ALGERIAN SPACE AGENCY
CENTRE OF SPACE TECHNIQUES

# On Selecting the Right Coordinate Transformation Method between WGS84 and Nord Sahara Geodetic Reference Systems

## Study Objectives

In this study, we explain how it is possible to determine exhaustive transformation parameters between the WGS84 and the local Nord Sahara 59 systems using the available collocated points with coordinates determined in both systems.

Giving a set of candidate models of transformation, the preferred model is chosen with the minimum RMS of differences between the source local coordinates of collocated points and the transformed coordinates of these points from WGS84.

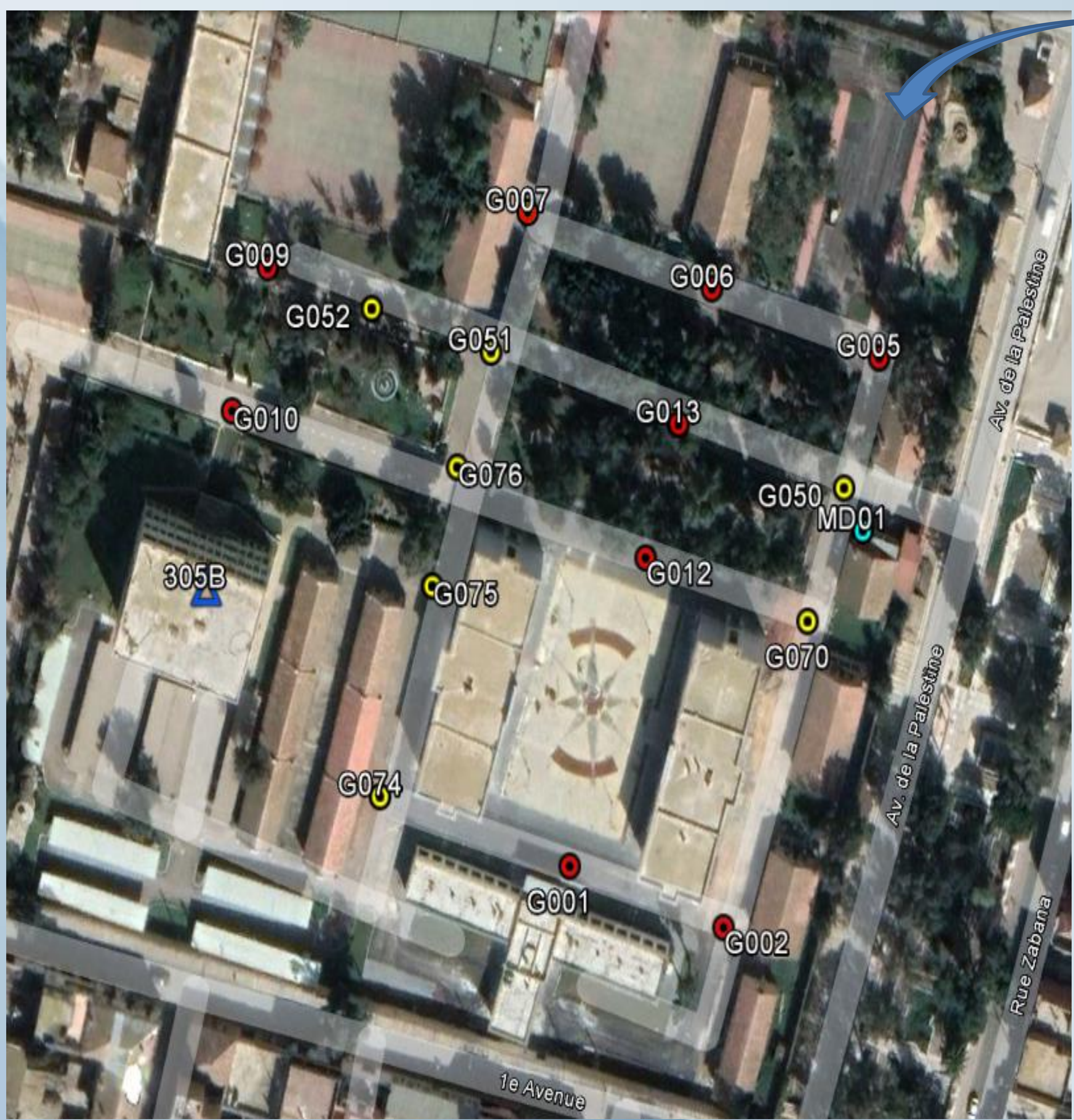
## Introduction

The geodetic system in use in Algeria, known as "Nord Sahara 1959 " or "NS 59", was defined under the Clarke 1880 A ellipsoid.

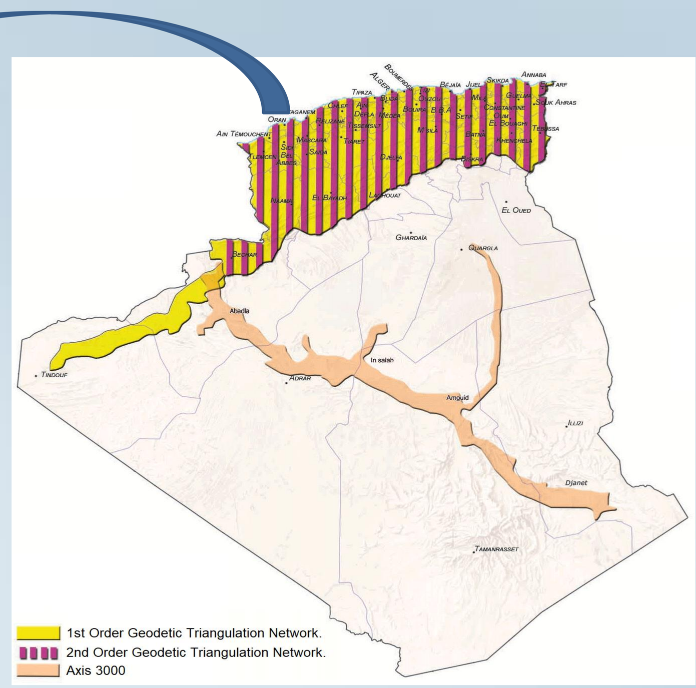
The rationale behind this system, which replaced the Voirol 1875 system in 1960, was to reconcile the two networks; the geodetic 1st and 2nd order calculated under the ED50 system (international Hayford ellipsoid) and the astronomical network that covers the southern part of Algeria.

Thus, a system of meridians and parallels was established on the Clarke 1880 A ellipsoid such that the average discrepancies between the geographic coordinates from the two networks (geodetic and astronomical) would be minimal. The resulting coordinate system was referred to as the **"**Nord Sahara 1959 geodetic system.**"**

The aim of this study is to evaluate the performance of seven (07) transformation models on the basis of the analysis of the RMS of the obtained residuals, as differences between the source local coordinates of collocated points and those obtained by transformation of WGS84 coordinates of these points using the estimated transformation parameters.




*Geographical distribution of collocated stations. Points in red: new GNSS stations, Points in yellow: old topographic stations.*




***NS59 geodetic network :***

- ***1st order** : 450 points, 30 to 45 Kms*
- ***2nd order** : 3291 points, 10 to 20 Kms*
- ***Axis 3000** : 170 points (Abadla-Adrar, Adrar-Amguid, Amguid-Ouargla, Amguid-Djanet)*

## Materials and Methods

A local topographic network within the National Higher School of Geodetic Sciences and Space Techniques (ENSSGTS-Arzew, Oran), situated in the NorthWest of Algeria, was observed on the one hand by the use of the GNSS technique and on the other hand by the use of topometry processes (classic observations of angles and levelling).

**Sixteen (16) collocated points** were thus determined under WGS84 and Nord Sahara 59 geodetic systems :

- 12 points were selected for the estimation of transformation parameters.
- The remaining 4 points will serve only as control points (don't used for the estimation of transformation parameters).

**Seven (07) different transformation models are tested :** *Geocentric Translation Model, Bursa-Wolf Model, Molodensky-Badekas Transformation, Abridged Molodensky transformation, Multiple Regression Equations (MRE), Quadratic Polynomial Transformation Model and Quadratic Polynomial Transformation Model with Legendre polynomials.*

## Results

***Table 1.** Bursa-Wolf model: Residuals in local coordinates of collocated points after transformation.*

| Point | Cartesian coordinates | | | Geographic coordinates | | Planes UTM 30 | | Height (m) | RMS Position (m) |
|---|---|---|---|---|---|---|---|---|---|
| | X(m) | Y (m) | Z (m) | Lat (sec) | Long (sec) | Northing (m) | Easting (m) | | |
| G001 | 0.006 | 0.003 | -0.021 | -0.000666 | 0.000121 | -0.020 | 0.003 | -0.008 | 0.022 |
| G002 | 0.000 | 0.002 | -0.019 | -0.000499 | 0.000080 | -0.015 | 0.002 | -0.012 | 0.019 |
| G006 | 0.040 | 0.007 | -0.039 | -0.001785 | 0.000288 | -0.055 | 0.008 | 0.009 | 0.056 |
| G007 | -0.021 | 0.140 | -0.011 | 0.000124 | 0.005575 | 0.007 | 0.140 | -0.024 | 0.142 |
| G009 | 0.023 | -0.038 | 0.011 | -0.000152 | -0.001509 | -0.006 | -0.037 | 0.026 | 0.046 |
| G010 | 0.012 | -0.048 | -0.006 | -0.000391 | -0.001910 | -0.013 | -0.048 | 0.007 | 0.050 |
| G012 | 0.008 | -0.009 | 0.002 | -0.000100 | -0.000357 | -0.003 | -0.009 | 0.008 | 0.012 |
| G050 | -0.026 | -0.026 | 0.042 | 0.001596 | -0.001042 | 0.048 | -0.027 | 0.004 | 0.055 |
| G051 | -0.032 | -0.008 | -0.011 | 0.000318 | -0.000326 | 0.009 | -0.008 | -0.033 | 0.035 |
| G070 | -0.006 | -0.042 | 0.044 | 0.001267 | -0.001675 | 0.038 | -0.043 | 0.021 | 0.061 |
| G074 | 0.004 | 0.036 | 0.004 | 0.000033 | 0.001436 | 0.002 | 0.036 | 0.005 | 0.036 |
| G076 | -0.008 | -0.017 | 0.004 | 0.000255 | -0.000679 | 0.008 | -0.017 | -0.004 | 0.019 |
| **RMS (m)** | 0.013 | 0.031 | 0.015 | 0.000538 | 0.001228 | 0.016 | 0.031 | 0.011 | |
| **Sigma0 (m)** = 0.037 | | | | | | | | | |

***Table 2.** Bursa-Wolf model: Residuals in local coordinates of the control points after transformation.*

| Point | Cartesian coordinates | | | Geographic coordinates | | Planes UTM 30 | | Height (m) | RMS Position (m) |
|---|---|---|---|---|---|---|---|---|---|
| | X(m) | Y (m) | Z (m) | Lat (sec) | Long (sec) | Northing (m) | Easting (m) | | |
| G005 | -0.161 | 0.049 | -0.108 | 0.000225 | 0.001918 | 0.009 | 0.048 | -0.194 | 0.200 |
| G013 | -0.073 | -0.011 | -0.114 | -0.001612 | -0.000454 | -0.050 | -0.010 | -0.126 | 0.136 |
| G052 | 0.016 | -0.053 | -0.035 | -0.001230 | -0.002109 | -0.040 | -0.051 | -0.008 | 0.065 |
| G075 | 0.047 | 0.003 | 0.063 | 0.000763 | 0.000129 | 0.024 | 0.003 | 0.075 | 0.079 |
| **RMS (m)** | 0.092 | 0.037 | 0.086 | 0.001089 | 0.001445 | 0.034 | 0.035 | 0.122 | |
| **Sigma0 (m)** = 0.131 | | | | | | | | | |

***Table 3.** RMS of the residuals in local coordinates of the used collocated points to estimate the transformation parameters.*

| Model | Cartesian coordinates | | | Geographic coordinates | | Planes UTM 30 | | Height (m) | Sigma0 (m) |
|---|---|---|---|---|---|---|---|---|---|
| | X(m) | Y (m) | Z (m) | Lat (sec) | Long (sec) | Northing (m) | Easting (m) | | |
| Geocentric Translation | 0.017 | 0.033 | 0.016 | 0.000630 | 0.001331 | 0.019 | 0.034 | 0.013 | 0.041 |
| Bursa-Wolf | 0.013 | 0.031 | 0.015 | 0.000538 | 0.001228 | 0.016 | 0.031 | 0.011 | 0.037 |
| Molodensky-Badekas | 0.013 | 0.033 | 0.016 | 0.000568 | 0.001297 | 0.017 | 0.033 | 0.011 | 0.039 |
| Abridged Molodensky | 0.017 | 0.033 | 0.016 | 0.000631 | 0.001329 | 0.020 | 0.033 | 0.013 | 0.041 |
| MRE – 1st order | 0.022 | 0.054 | 0.026 | 0.000935 | 0.002174 | 0.029 | 0.055 | 0.019 | 0.064 |
| MRE – 2nd order | 0.026 | 0.038 | 0.021 | 0.000927 | 0.001519 | 0.029 | 0.038 | 0.017 | 0.050 |
| MRE – 3th order | 0.025 | 0.039 | 0.006 | 0.000523 | 0.001560 | 0.017 | 0.039 | 0.020 | 0.047 |
| Quadratic Polynomial | 0.023 | 0.047 | 0.013 | 0.000661 | 0.001869 | 0.021 | 0.047 | 0.017 | 0.054 |
| Legendre polynomials | 0.023 | 0.047 | 0.013 | 0.000661 | 0.001869 | 0.021 | 0.047 | 0.017 | 0.054 |

***Table 4.** Doubtful points according to the different models and discrepancies between the source local UTM plane coordinates and those obtained from transformation.*

| Model | In Northing component | | In Easting component | |
|---|---|---|---|---|
| | Station | Difference (m) | Station | Difference (m) |
| Geocentric Translation | G006 | -0.064 | G007 | 0.168 |
| Bursa-Wolf | G006 | -0.055 | G007 | 0.140 |
| | G050 | 0.048 | | |
| Molodensky-Badekas | G006 | -0.055 | G007 | 0.140 |
| | G050 | 0.048 | | |
| Abridged Molodensky | G006 | - 0.064 | G007 | 0.168 |
| MRE - 1st order | G006 | -0.063 | G007 | 0.130 |
| MRE - 2nd order | G006 | -0.047 | G006 | -0.049 |
| | | | G007 | 0.062 |
| MRE - 1st order | - | - | - | - |
| Quadratic Polynomial | G001 | 0.015 | G007 | 0.034 |
| | | | G051 | -0.041 |
| Legendre polynomials | G001 | 0.015 | G007 | 0.034 |
| | | | G051 | -0.041 |

***Table 5.** RMS of the residuals in the local coordinates of control points.*

| Model | Cartesian coordinates | | | Geographic coordinates | | Planes UTM 30 | | Height (m) | Sigma0 (m) |
|---|---|---|---|---|---|---|---|---|---|
| | X(m) | Y (m) | Z (m) | Lat (sec) | Long (sec) | Northing (m) | Easting (m) | | |
| Geocentric Translation | 0.089 | 0.045 | 0.104 | 0.001222 | 0.001787 | 0.037 | 0.045 | 0.131 | 0.144 |
| Bursa-Wolf | 0.092 | 0.037 | 0.086 | 0.001089 | 0.001445 | 0.034 | 0.035 | 0.122 | 0.131 |
| Molodensky-Badekas | 0.092 | 0.037 | 0.086 | 0.001089 | 0.001445 | 0.034 | 0.035 | 0.122 | 0.131 |
| Abridged Molodensky | 0.089 | 0.045 | 0.104 | 0.001211 | 0.001789 | 0.037 | 0.045 | 0.132 | 0.144 |
| MRE – 1st order | 0.087 | 0.036 | 0.091 | 0.001140 | 0.001429 | 0.036 | 0.035 | 0.121 | 0.131 |
| MRE – 2nd order | 0.096 | 0.038 | 0.094 | 0.001117 | 0.001514 | 0.035 | 0.037 | 0.130 | 0.140 |
| MRE – 3th order | 0.099 | 0.049 | 0.089 | 0.001132 | 0.001923 | 0.035 | 0.048 | 0.129 | 0.142 |
| Quadratic Polynomial | 0.153 | 0.081 | 0.121 | 0.000977 | 0.003194 | 0.030 | 0.081 | 0.193 | 0.211 |
| Legendre polynomials | 0.153 | 0.081 | 0.121 | 0.000977 | 0.003194 | 0.030 | 0.081 | 0.193 | 0.211 |

## Conclusion

The 7-parameter Bursa Wolf and 10-parameter Molodensky-Badekas transformation models obtained the best transformation results : an RMS in the Northing component of ±1.6 cm, in the Easting component of ±3.1 cm and in the vertical component of ±1.1 cm, while the quadratic polynomial models obtain the worst transformation results.

The transformation accuracies of the 3D multiple regression equations of 1st, 2nd and 3rd order are ranged from ±1.7 to ±2.9 cm in the Northing component and from ±3.8 to ±5.5 cm in the Easting component.

The validation results show acceptable RMS in transformed coordinate of the control stations using the three 3D transformation models of Bursa-Wolf and Molodensky-Badekas that are of ±3.4 cm and ±3.5 cm in the Northing and the Easting components, respectively.

In the future, we hope to be able to test the performance of other transformation models with data from a denser geodetic network, in order to draw more exhaustive conclusions as to the best or worst model.

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