Precise Orbit Determination and Spacetime Transformation for Autonomous Navigation in the Earth-Moon Space

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IGS 2026 WORKSHOP
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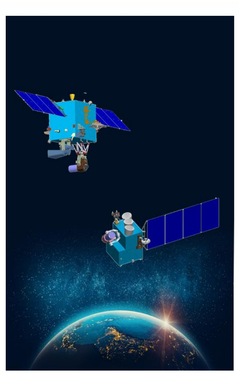

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4 POD of GNSS-lunarSat network
5 Conclusions and Prospects

Requirements and Challenges of Earth-Moon Navigation

cislunar space
- Regions dominated by the gravitational forces of the Earth and the Moon, including cislunar near-Earth space, near-lunar space and Earth-Moon transfer space
- The development of cislunar space has attracted great attention from major space-faring nations worldwide

High-precision and high-timing-efficiency Positioning, Navigation and Timing (PNT) services are required at all stages of cislunar space exploration/deep space exploration.

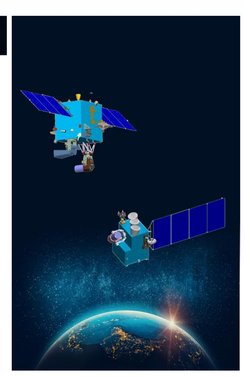
- Limited deep-space measurement and control resources
- Constrained observation geometry
- Long communication delay

Challenge 1: Variations in Dynamic Environment and Reference Frames
- The Earth-Moon space cannot be simply regarded as a near-Earth two-body problem.
- Changes in reference frames affect dynamic modeling, time-scale transformation, observation equation construction, and parameter estimation architecture.
- A shift from the near-Earth local framework to a globally unified dynamic background is required.

Challenge 2: Cross-Regional Observation Geometry Determines Observability
- Though Earth-Moon space ISL autonomous navigation utilizes gravitational field asymmetry, its performance is highly sensitive to orbital configuration, link geometry, and line-of-sight conditions.
- The large spatial scale of Earth-Moon space leads to unbalanced geometric sensitivity in different directions.
- Weak observability in normal directions, and geometric degradation of near-Moon links due to line-of-sight limitations.

Challenge 3: Traceability and Unification of Space-Time Datums
- High-precision orbit determination and time service for near-Earth space are well-established.
- Multi-reference-frame issues (BCRS/GCRS/LCRS) need to be addressed.
- IAU defines LCRS and TCL, yet fails to clarify the fundamental datum reference.
- Navigation systems covering near-Earth, transfer and near-Moon regions require more than local reference-frame definitions.

How to achieve high-precision orbit determination under complex dynamics, multiple reference frames, constrained observation geometry and engineering constraints?



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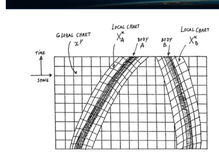
Lunar Celestial Reference System



One global and N local coordinate systems are used for the description of the gravitational N-body problem (Image credit: Damour et al. 1992b)

1. the same techniques used to construct the GCRS be used to construct an analogous Lunar Celestial Reference System (LCRS), with its time coordinate designated Lunar Coordinate Time (TCL);
2. the LCRS metric tensor, the LCRS gravitational potentials, and the transformation between the BCRS and the LCRS be defined exactly as given in the relations in IAU Resolution B1.3 of the XXIVth General Assembly (2000), with the substitution of quantities related to the Moon for those related to the Earth;
3. the unit of measurement of TCL be consistent with the SI second;
4. the reading of TCL be 1977 January 1, $0^h0^m32.184^s$ exactly when the reading of TCB be 1977 January 1, $0^h0^m32.184^s$ at the center of the Moon, and
5. the transformation between TCL and TCB be given by the relations in IAU Resolution B1.5 of the XXIVth General Assembly (2000), with the substitution of quantities related to the Moon for those related to the Earth.

- The dynamical behavior of N mutually gravitating bodies is core to celestial mechanics.
- The gravitational N-body problem has three parts: local problem, global problem, and their connection.
- Local problem: internal physics of individual bodies, analyzed in body-fixed local coordinates.
- Global problem: motion of all bodies in a global coordinate system.
- Astronomical reference system theory connects local/global systems, easy in Newtonian spacetime but hard in relativity.
- This chapter presents the reference system theory at first post-Newtonian approximation, based on DSX-I (1991).

$$G_{M,00} = -1 + \frac{2W_M}{c^2} - \frac{2W_M}{c^4} + O(c^{-5})$$
$$G_{M,0i} = -\frac{4}{c^3}W_M^i + O(c^{-5})$$
$$G_{M,ij} = \delta_{ij}\left(1 + \frac{2W_M}{c^2}\right) + O(c^{-4})$$

$$W_M = U_M + U_{M,EXT}$$
$$ds^2_{LCRS} = G_{M,\mu\nu}(\mathcal{T}, \mathcal{X}^i)d\mathcal{X}^\mu d\mathcal{X}^\nu$$

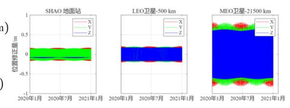
spacetime reference frame--coordinate

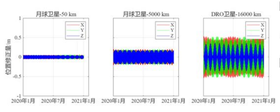
GCRS→BCRS

$$\mathbf{x}_{TT} = \left(1 + L_C + c^{-2}\sum_{B\neq E} U_B\right)\mathbf{r}_{E,TDB} + \frac{1}{2c^2}(\mathbf{v}_E \cdot \mathbf{r}_{E,TDB})\mathbf{v}_E + O(10^{-6}\,\mathrm{m})$$

BCRS→GCRS

$$\mathbf{r}_{E,TDB} = \left(1 - L_C - c^{-2}\sum_{B\neq E} U_B\right)\mathbf{x}_{TT} - \frac{1}{2c^2}(\mathbf{v}_E \cdot \mathbf{x}_{TT})\mathbf{v}_E + O(10^{-6}\,\mathrm{m})$$





LCRS→BCRS

$$\mathbf{x}_{TCL} = \left(1 + L_B + c^{-2}\sum_{B\neq M} U_B\right)\mathbf{r}_{M,TDB} + \frac{1}{2c^2}(\mathbf{v}_M \cdot \mathbf{r}_{M,TDB})\mathbf{v}_M + O(10^{-7}\,\mathrm{m})$$

BCRS→LCRS

$$\mathbf{r}_{E,TDB} = \left(1 - L_B - c^{-2}\sum_{B\neq M} U_B\right)\mathbf{x}_{TCL} - \frac{1}{2c^2}(\mathbf{v}_M \cdot \mathbf{x}_{TCL})\mathbf{v}_M + O(10^{-7}\,\mathrm{m})$$

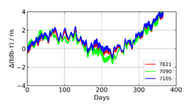
spacetime reference frame--time

GCRS ↔ BCRS (TT↔TCG ↔TCB ↔TDB):

$$t_{TDB} - T_{TT}$$
$$= \mathrm{TDB}_0 - L_C \times (\mathrm{JD}_{TT} - \mathrm{T}_0) \times 86400 + \frac{1}{c^2}\left[\int_{T_0}^{T_{TT}}\left(\frac{v_E^2}{2} + \sum_{B\neq E} U_B\right)\mathrm{d}T + \boldsymbol{v}_E \cdot \boldsymbol{x}_{TT}\right]$$
$$+ O(c^{-4})$$

Or SOFA :

$$t_{TDB} - T_{TT} = \mathrm{TDB}_0 + \boldsymbol{P}(T_{TT}) - \boldsymbol{P}(\mathrm{T}_0) + \frac{1}{c^2}(\boldsymbol{v}_E \cdot \boldsymbol{x}_{TT}) + O(c^{-4})$$



Differences between numerical and series solutions of TT-TDB for different SLR stations

LCRS ↔ BCRS(TL ↔ TCL ↔TCB ↔TDB):

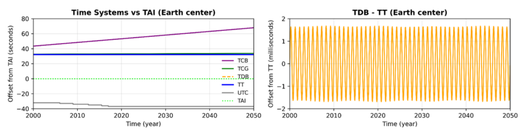
TCL ↔TDB:

$$t_{TDB} - \mathcal{T}_{TCL} = \mathrm{TDB}_0 - L_B \times (\mathrm{JD}_{TCL} - \mathcal{J}_0) \times 86400 + \frac{1}{c^2}\left[\int_{\mathcal{J}_0}^{\mathcal{T}_{TCL}}\left(\frac{v_M^2}{2} + \sum_{B\neq M} U_B\right)\mathrm{d}\mathcal{T} + \boldsymbol{v}_M \cdot \boldsymbol{x}_{TCL}\right] + O(c^{-4})$$

TL ↔TDB:

$$t_{TDB} - \mathcal{T}_{TL} = \mathrm{TDB}_0 - \frac{L_B - L_L}{1 - L_L} \times (\mathrm{JD}_{TL} - \mathcal{J}_0) \times 86400 + \frac{1}{c^2}\left[\int_{\mathcal{J}_0}^{\mathcal{T}_{TL}}\left(\frac{v_M^2}{2} + \sum_{B\neq M} U_B\right)\mathrm{d}\mathcal{T} + \boldsymbol{v}_M \cdot \boldsymbol{x}_{TL}\right] + O(c^{-4})$$

spacetime reference frame--time



The time system must be consistent with the spatial system.

Periodic orbits obtained by differential correction

$$L(x,y,z,\dot{x},\dot{y},\dot{z}) = \frac{1}{2}\left[(\dot{x}-y)^2 + (\dot{y}+x)^2 + \dot{z}^2\right] - U(x,y,z)$$
$$U(x,y,z) = -\frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} - \frac{1}{2}\mu_1\mu_2$$
$$H = \frac{1}{2}\left[(p_x+y)^2 + (p_y-x)^2 + p_z^2\right] + \tilde{U}$$
$$\tilde{U} = -\frac{1}{2}(x^2+y^2) + U$$

Initial valule x_0, Γ $\quad \boldsymbol{X}_0 = [x_0, 0, 0, \dot{y}(\Gamma, x_0)]$

Nominal orbit $\quad \boldsymbol{X}_\tau^* = [x_0, 0, 0, \dot{y}(\Gamma, x_0)]$

First Variation Equation $\quad \frac{d\boldsymbol{\Phi}}{dt} = \boldsymbol{A}\boldsymbol{\Phi}$

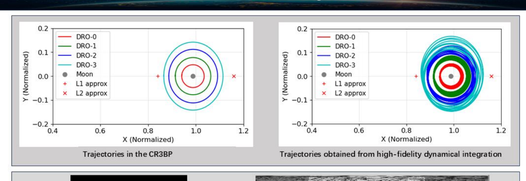
$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{\partial F_3}{\partial x} & \frac{\partial F_3}{\partial y} & 0 & 2 \\ \frac{\partial F_4}{\partial x} & \frac{\partial F_4}{\partial y} & -2 & 0 \end{bmatrix}$$

Nonlinear Phase Flow Equations for Periodic Orbits

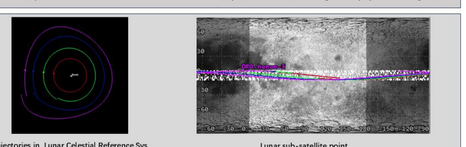
$$\boldsymbol{G}(\boldsymbol{X}) \overset{def}{=} \begin{bmatrix} \varphi_x(x_0,0,0,\dot{y}_0,\tau) - x_0 \\ \varphi_y(x_0,0,0,\dot{y}_0,\tau) \\ \varphi_z(x_0,0,0,\dot{y}_0,\tau) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-\boldsymbol{G}(\boldsymbol{X}) = \frac{\partial \boldsymbol{G}}{\partial[x,\tau]}\Delta\begin{bmatrix} x \\ \tau \end{bmatrix} \qquad \frac{\partial \boldsymbol{G}}{\partial\begin{bmatrix} x \\ \tau \end{bmatrix}^T} = \begin{bmatrix} \frac{\partial \varphi_x}{\partial x} - 1 & \dot{x} \\ \frac{\partial \varphi_y}{\partial x} & \dot{y} \\ \frac{\partial \varphi_{\dot{x}}}{\partial x} & f_3 \end{bmatrix}$$

Evolution of DRO under full perturbation forces



Trajectories in the CR3BP — Trajectories obtained from high-fidelity dynamical integration



Trajectories in Lunar Celestial Reference Sys. — Lunar sub-satellite point

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Orbit Determination Applications for Low Earth Orbits

Spacecraft **O**rbit **D**etermination **A**nalyzer.

- The current objective of this software is **on-board embedded real-time** processing of satellite-borne GNSS.
- According to requirements, the software is divided into a laboratory version and an on-board embedded version.
- Currently, the laboratory version is utilized via the command line. Subsequently, adding a user interface and satellite flight visualization are being considered.

- This software is independently developed by the Astronomical Geodynamics Research Center of the Shanghai Astronomical Observatory. To meet the needs of real-time processing of satellite-borne GNSS, the software is written in standard **C99 language** (ISO/IEC 9899:1999).
- At present, it mainly includes functions such as **precise orbit determination, orbit prediction, SLR laser validation, and virtual reference station network RTK.** The latter two, as separate modules, can be independent software.

Integrated Component of Spaceborne POD

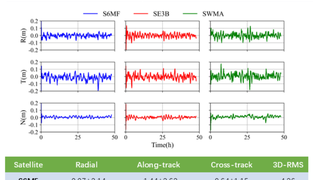
- Enables **real-time autonomous orbit determination** on board, reducing reliance on ground stations.
- **Improves response speed** and operational safety by supporting immediate orbit correction.
- **Provides highly stable and precise orbital** data for observation, navigation and mission execution.

模块名称	指标参数
通道	不小于24通道
实时定位精度	0.1m/1cm/30m（三频、1cl （三种配置选择），同时支持分米级和厘米级
后时测速精度	≤0.2m/s（三频、1cl
实验测速精度	≤0.1m/s（三频、1cl
捕获灵敏度	-163dBHz，跟踪灵敏度：-166dBW
首次定位时间	小于60s，失锁重新捕获：优于20s
时间精度	定位状态下优于50ns（1σ）
动态性能	最大速度1Km/s
数据更新率	1Hz
功耗	5W-12W(根据实际模式确定)
在轨寿命	不小于5年
地面灯海寿命	不小于1年
重量	不大于10kg，并结重量根据不同选型确定
尺寸	不大于≤200mm（长）×≤100mm（宽）×≤40mm（高）

- **Earth observation satellites** for high-precision imaging and data acquisition.
- **Navigation and positioning** satellites to ensure accurate orbital service.
- **Deep space probes and scientific exploration** satellites for autonomous orbit control and long-distance mission execution.

Sentinel-6 (S6MF), Sentinel-3B (SE3B) and Swarm-A (SWMA)

	S6MF	SE3B	SWMA
Altitude	1336 km	814.5 km	462 km
Inclination	66 deg	98.65 deg	87.35 deg
Launch date	21 November 2020	25 April 2018	22 November 2013
Obs. type	C1C/C2L/L1C/L2L	C1W/C2W/L1C/L1CW	
Obs. Duration	From 1 September 2021 to 3 September 2021		

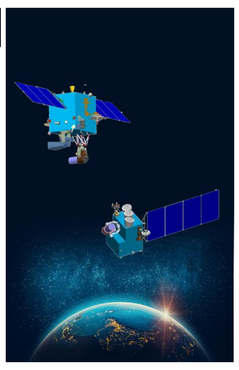


Satellite	Radial	Along-track	Cross-track	3D-RMS
S6MF	-0.07±2.14	-1.44±3.62	0.64±1.15	4.36
SE3B	-0.00±2.55	-0.33±3.34	0.19±1.47	4.45
SWMA	0.11±3.02	0.56±3.70	1.08±1.64	5.05

Satellite	Direction	time	Peak value	Mean±STD	RMS
S6MF	R	10 min	-8.41 cm	-0.60±2.47 cm	2.54 cm
		20 min	-8.15 cm	-0.80±2.33 cm	2.47 cm
	T	10 min	-28.43 cm	-3.03±5.96 cm	6.69 cm
		20 min	-32.66 cm	-4.27±7.65 cm	8.88 cm
	N	10 min	4.60 cm	-0.00±1.60 cm	1.60 cm
		20 min	-5.07 cm	-0.43±1.90 cm	1.95 cm
SE3B	R	10 min	-14.11 cm	-0.84±3.75 cm	3.84 cm
		20 min	-13.62cm	-1.09±3.99 cm	4.13 cm
	T	10 min	-34.32 cm	-2.54±7.72 cm	8.12 cm
		20 min	-49.51 cm	-4.38±11.06 cm	11.90 cm
	N	10 min	6.18 cm	0.05±2.15 cm	2.15 cm
		20 min	-7.09 cm	0.03±2.54 cm	2.54 cm
SWMA	R	10 min	-22.81 cm	-0.19±5.28 cm	5.29 cm
		20 min	-38.74 cm	-0.52±6.24 cm	6.26 cm
	T	10 min	38.07 cm	0.12±9.73 cm	9.74 cm
		20 min	60.90 cm	-0.50±15.22 cm	15.23 cm
	N	10 min	11.50 cm	0.48±2.57 cm	2.61 cm
		20 min	13.06 cm	-0.16±3.32 cm	3.32 cm

Spacecraft Orbit Determination Analyzer

Space Target Types	Orbit Types	Supported Data Types	Measurement Platforms
• Earth satellites • lunar satellites • space debris • Rocket • artificial probes • natural satellites • asteroids	• Orbiting satellites • periodic orbits in the restricted three-body problem (DRO, HALO, NRHO), • interplanetary transfer trajectories	• optics • radar • VLBI • SLR • ranging (one-way, two-way, three-way) • azimuth, elevation • right ascension, declination • GNSSISL	• Ground-based • space-based • sea-based • inter-satellite links • inter-satellite optical measurement



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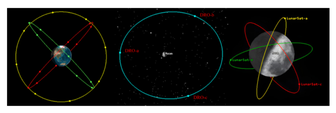
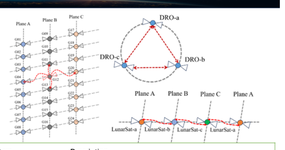
On-Board ISL Measurement Simulation for Earth-Moon Navigation



Constellation Configuration
- 24 MEO satellites (Walker constellation, 24/3/8, semi-major axis: 27 900 km)
- 3 DRO spacecraft (covering Earth-Moon region, orbital period: ~6 days)
- 3 LunarSat satellites (orbital period: 5 h, inclination: 95°, eccentricity: 0.3)

Simulation Duration
- 120 days



Simulation Scheme Settings

Name	Description
S	Only GNSS satellites participate in link construction
SD	Only GNSS satellites and DRO spacecraft participate in link construction
SDL	GNSS satellites, DRO spacecraft and LunarSats all participate in link construction

Item	Description
JPL Ephemeris	DE440
Solar Radiation Pressure	GNSS: ECOM5 model; DRO & LunarSat: Cannon-ball model
Antenna Half-cone Angle	45°
Observation Noise	GNSS-GNSS: Normalized inter-satellite ranging (0.01 m) GNSS-DRO: Two-way one-way inter-satellite ranging (1.0 m) DRO-DRO: Two-way one-way inter-satellite ranging (0.03 m) DRO-LunarSat: Two-way one-way inter-satellite ranging (0.10 m) LunarSat-LunarSat: Two-way one-way inter-satellite ranging (0.03 m)

inter-satellite measurement model

Observations modeled in the BCRS framework (inter-satellite one-way ranging, inter-satellite two-way ranging, normalized inter-satellite ranging, VLBI, etc.)

Inter-satellite One-way Ranging (OWR) :

$$\rho_{12} = (\mathrm{r}_2 - \mathrm{r}_1)\cdot c$$
$$= |\mathrm{r}_B(t_2) - r_A(t_1)| + (\delta t_{A1} - \delta t_{B2})\cdot c + (\Delta t_{B2} - \Delta t_{A1})\cdot c + \left(D_{B2}^{recv} - D_{A1}^{tran}\right)\cdot c + \Delta R_{12} + \epsilon_{12}$$

Shapiro dealy:

$$\Delta R_{rel} = \frac{2GM_S}{c^2}ln\left\{\frac{r_1^S + r_2^S + r_{12}^S + \frac{2GM_S}{c^2}}{r_1^S + r_2^S - r_{12}^S - \frac{2GM_S}{c^2}}\right\} + \sum_B \frac{2GM_B}{c^2}ln\left\{\frac{r_1^B + r_2^B + r_{12}^B}{r_1^B + r_2^B - r_{12}^B}\right\}$$

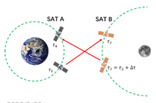
Inter-satellite Dual One-way Ranging (DOWR) :

Summation combination :

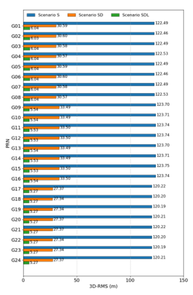
$$\rho_{12} + \rho_{34} = (\mathrm{r}_2 - \mathrm{r}_1 + \mathrm{r}_4 - \mathrm{r}_3)\cdot c$$
$$= (\mathrm{r}_4 - \mathrm{r}_1 - \Delta\mathrm{r})\cdot c$$
$$= |\mathrm{r}_B(t_2) - r_A(t_1)| + |\mathrm{r}_A(t_4) - \mathrm{r}_B(t_3)| + (\delta t_{A1} - \delta t_{A4})\cdot c + (\delta t_{B3} - \delta t_{B2})\cdot c + (\Delta t_{B2} - \Delta t_{B3})\cdot c + (\Delta t_{A4} - \Delta t_{A1})\cdot c + \left(D_{B2}^{recv} - D_{A1}^{tran}\right)\cdot c + \left(D_{A4}^{recv} - D_{B3}^{tran}\right)\cdot c + \Delta R_{12} + \Delta R_{34} + \epsilon_{12} + \epsilon_{34}$$

Difference combination :

$$\rho_{12} - \rho_{34} = (\mathrm{r}_2 - \mathrm{r}_1 - (\mathrm{r}_4 - \mathrm{r}_3))\cdot c$$
$$= |\mathrm{r}_B(t_2) - r_A(t_1)| - |\mathrm{r}_A(t_4) - \mathrm{r}_B(t_3)| + (\delta t_{A1} + \delta t_{A4})\cdot c - (\delta t_{B3} + \delta t_{B2})\cdot c + (\Delta t_{B2} + \Delta t_{B3})\cdot c - (\Delta t_{A4} + \Delta t_{A1})\cdot c + \left(D_{B2}^{recv} + D_{A1}^{tran}\right)\cdot c - \left(D_{A4}^{recv} + D_{B3}^{tran}\right)\cdot c + \Delta R_{12} - \Delta R_{34} + \epsilon_{12} - \epsilon_{34}$$

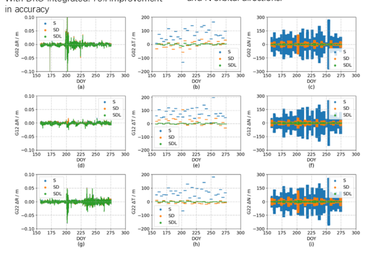


Earth-Moon Space POD with ISL Measurements

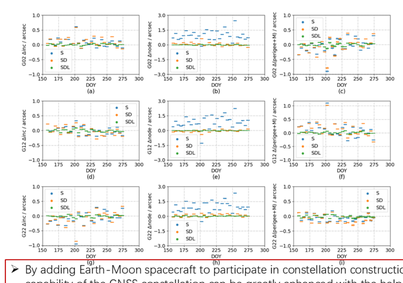


GNSS-only Link Establishment: Orbit determination accuracy at the hundred-meter level
With DRO integrated: 75% improvement in accuracy

Add LunarSat for an additional 80% improvement in precision.
Autonomous constellation rotation is reflected in the T and N orbital directions.



Earth-Moon Space POD with ISL Measurements-GNSS



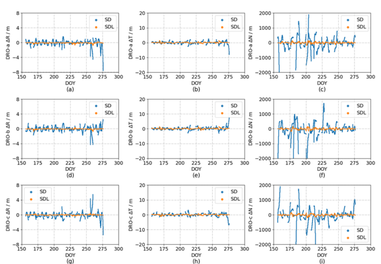
Orbital Elements Accuracy Statistics

PRN	Scenario	Inc (arcsec)	Node (arcsec)	Perigee+M (arcsec)
G02	S	0.161	1.083	0.252
	SD	0.176	0.172	0.248
	SDL	0.028	0.024	0.049
G12	S	0.133	1.087	0.226
	SD	0.147	0.116	0.275
	SDL	0.040	0.019	0.035
G22	S	0.244	1.102	0.118
	SD	0.236	0.152	0.156
	SDL	0.035	0.037	0.042

GNSS-only Link Establishment: The right ascension of ascending node error is significantly larger than other orientation orbital elements
Incorporating DRO: The accuracy of right ascension of ascending node is improved by 75%
Further incorporating LunarSat: Compared with the SD scenario, the accuracy improvement of all orientation parameters exceeds 85%

- By adding Earth-Moon spacecraft to participate in constellation construction, the autonomous orientation capability of the GNSS constellation can be greatly enhanced with the help of lunar gravity and known Earth-Moon pointing directions.

Earth-Moon Space POD with ISL Measurements-DRO



In the SD scenario:
The orbit determination accuracy of DRO is at the hundred-meter level, with some reaching the kilometer level, and errors are concentrated in the N direction.
This is related to weak constraints in the orbital normal direction.
It is also associated with more stringent chain-building conditions (concentrated between the Earth-Moon line).

In the SD scenario:
The orbit determination accuracy of DRO is at the hundred-meter level, with some reaching the kilometer level, and errors are concentrated in the N direction.
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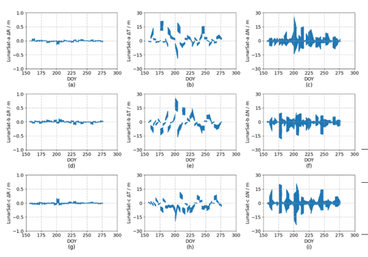
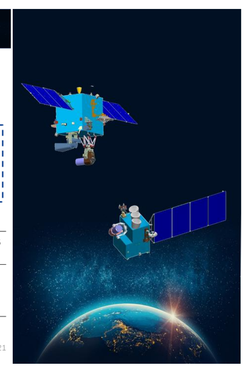
Earth-Moon Space POD with ISL Measurements-LunarSat



- The orbit determination accuracy of LunarSat is at the 10-meter level, with errors concentrated in the along-track (T) and normal (N) directions, exhibiting strong oscillations.

PRN	scenario	R-RMS (m)	T-RMS (m)	N-RMS (m)
LunarSat-a	SDL	0.023	6.093	4.299
LunarSat-b	SDL	0.028	6.446	4.049
LunarSat-c	SDL	0.026	4.829	5.167



1 introduction
2 Spacetime of cislunar
3 SODA for On-Board processing
4 POD of GNSS-lunarSat network
5 Conclusions and Prospects

Conclusions and Prospects

- Cislunar space features multi-body gravitational coupling, requiring a unified global space-time datum for cross-regional precise orbit determination.
- Large-scale cislunar space causes geometric imbalance and lunar-side link degradation, which can be mitigated by the GNSS-DRO-LunarSat hybrid constellation.
- The SDL scheme outperforms single GNSS constellations, enabling high-precision global autonomous orbit determination.
- High-precision orbit determination software SODA is developed for cislunar and interplanetary environments, supporting ISL-based simulation and validation.
- Future work will optimize constellation configuration, link topology and datum unification for engineering applications.

THANKS FOR WATCHING!

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Any feedback is highly appreciated. Prospective graduate students and postdocs welcome.

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