Taming the Invisible: All-Force Radiation Pressure Modeling for Multi-GNSS

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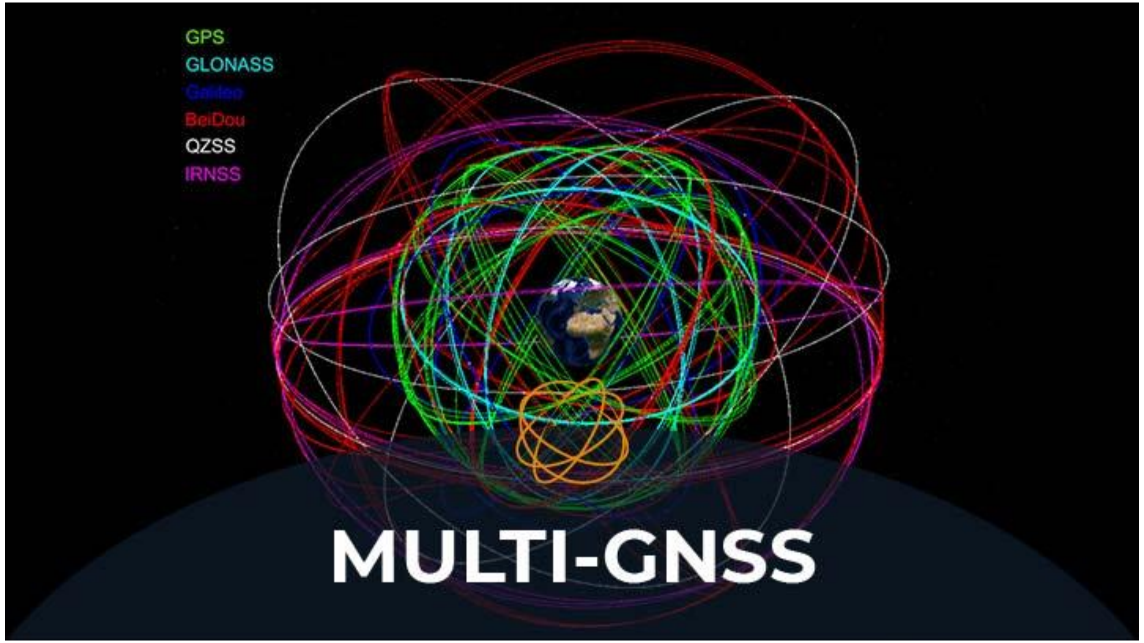
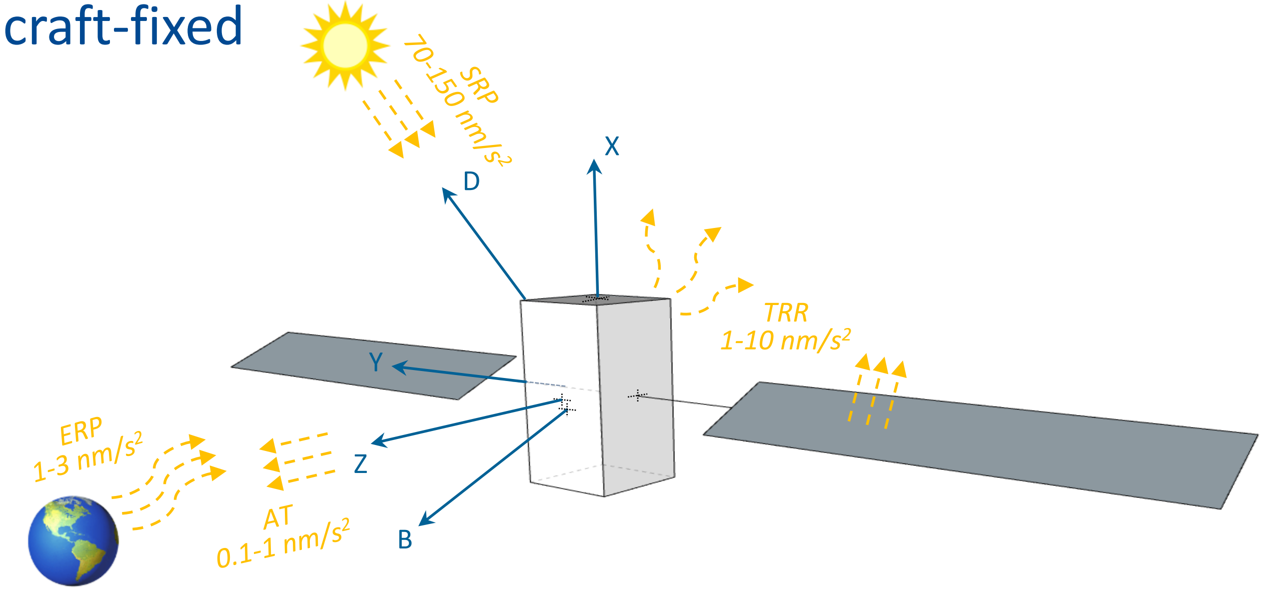
Complexity of GNSS Orbit Modelling



- Numerous GNSS space vehicles (SVs) with varied responses to radiation pressure
  - Six systems, 270 SVs launched, 110+ operational
- Over a dozen spacecraft types:
  - GPS: Block I, II, IIA, IIR, IIR-M, IIF, III
  - Galileo: IOV, FOC
  - GLONASS: M, M+, K1, K2
  - BeiDou: Block I, II, III SECM, III CAST
  - QZSS: Block I, II, IIA
- Physics-based analytical modeling requires:
  - Detailed knowledge of satellite characteristics (attitude, geometry, surface properties, thermal design)
  - Accurate Earth albedo/IR models
- Pre-launch engineering data often inaccurate, incomplete, or unavailable

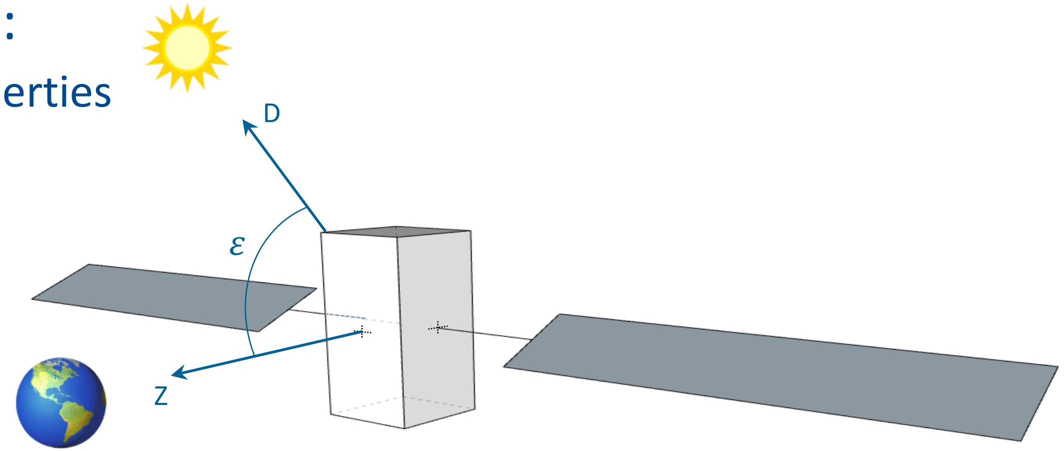
Complexity of GNSS Orbit Modelling

- Key non-gravitational forces acting on GNSS SVs:
  - Solar radiation pressure (SRP) – the force from Sun's electromagnetic radiation
  - Thermal re-radiation (TRR) – the force from anisotropic heat emission by the spacecraft
  - Earth radiation pressure (ERP) – the force from Earth's reflected visible and emitted IR light
  - Antenna thrust (TR) – the force from navigation signal transmission
- Modeling shortcomings commonly addressed by solving for constant or time-varying parameters in Sun-fixed (DYB) or spacecraft-fixed (XYZ) reference frame



Empirically-derived Multi-GNSS Radiation Pressure Model

- Development similar to JPL's GSPM approach (Bar-Sever and Kuang, 2004)
  - Dynamical long (5-day) arc fitting to precise orbit data
  - Force represented as truncated Fourier expansion about Earth-Satellite-Sun angle (Fliegel and Gallini, 1992)
  - Iterative adjustment of Fourier coefficients along with orbit state, Y-bias and along-track CPR parameters
  - Long time series combination at normal equation level for robust satellite- or block-specific force models
- Advantages over physics-based analytical models:
  - No need for surface dimensions or optical/thermal properties
  - Computationally efficient and easy to implement
  - Accurately reflects satellite in-orbit behavior
  - All in one – SRP, ERP, TRR, AT lumped together
  - Eliminates the need for separate ERP models



- Disadvantage:
  - Fourier parameters may absorb effects of other unmodelled or mismodelled processes (e.g. Earth rotation, geocenter variation)

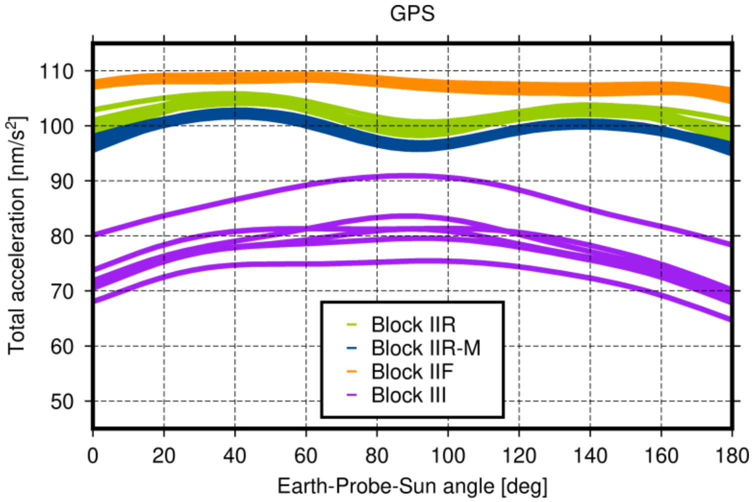
Empirically-derived Multi-GNSS Radiation Pressure Model

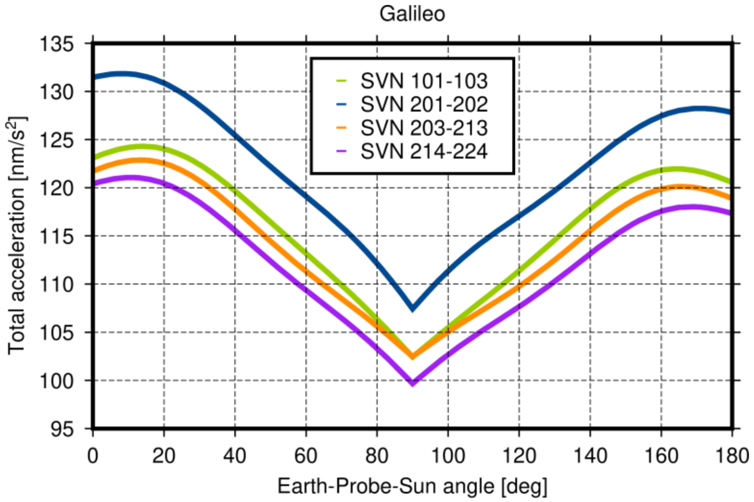
- Model created based upon one year of daily GNSS orbits from CODE MGEX processing
  - Galileo: Group-specific, 11 parameters each for SVN 101-103, 201-202, 203-213, and 214-223
  - Otherwise, satellite-specific with 12 parameters

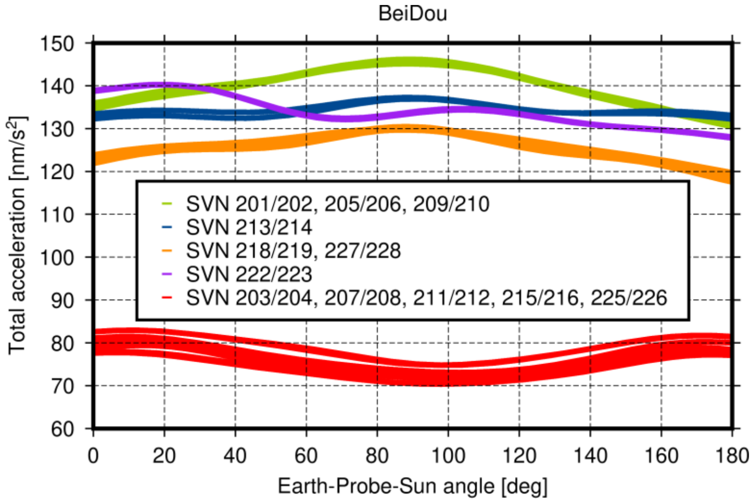
Galileo	$X = XS1 \cdot \sin(\epsilon) + XS2 \cdot \lvert\sin(2\epsilon)\rvert + XS3 \cdot \sin(3\epsilon) + XS4 \cdot \sin(4\epsilon)$ $\mathrm{Y} = Y0 + YS1 \cdot \sin(\epsilon) + YC1 \cdot \cos(\epsilon)$ $Z = \mathrm{Z}0 + ZC1 \cdot \cos(\epsilon) + ZS2 \cdot \sin(2\epsilon) + ZS4 \cdot \sin(4\epsilon)$
Others	$X = XS1 \cdot \sin(\epsilon) + XS2 \cdot \sin(2\epsilon) + XS3 \cdot \sin(3\epsilon) + XS4 \cdot \sin(4\epsilon) + XS5 \cdot \sin(5\epsilon)$ $\mathrm{Y} = Y0 + YS1 \cdot \sin(\epsilon) + YC1 \cdot \cos(\epsilon)$ $Z = \mathrm{Z}0 + ZC1 \cdot \cos(\epsilon) + ZS2 \cdot \sin(2\epsilon) + ZS4 \cdot \sin(4\epsilon)$

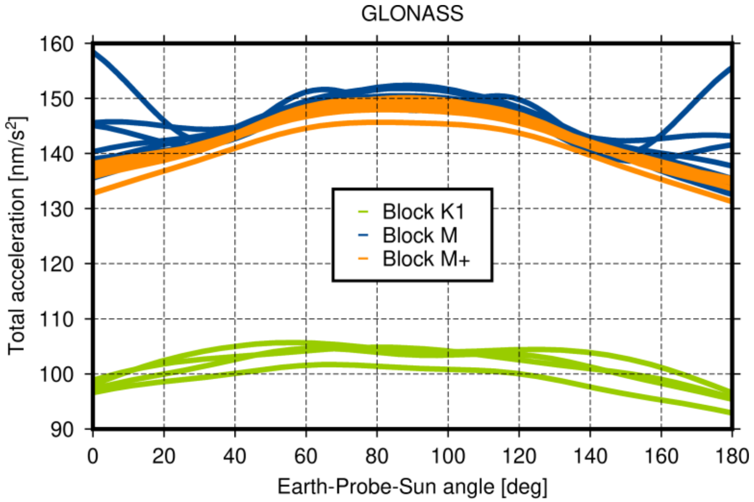
- Force in Y of thermal origin, applicable both inside and outside eclipse

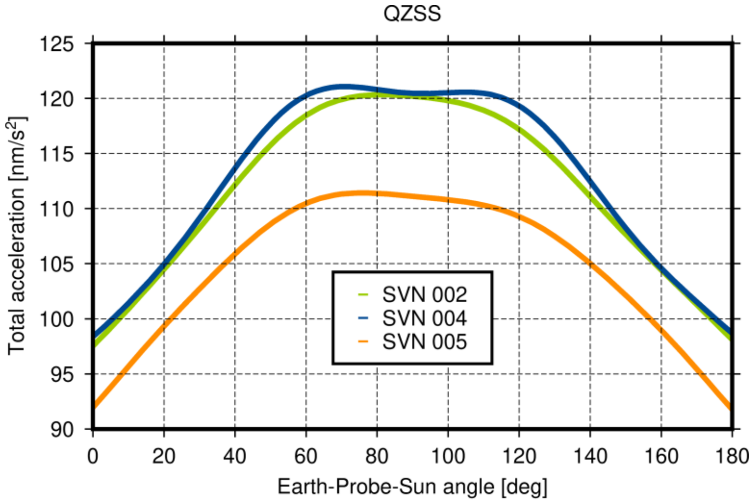
Empirically-derived Multi-GNSS Radiation Pressure Model











Model Evaluation

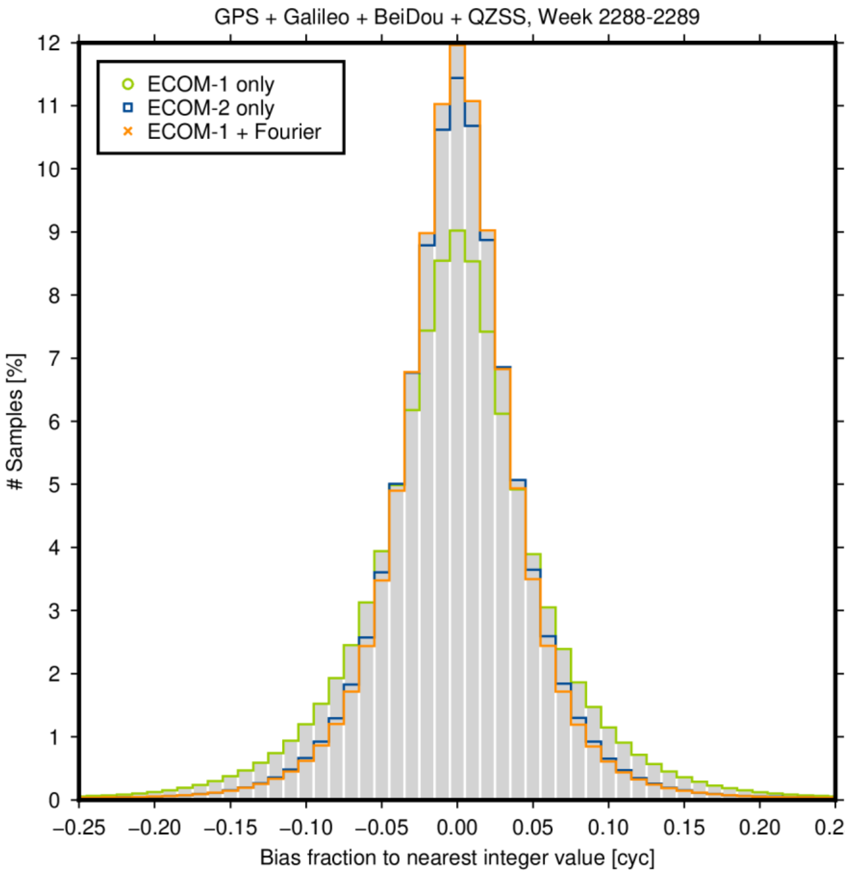
- Generate 24-hour arc solutions spanning January 1 to December 31, 2023
  - 150 station global network
  - Three solutions based on identical data, differing only by the force model

	Force model	Description
1	ECOM-1 only	Five-parameter ECOM, three tightly constrained along-track CPRs, no a-priori model
2	ECOM-2 only	Same as strategy 1 but with additional twice-per-revolution terms
3	ECOM-1 + Fourier	Same as strategy 1 but with a-priori Fourier model

- Evaluation by means of several internal and external performance metrics
  - Narrow-lane double-difference residuals, empirical radiation pressure estimates, laser range residuals, day-boundary orbit differences, orbit prediction errors

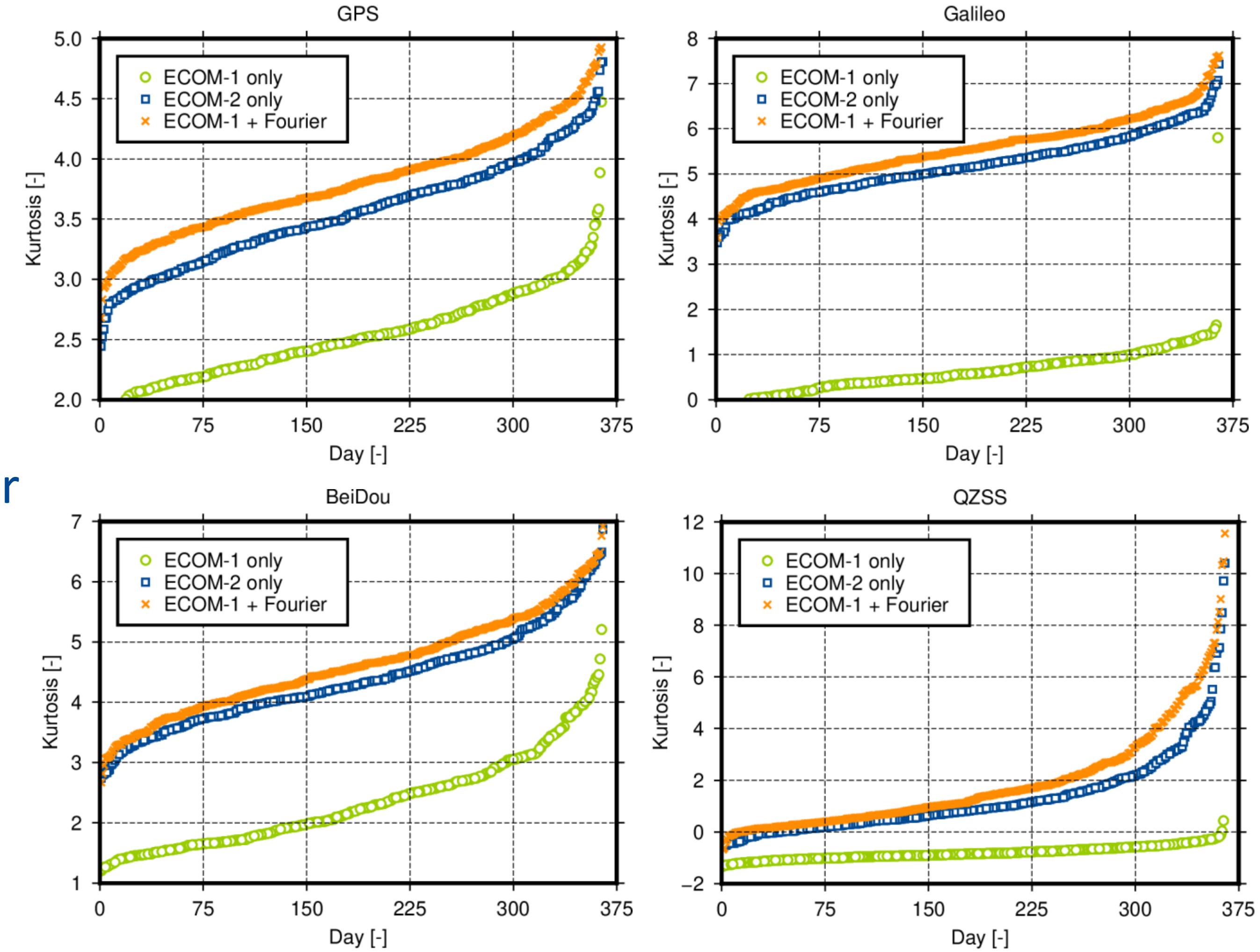
Distribution of Narrow-Lane Fractionals

- Fractional part of unresolved double-difference narrow-lane (NL) ambiguities
- Strong indicator of orbit model accuracy
- Background:
  - Orbit errors do not fully chancel out in double differencing
  - Residual orbit error grows with baseline length (Bauersima, 1983)
  - More accurate orbit model leading to smaller NL fractionals
- Histogram demonstrates the Fourier-based approach's superiority over purely empirical modeling



Kurtosis of Narrow-Lane Fractionals

- Measure of distribution’s tailedness
- The larger the kurtosis, the …
  - … fewer values in the tails
  - ... more peaked the distribution
  - ... closer fractionals cluster around zero
  - ... more ambiguities can be resolved
  - … more accurate the resolved solution
- Highest kurtosis for ECOM-1 + Fourier
  - Factor 1.9 compared to ECOM-1 alone
  - Factor 1.1 compared to ECOM-2

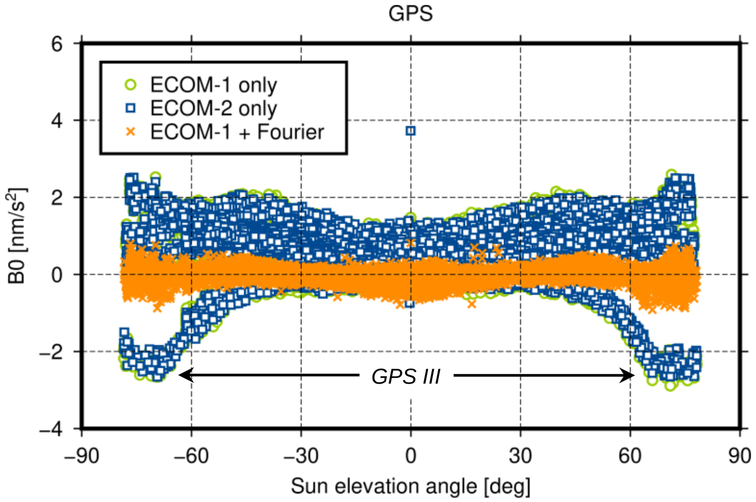


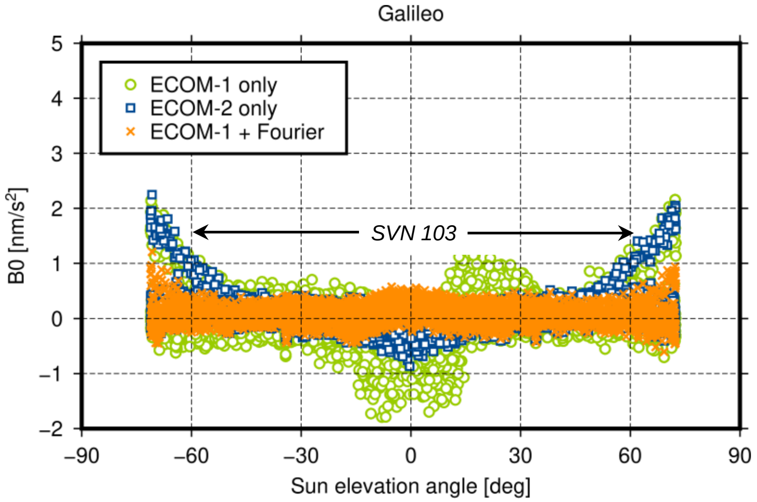
ECOM Parameter Estimates

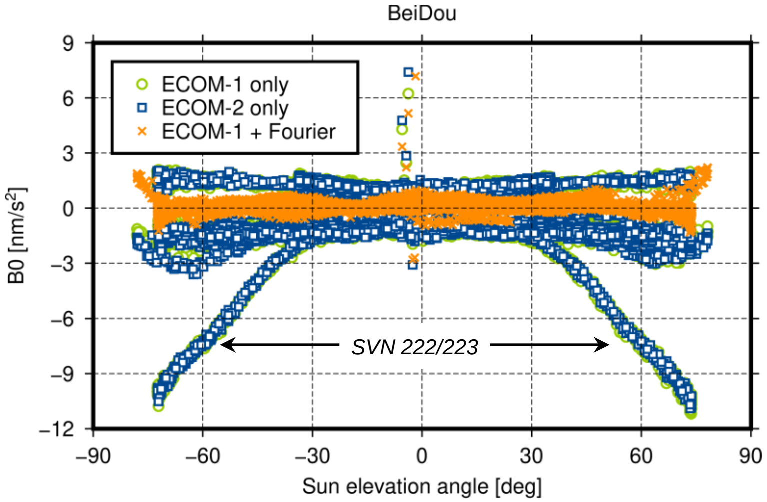
- Large systematics in ECOM-only estimates, particularly for B-constant (B0) and B-cosine (BC)
- “Gull-wing” pattern observed in:
  - B0 term of GPS III satellites, Galileo SVN 103, BeiDou SVN 222/223 (Zajdel et al. 2022)
  - BC term of Galileo satellites
- Estimates significantly smaller and less systematic with Fourier a-priori model applied
  - RMS reduction for B0 and BC by factor of 3 to 4
- Table below shows RMS computed across all GNSS satellites

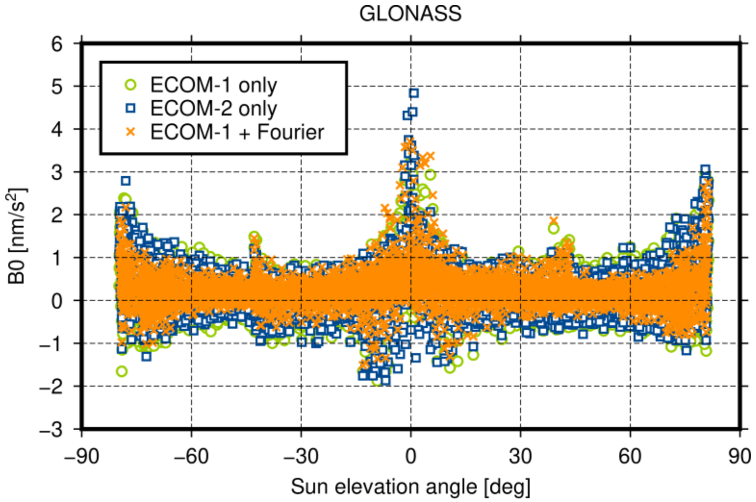
Force Model	RMS D0 [nm/s^2]	RMS Y0 [nm/s^2]	RMS B0 [nm/s^2]	RMS BC [nm/s^2]	RMS BS [nm/s^2]
ECOM-1 only	114.55	0.44	1.13	2.59	0.37
ECOM-2 only	114.70	0.42	1.13	3.20	0.47
ECOM-1 + Fourier	0.25	0.11	0.26	0.78	0.36

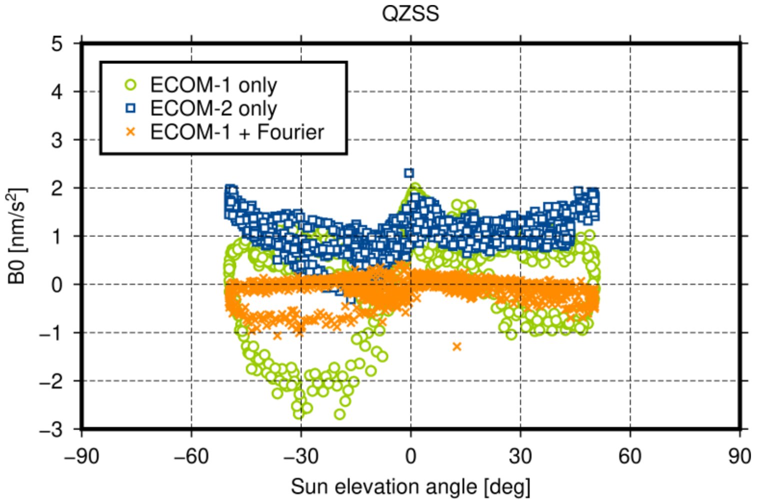
B0 Estimates vs. Beta Angle



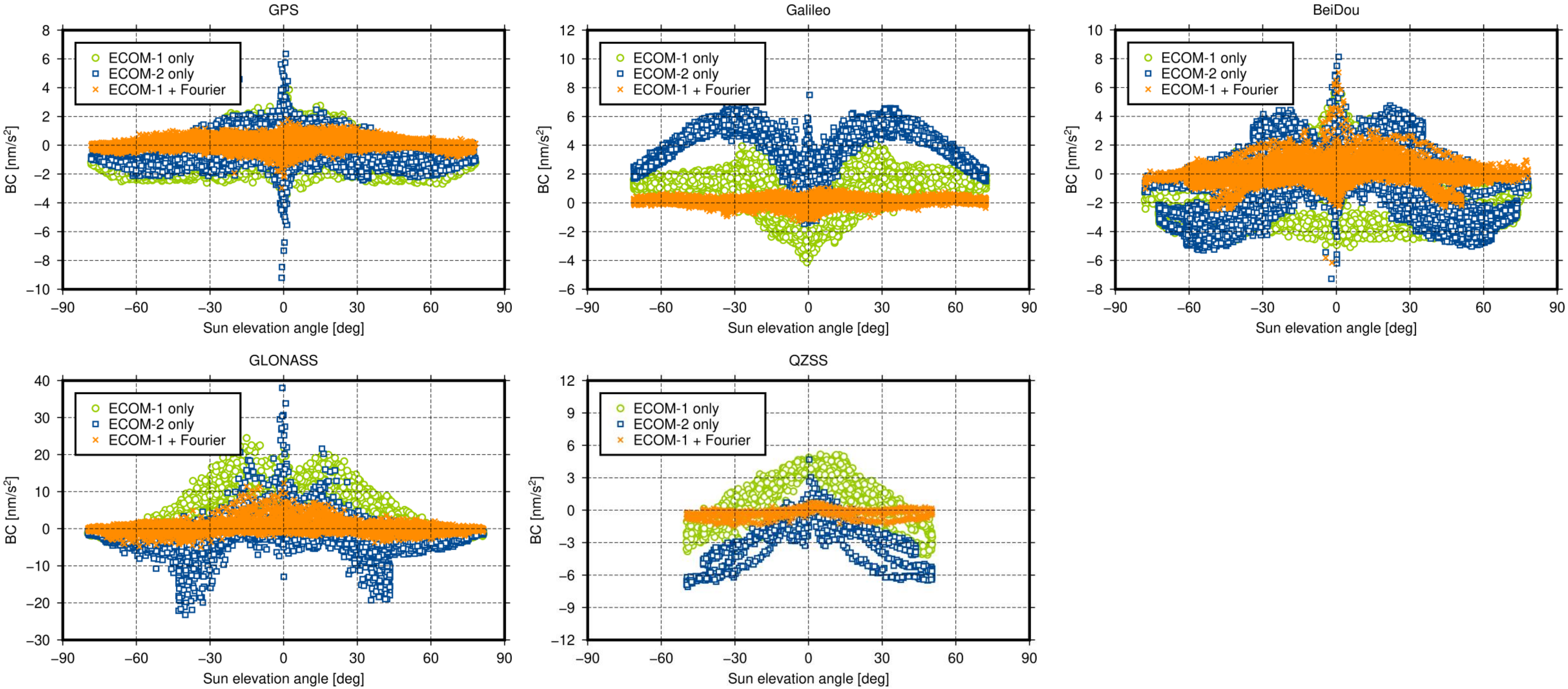








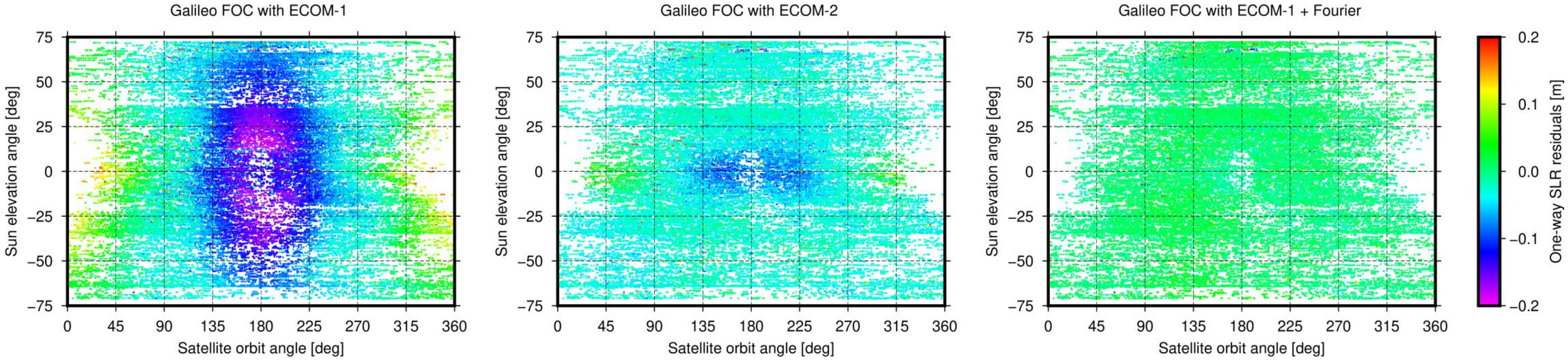
BC Estimates vs. Beta Angle



SLR Residuals

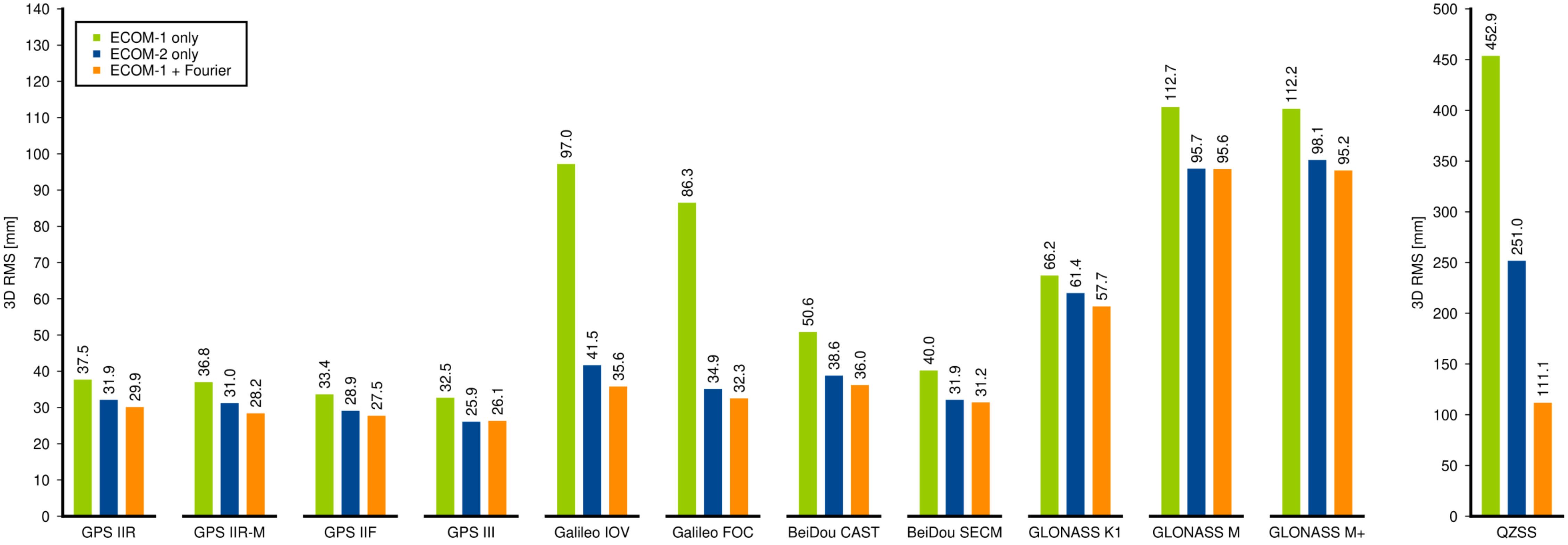
- Strong reduction in SLR residuals for most constellations, especially Galileo and QZSS

Force Model	Galileo IOV Bias ± STD [mm]	Galileo FOC Bias ± STD [mm]	BeiDou CAST Bias ± STD [mm]	BeiDou SECM Bias ± STD [mm]	QZSS Bias ± STD [mm]
ECOM-1 only	-64.0 ± 79.2	-59.5 ± 67.6	25.5 ± 46.1	-65.0 ± 30.1	179.4 ± 132.3
ECOM-2 only	-30.2 ± 27.3	-30.3 ± 26.6	18.5 ± 45.0	-49.2 ± 25.7	26.3 ± 109.1
ECOM-1 + Fourier	2.9 ± 22.4	6.0 ± 20.3	33.8 ± 37.8	-25.8 ± 20.1	9.1 ± 37.2



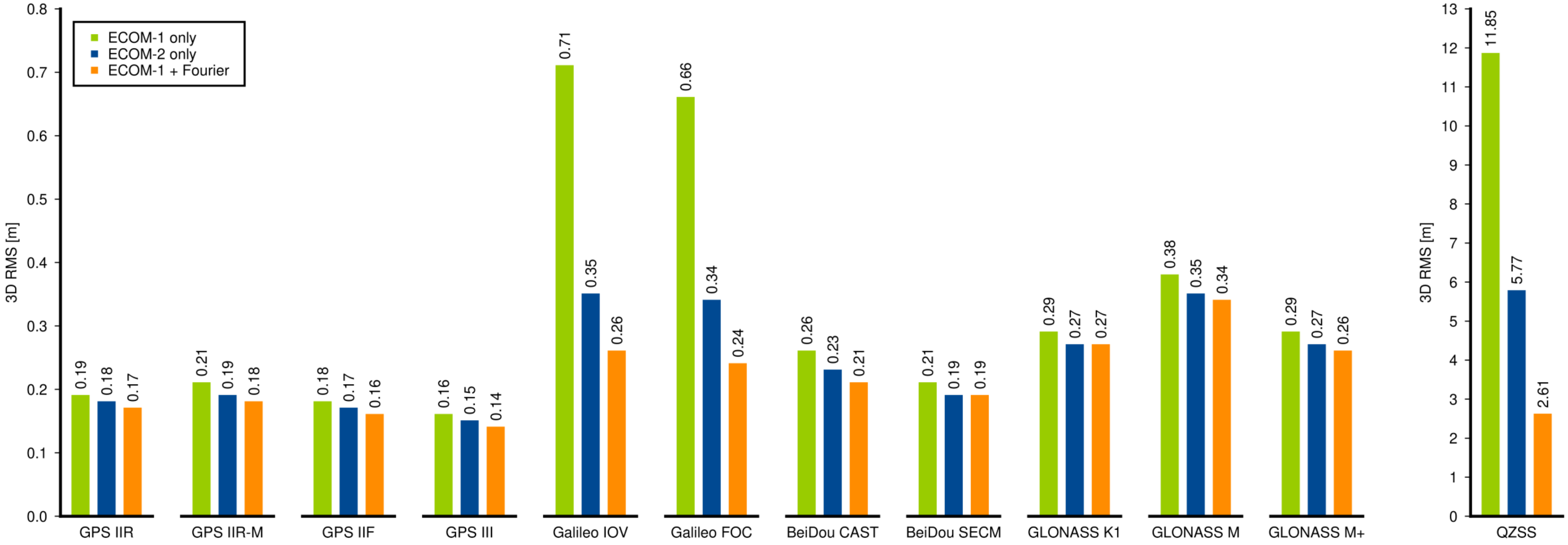
Day-Boundary Orbit Differences

- Comparison of consecutive arcs at midnight epoch as measure for internal orbit consistency
- ECOM-1 + Fourier outperforms ECOM-1 significantly and generally surpasses ECOM-2



One Day Orbit Prediction Error

- Comparison of predicted orbit at the 24-hour epoch against precise reference orbit
- ECOM-1 + Fourier again superior to ECOM-1 alone and generally better than ECOM-2



Conclusions

- Simple radiation force background model for all GNSS derived and thoroughly tested
- Comparison against ECOM approaches without a-priori model
  - Superior to ECOM-1 in every respect (SLR residuals, ambiguity resolution, orbit overlaps, etc.)
  - Performs on par with, or even surpasses, ECOM-2
- Future work:
  - Refine model
  - Compare against “box-wing”
  - Improve eclipse handling
  - Make model available to everyone, free of charge

The invisible is tamed, but not yet defeated…

Abstract

Precise orbit determination and propagation of global navigation satellite system (GNSS) spacecraft hinge on accurate modeling of non-gravitational forces, such as solar radiation pressure (SRP), Earth radiation pressure (ERP), thermal re-radiation (TRR), and antenna thrust (AT). Modeling these forces is intricate, as it requires detailed knowledge of spacecraft characteristics (attitude, geometry, surface properties, thermal design), as well as accurate Earth albedo and Earth infrared models. Even slight uncertainties, as small as a few nanometers per second squared, will significantly impact satellite trajectories, altering them by tens of centimeters over a day. In GNSS practice, these modeling shortcomings are addressed by estimating a set of constant or time-varying parameters in Sun-fixed or spacecraft-fixed reference frames. With an accurate, all-force radiation pressure model in the background, the number of solve-for parameters can be reduced, or their stochastic constraints can be tightened. In this presentation, we outline the development and testing of a new empirically derived radiation pressure model applicable to all GNSS and created independently of any spacecraft metadata. Our methodology involves dynamic long-arc fitting to precise orbit data and the utilization of low-order Fourier functions of the Earth-spacecraft-Sun angle to represent the combined effect of the SRP, ERP, TRR, and AT forces as accurately as possible. The parameterization is similar to that of existing models, such as the "good old" GPS ROCK or JPL's GSPM model. We iteratively adjust the Fourier coefficients along with state vector, Y-bias, and 1-cycle per revolution (1-cpr) along-track parameters to best fit the data in a least-squares sense. Subsequently, we rigorously combine all N-day arc solutions at the normal equation level to construct a robust force model. This new model offers several benefits: it is easy to implement into any GNSS software, accurately reflects the satellites' in-orbit behavior, eliminates the need for an additional Earth albedo/infrared model, and improves the accuracy of both the estimated and predicted orbits. The improved performance is evident in empirical radiation pressure estimates, laser range residuals, day-to-day orbit overlaps, Earth orientation parameters, and narrow-lane double-difference residuals. The model will be publicly available and updated regularly.