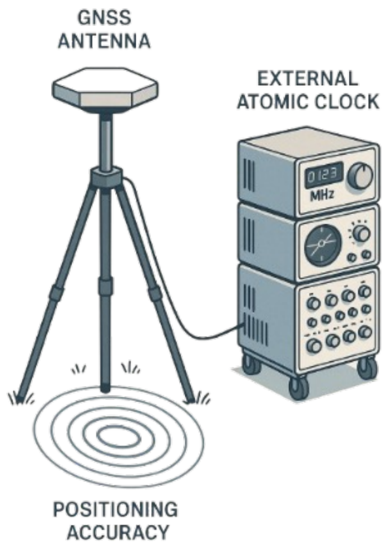
The bold values correspond to the elevation cut-off angle of 5° as used in this study

$$N = \begin{bmatrix} \int_{z_{\min}}^{z_{\max}} \sin(z)^2 \cos(z)^2 dz & -\int_{z_{\min}}^{z_{\max}} \sin(z)^2 dz & -\int_{z_{\min}}^{z_{\max}} \sin(z)^2 \cos(z) dz \\ -\int_{z_{\min}}^{z_{\max}} \sin(z)^2 dz & \int_{z_{\min}}^{z_{\max}} \sin(z)^2 \cos(z)^{-2} dz & \int_{z_{\min}}^{z_{\max}} \sin(z)^2 \cos(z)^{-1} dz \\ -\int_{z_{\min}}^{z_{\max}} \sin(z)^2 \cos(z) dz & \int_{z_{\min}}^{z_{\max}} \sin(z)^2 \cos(z)^{-1} dz & \int_{z_{\min}}^{z_{\max}} \sin(z)^2 dz \end{bmatrix}$$

Mikoś, M., Kazmierski, K. & Sośnica, K. (2025) Multi-GNSS time transfer with different modeling of station coordinates and troposphere delays. *GPS Solut* **29**, 192. https://doi.org/10.1007/s10291-025-01950-5

Introduction – GNSS Receiver Clock

- **GNSS Receiver Clock**
  - **Clock / Oscillator**
    - Internal
      - Crystal Oscillator (XO)
    - External
      - Rubidium (Rb)
      - Cesium (Cs)
      - Hydrogen Maser (HM)
  - **Estimated Clock Parameter**
    - Parameterization ⟵ Modeling ⟶ Constraining
    - Parameterization
      - Individual Clocks (G, R, E, C)[1)]
      - Master Clock for G + ISB[2)]
    - Constraining
      - White Noise ← reference solution (RV)
      - Random Walk ← test solution (SV)



1) **G** (GPS), **R** (GLONASS), **E** (Galileo), **C** (BeiDou)
2) **ISB** (Inter-System Biases)

Stochastic Modeling of the Receiver Common Clock Parameter (ClcG)

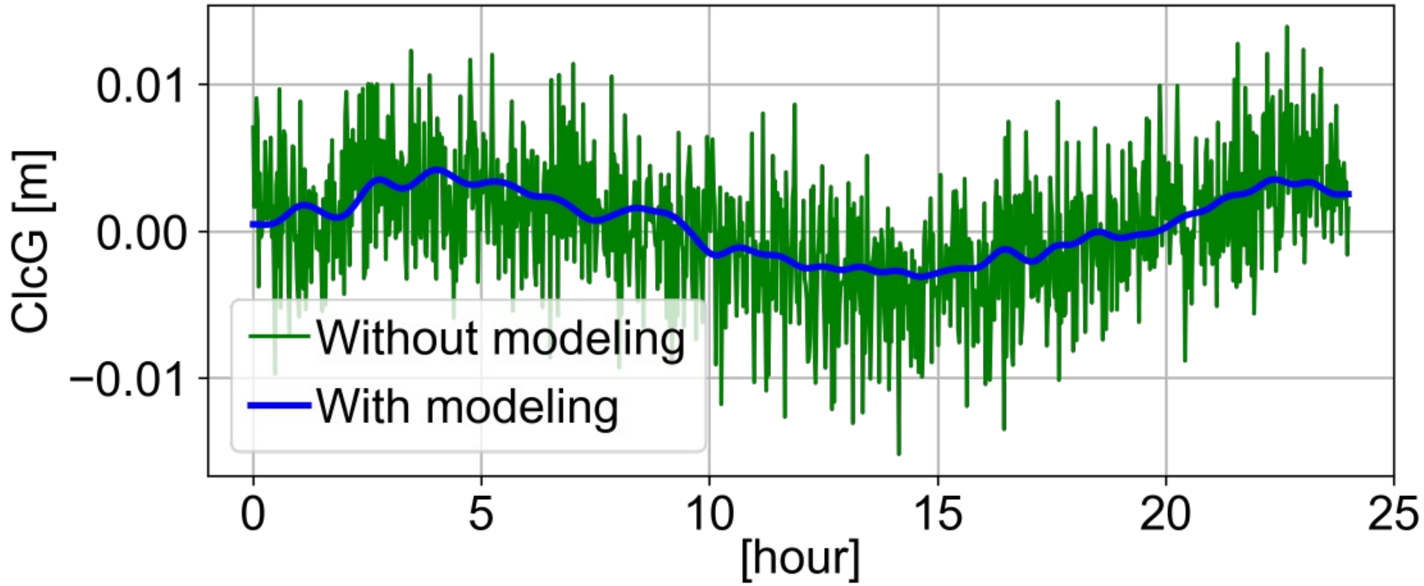
Stochastic Modeling => random walk (RW) on the receiver common clock parameter by modifying the variance-covariance matrix (VCM) within the sequential least squares (LSQ) method (relative time constranits)

$$\sigma_{RW}^2 = \left(\frac{val.}{\sqrt{t}}\right)^2$$

val. – estimated value

t – interval between observations, 30s

Mikoś et al. (2024), the optimal constraint parameter for ClcG **val. = *0.015 m***



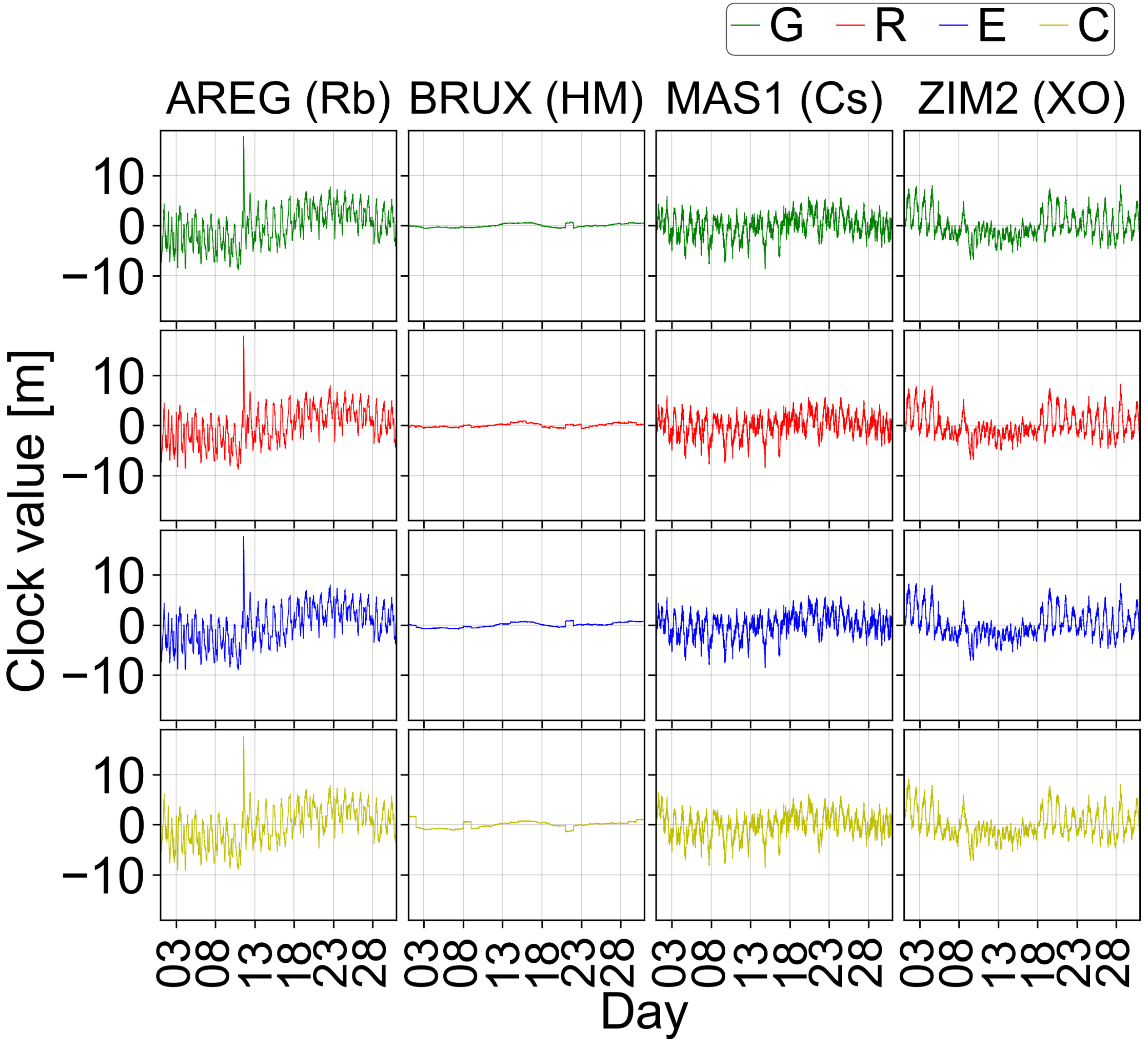
Assuming:

1) **Time correlations for the receiver clock**
2) **Clock variability as the random walk proces with a pre-defined variation**

Stochastic Modeling of the Receiver Common Clock Parameter (ClcG)

Hydrogen masers (HM) allow for an efficient clock modeling and thus for stabilizing the GNSS PPP solutions via clock constraints.

Stability of Rubidium (Rb), Cesium (Cs), or Quartz (XO) is typically insufficient.

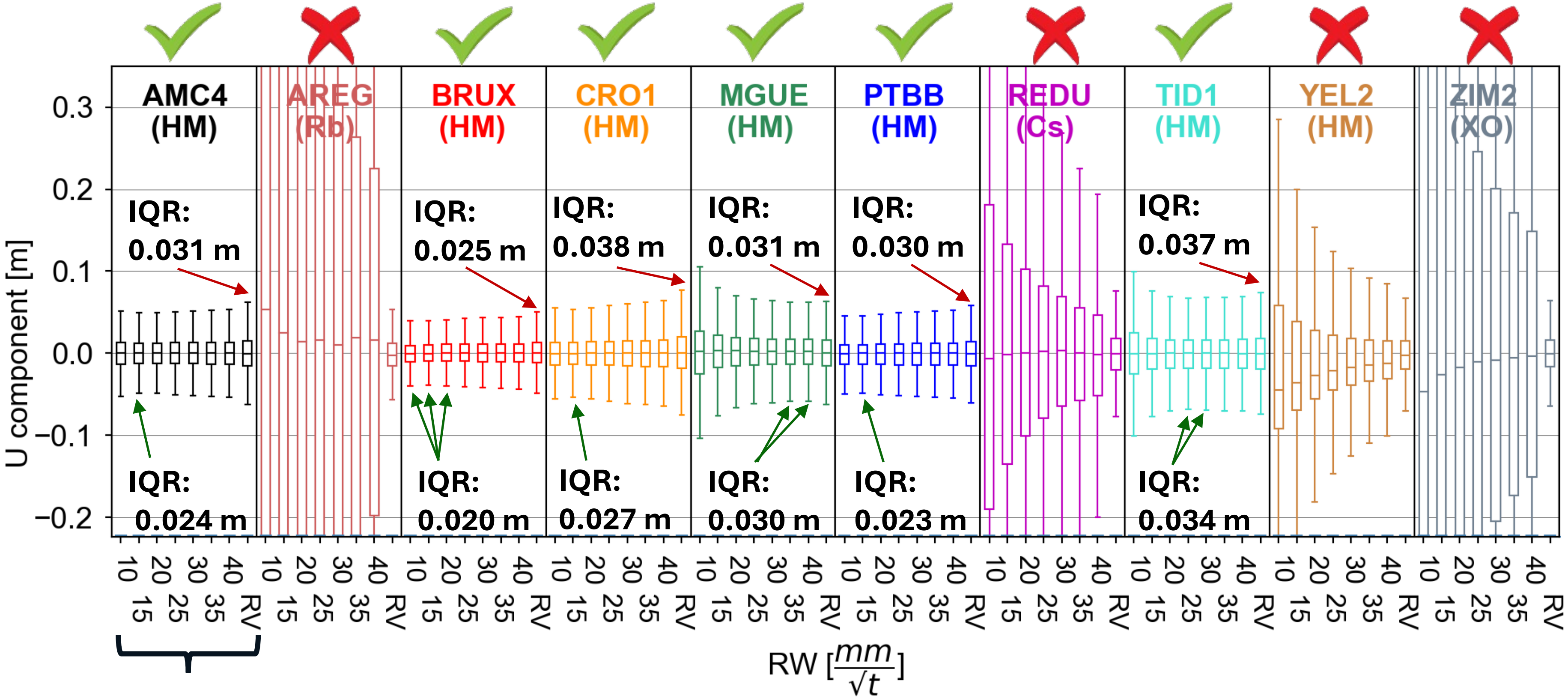


Mikoś M., Kazmierski K., Sośnica K. (2023)
Characteristics of the IGS receiver clock performance from multi-GNSS PPP solutions
GPS Solutions, Vol. 27 No. 55,
https://link.springer.com/article/10.1007/s10291-023-01394-9

Stability of the Up Component

Mikoś, M., *et al.* (2024) Stochastic modeling of the receiver clock parameter in Galileo-only and multi-GNSS PPP solutions. *GPS Solut* **28**, 14. https://doi.org/10.1007/s10291-023-01556-9

1. The improvement of the solution using RW is visible only for IGS stations equipped with HM clocks.
2. The most favorable RW constraints are 15 and 20 mm.
3. Despite utilizing the HM clock, the IGS station - YEL2 shows high instability due to significant clock drift.



RW_G and RV strategies from kinematic solutions

$$\text{RandWlk} = \frac{\text{RW}}{\sqrt{t}}$$

RW [mm]: 10 - 40

Reference Value (RV)

Test for stations with HMs

Post-processing

CODE

Real-time

HAS | IGS | CNES

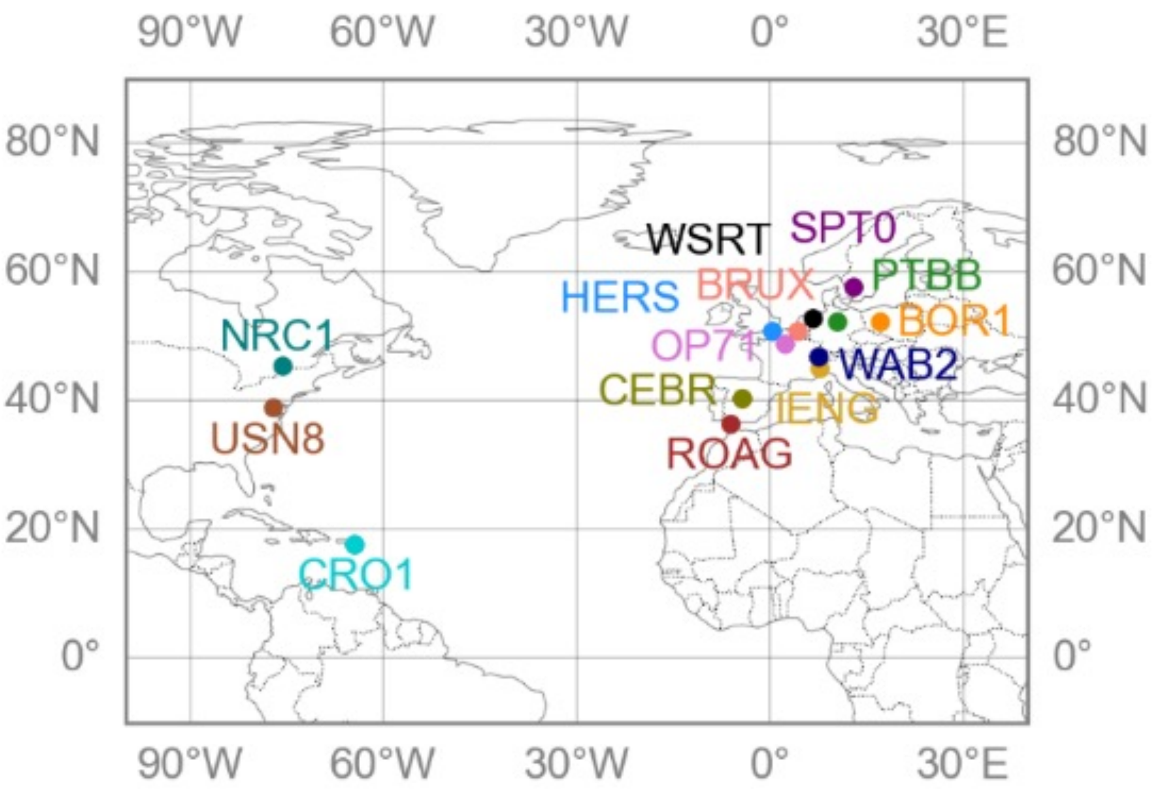
Strategy

PPP

GPS + Galileo

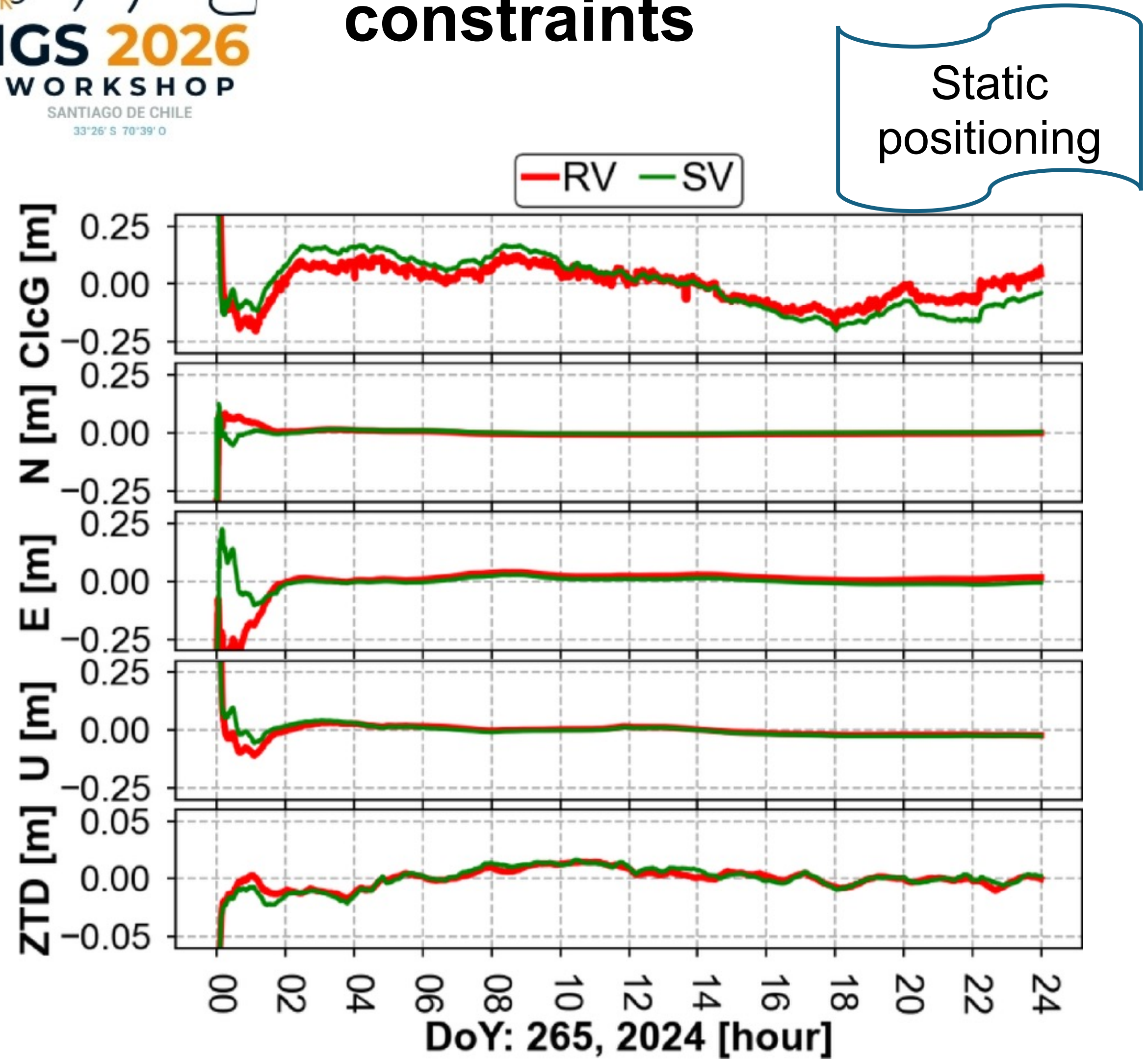
Static Positioning (Week of Data)

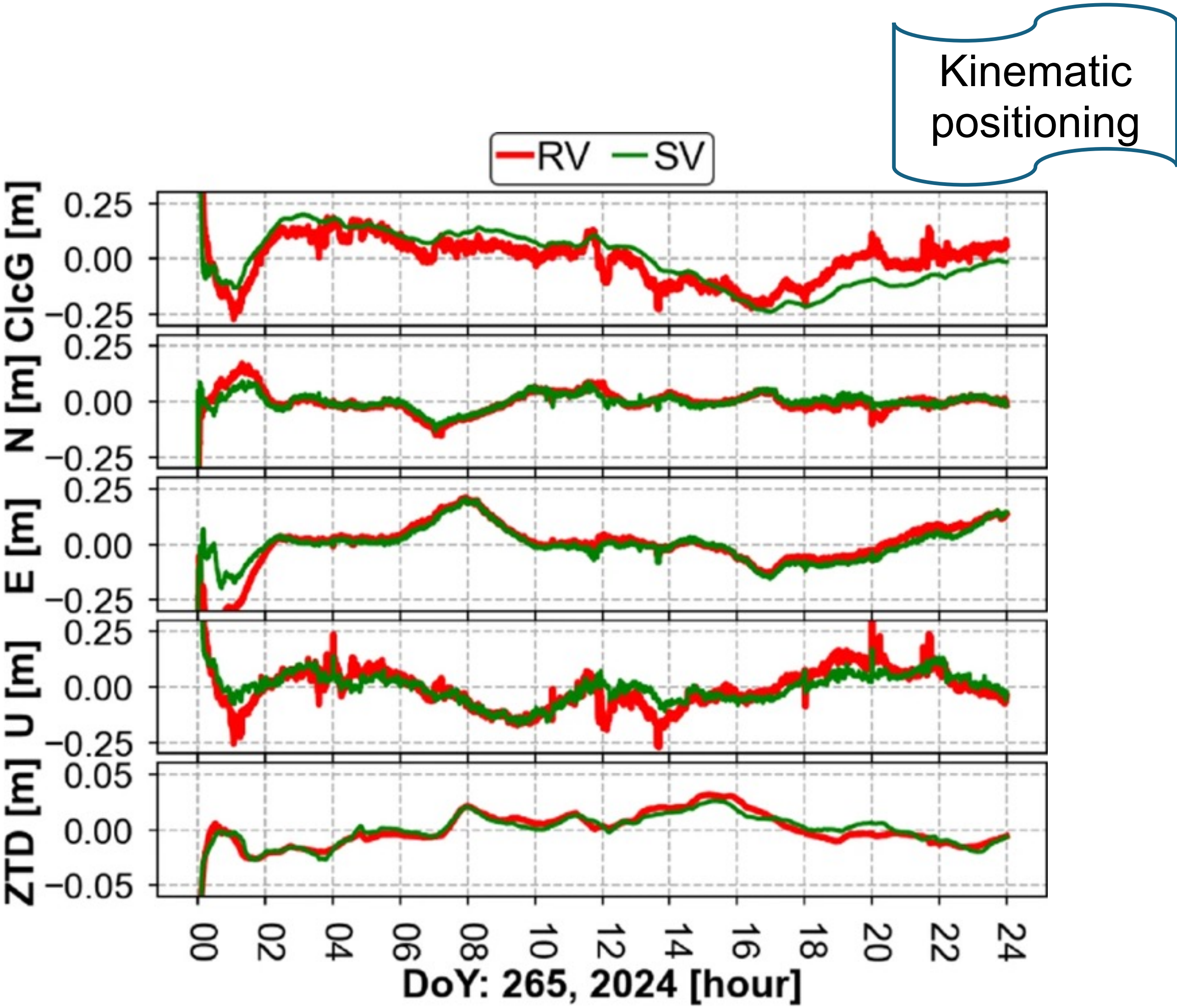
Kinematic Positioning (Week of Data)



GNSS-WARP
(**W**roclaw **A**lgorithms for **R**eal-time **P**ositioning)

Estimated parameters from PPP with and without clock constraints





RV – reference solution without additional constraints

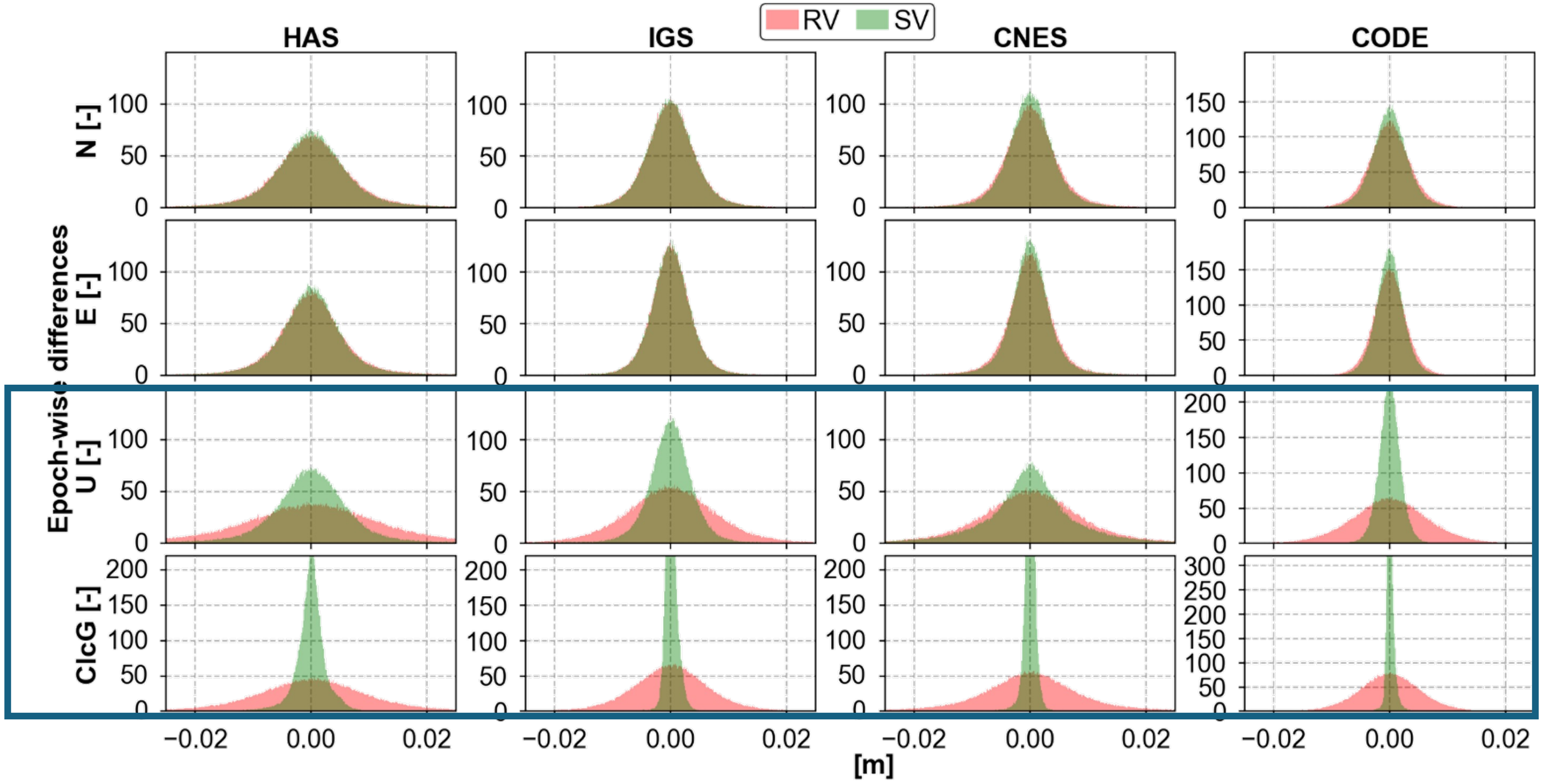
SV – test solution with additional constraints

Histograms of Epoch-wise Differences – HMs IGS Stations

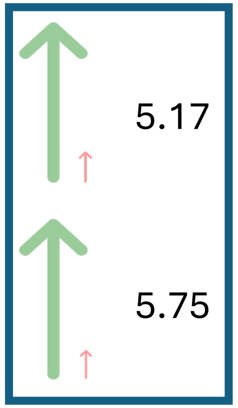
North coordinate (**N**), East coordinate (**E**), Height coordinate component (**U**),
Receiver common clock parameter (**ClcG**)

The impact of the SV solution on the stability of the solution

$$\text{Impact ratio} = \frac{\text{STD 1)}_{\text{RV}}}{\text{STD 1)}_{\text{SV}}}$$



RV – reference solution without additional constraints
SV – test solution with additional constraints



The histograms include all the analyzed stations.

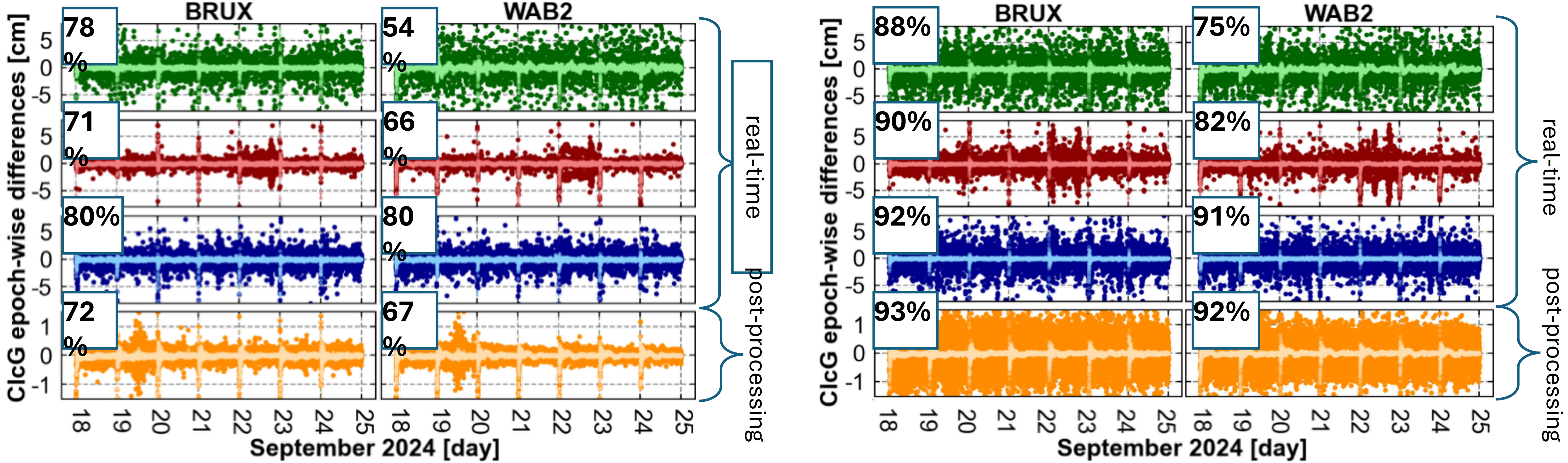
1) **STD** (standard deviation)

Epoch-Wise Differences of Receiver Clock Parameter (ClcG)

Static positioning

(1) - RV
(2) - SV

Kinematic positioning



78% - improvement of the SV solution compared to the RV solution

RV – reference solution without additional constraints

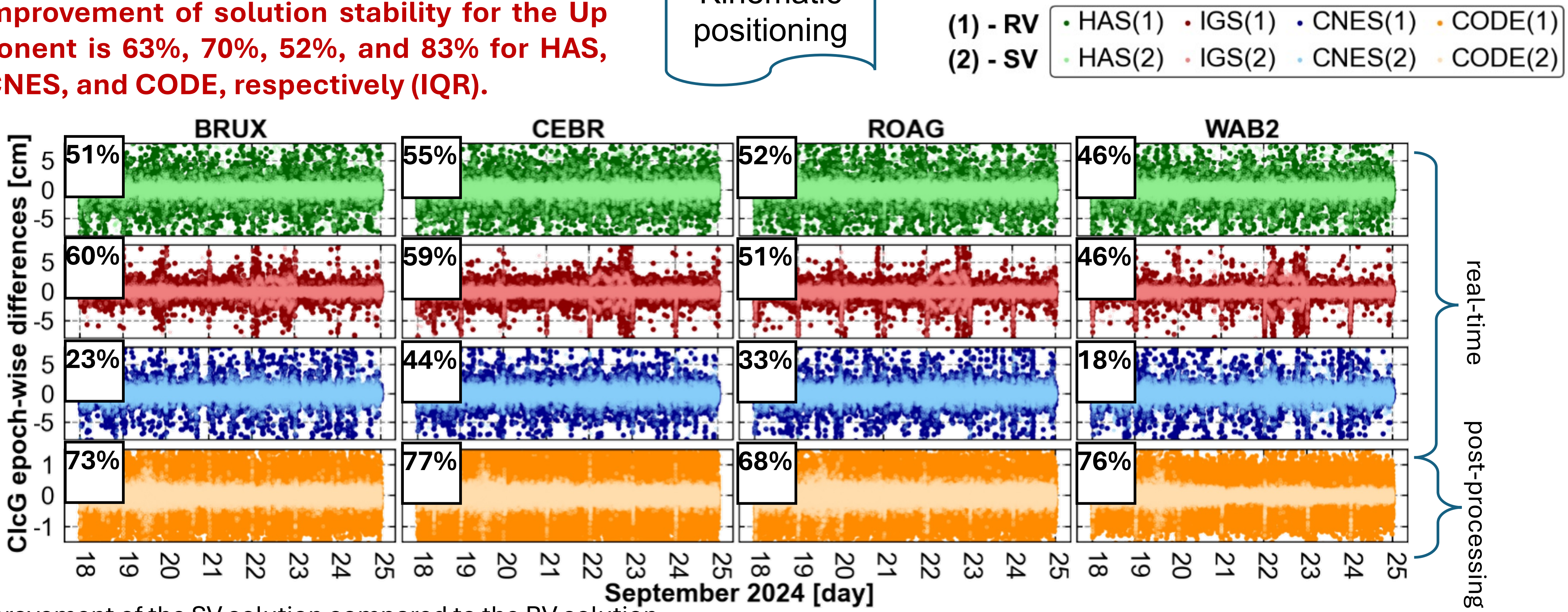
SV – test solution with additional constraints

Epoch-Wise Differences for the Height Component (U)

The improvement of solution stability for the Up component is 63%, 70%, 52%, and 83% for HAS, IGS, CNES, and CODE, respectively (IQR).

Kinematic positioning

(1) - RV
(2) - SV



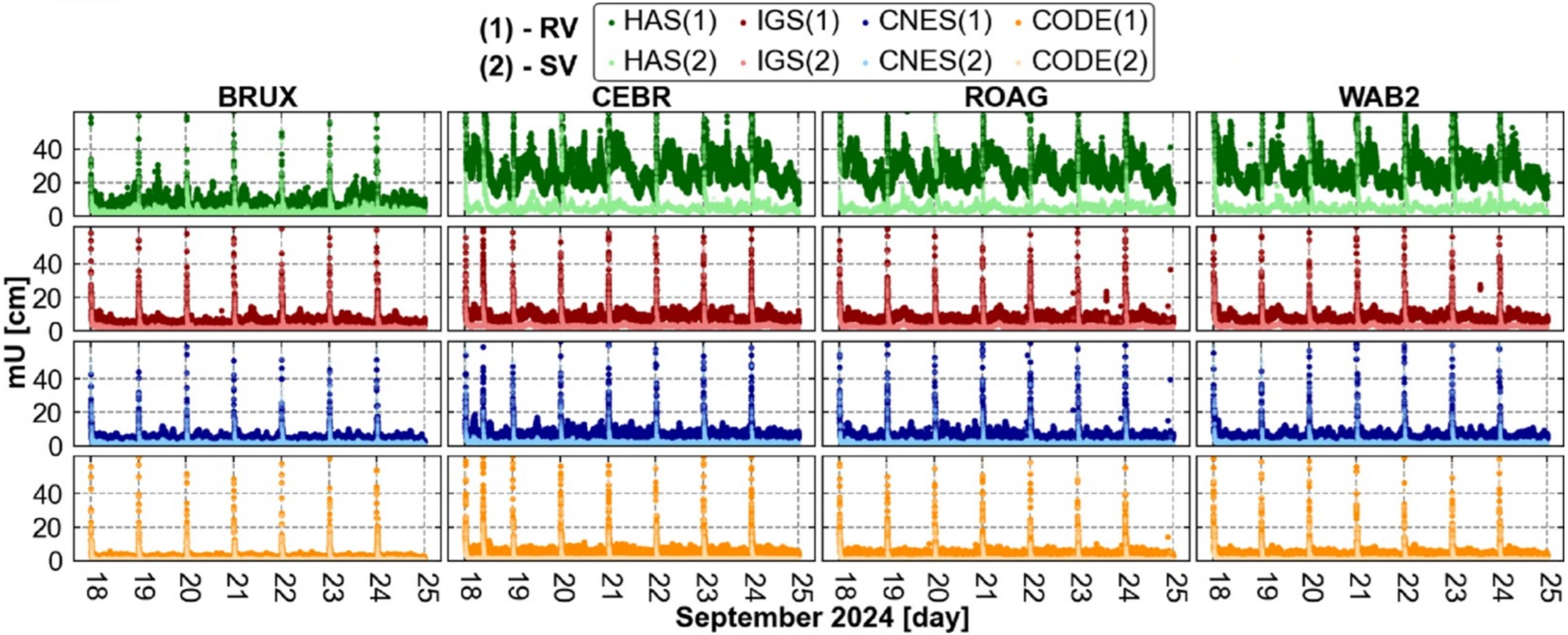
51% - improvement of the SV solution compared to the RV solution

RV – reference solution without additional constraints

SV – test solution with additional constraints

Formal errors for the Height Component (U)

Kinematic positioning



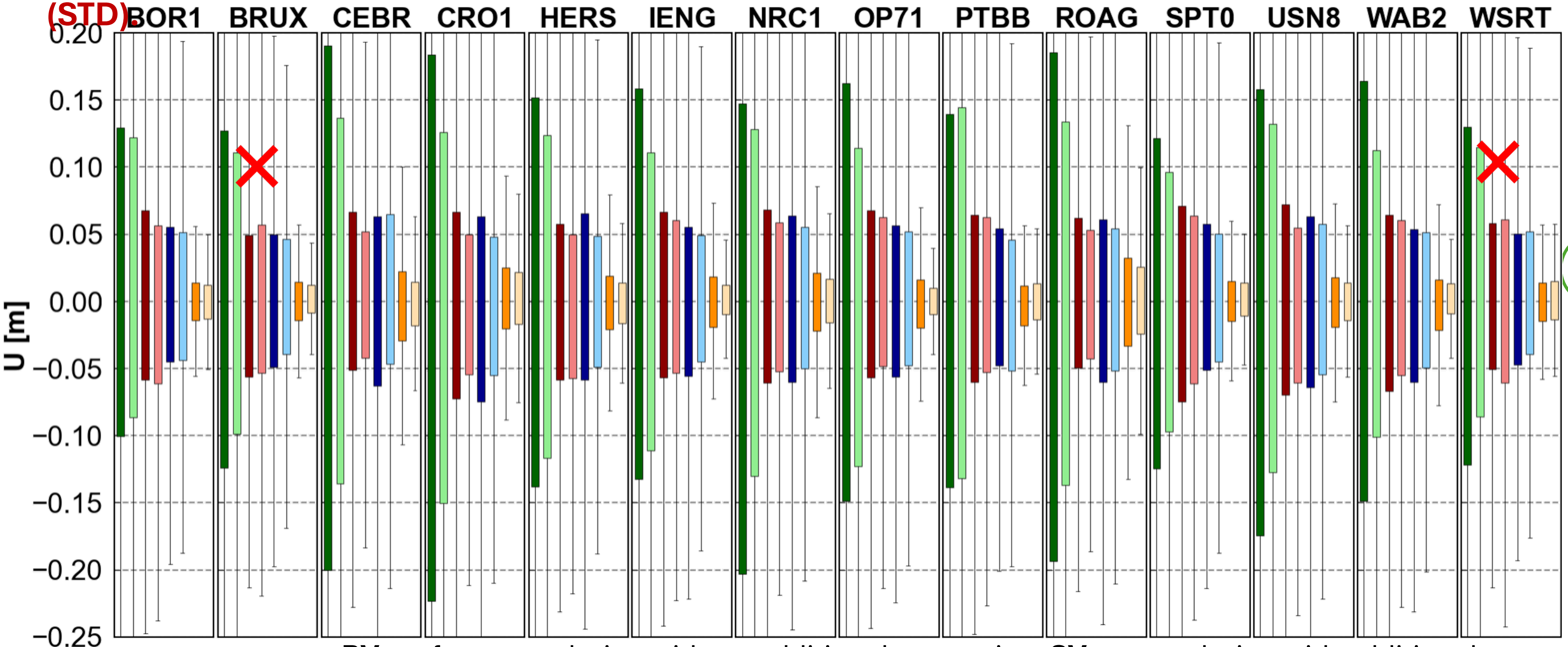
RV – reference solution without additional constraints
SV – test solution with additional constraints

The Quality of the Height Coordinate Component (U)

The improvement in the position precision for the Up component is 38%, 50%, 18%, and 26% for HAS, IGS, CNES, and CODE, respectively (STD)

Kinematic positioning

(1) - RV · HAS(1) · IGS(1) · CNES(1) · CODE(1)
(2) - SV · HAS(2) · IGS(2) · CNES(2) · CODE(2)



SV (bright color) solutions achieve greater stability for U

The RV strategy provides a more stable solution than the SV strategy (2 cases vs. 54)

RV – reference solution without additional constraints **SV** – test solution with additional constraints

Summary and Outlook

1. **Stochastic modeling of the clock parameter:**
   - applying a random walk (RW) constraint limits the clock estimates variations and considers the time correlations between estimated (epoch-wise) clock parameters,
   - provides the best solution by imposing constraints through stochastic modeling for the common clock of HMs, using the **RW** constraint for **15mm** (30s sampling)
   - is beneficial for IGS stations equipped with HM clocks, but deteriorates solutions for stations with inferior clocks,
   - for the **epoch-wise differences** for height coordinate component (**U**), for the interquartile range (IQR), the improvement is up to **63%**, **70%**, **52%**, and **83%** for **HAS**, **IGS**, **CNES**, and **CODE**, respectively (**solution STABILITY / REPEATABILITY ~68% of improvement**)
   - for the height coordinate component (**U**), for the standard deviation, the improvement is up to **38%**, **50%**, **18%**, and **26%** for **HAS**, **IGS**, **CNES**, and **CODE**, respectively (**solution PRECISION ~ 31% of improvement**).
2. **Outlook:**
   - Other clock noise models (not only random walk, adjustable filters, etc.)

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Mikoś, M., Kazmierski, K., Schön, S. Sośnica, K. (2026). Improving the stability of real-time PPP solutions by receiver clock modeling. *GPS Solut* **30**, 82 . https://doi.org/10.1007/s10291-026-02048-2

Thank you for your attention!

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Back-up

Median of Daily Standard Deviations for Coordinates and ZTD

North coordinate (**N**),
East coordinate (**E**),
Height coordinate component (**U**),
Zenith Total Delay (**ZTD**)

The SV solution achieves an improvement, particularly for the U component.

[mm]

HAS (color scale 0–200)

	BOR1 RV	BOR1 SV	BRUX RV	BRUX SV	CEBR RV	CEBR SV	CRO1 RV	CRO1 SV	HERS RV	HERS SV	IENG RV	IENG SV	NRC1 RV	NRC1 SV	OP71 RV	OP71 SV	PTBB RV	PTBB SV	ROAG RV	ROAG SV	SPT0 RV	SPT0 SV	USN8 RV	USN8 SV	WAB2 RV	WAB2 SV	WSRT RV	WSRT SV
N	67	79	97	84	98	81	101	90	96	88	79	79	162	90	116	87	128	81	87	77	82	87	107	106	100	90	100	89
E	109	125	131	154	243	213	174	145	153	156	285	164	240	169	246	193	219	142	244	186	106	133	185	192	234	147	123	155
U	190	170	214	151	318	206	296	198	216	179	270	183	391	169	318	194	303	190	271	192	206	166	229	176	266	168	210	172
ZTD	20	18	22	16	24	23	30	25	18	16	17	17	23	17	20	19	19	19	26	31	21	18	30	24	18	20	18	25

IGS (color scale 0–100)

	BOR1 RV	BOR1 SV	BRUX RV	BRUX SV	CEBR RV	CEBR SV	CRO1 RV	CRO1 SV	HERS RV	HERS SV	IENG RV	IENG SV	NRC1 RV	NRC1 SV	OP71 RV	OP71 SV	PTBB RV	PTBB SV	ROAG RV	ROAG SV	SPT0 RV	SPT0 SV	USN8 RV	USN8 SV	WAB2 RV	WAB2 SV	WSRT RV	WSRT SV
N	61	57	49	45	48	52	37	38	51	49	50	54	52	46	53	51	50	54	56	48	49	48	46	43	54	54	44	49
E	88	78	77	85	122	100	59	59	91	80	82	78	108	112	96	83	82	83	88	84	68	64	105	97	103	86	69	77
U	100	86	83	76	95	80	95	78	89	73	95	86	80	75	99	81	96	88	96	83	98	79	93	67	111	90	87	80
ZTD	11	11	11	9	16	15	21	20	13	13	10	10	13	13	15	13	9	8	18	18	14	14	23	24	12	10	12	16

CNES (color scale 0–100)

	BOR1 RV	BOR1 SV	BRUX RV	BRUX SV	CEBR RV	CEBR SV	CRO1 RV	CRO1 SV	HERS RV	HERS SV	IENG RV	IENG SV	NRC1 RV	NRC1 SV	OP71 RV	OP71 SV	PTBB RV	PTBB SV	ROAG RV	ROAG SV	SPT0 RV	SPT0 SV	USN8 RV	USN8 SV	WAB2 RV	WAB2 SV	WSRT RV	WSRT SV
N	43	45	37	37	39	38	25	25	40	41	42	36	44	44	35	38	39	36	36	37	45	42	38	40	38	40	39	41
E	62	72	60	75	81	84	51	50	53	70	68	84	82	76	67	80	77	61	73	79	53	61	72	85	77	80	60	62
U	80	64	72	67	97	81	107	88	79	72	78	68	85	74	93	74	75	74	97	79	84	71	107	88	80	67	71	70
ZTD	11	11	12	12	16	15	22	20	14	14	11	9	15	14	12	13	10	11	17	19	12	12	23	22	12	11	12	13

CODE (color scale 0–30)

	BOR1 RV	BOR1 SV	BRUX RV	BRUX SV	CEBR RV	CEBR SV	CRO1 RV	CRO1 SV	HERS RV	HERS SV	IENG RV	IENG SV	NRC1 RV	NRC1 SV	OP71 RV	OP71 SV	PTBB RV	PTBB SV	ROAG RV	ROAG SV	SPT0 RV	SPT0 SV	USN8 RV	USN8 SV	WAB2 RV	WAB2 SV	WSRT RV	WSRT SV
N	10	9	10	10	15	12	16	12	12	14	15	8	15	12	13	8	15	9	14	9	11	10	13	12	17	8	11	17
E	16	16	12	17	46	22	49	25	21	20	35	13	29	17	37	13	45	11	31	19	25	11	24	24	56	10	27	26
U	20	16	20	13	39	28	38	25	29	26	28	13	31	26	28	15	28	20	49	36	20	15	26	23	37	14	22	27
ZTD	8	7	10	9	11	9	16	16	10	11	7	7	10	9	12	12	7	6	19	20	11	12	21	22	7	8	9	11

RV – reference solution without additional constraints
SV – test solution with additional constraints

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