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# *GNSS positioning accuracy for referential frames transformation parameters determination: Case of NS59*

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**Abstract:** Multi GNSS and Multi frequencies are nowadays an important technic for swift establishment of geodetic networks accurately. In one hand it allow multiple combination between a different constellation taking the best properties of each one, in other hand, combining them allowing to benefit of these properties, such as accuracy, error balance reduction, an excellent GDOP, mitigation of multiple frequencies. That is mean the possibility of correction or elimination of different errors sources (Ionosphere, Orbit, Clock). These corrections are brought either from global network (IGS stations) or from a local stations network involving middle or long Baselines. Thus, we can in one hand, evaluate the required time for ambiguity fixation resultant from the first approach and their precisions as well, in other hand, we apply and analyze with details the second approach for determining final coordinates of network double points contribying to the transformation parameters estimation.

We deal hereby with an important theoretical problem of geodesy: we are looking for a mathematical dependency between two spatial systems using common pairs of points whose coordinates are given in both systems. In geodesy, the most often used procedures to move from one coordinates system to the other, is the 3D, 7 parameters (Bursa Wolf, Molodensky-Badekas, and Helmert) similarity transformations.

We discusses various used methods for estimation of the usual transformation parameters with taking in count some assumptions and the accuracy in coordinates brought by using GNSS survey in order to compensate the lack of precision due to lack of data (non exhaustive number of common points), or the non linear models cases (Jacobi method, Levenberg-Marquardt, Steepest descent ). Compare solutions based on seven parameters with those based on interpolation of data in Planimetric and Altimetric Grid using B-spline model, Conjugated Gradient for example.

## Problematic

Referential frames transformations are usually applied to convert coordinates related to one geodetic datum to another; this operation is commonly known as datum transformation. In such applications, the rotations between the two 3D coordinates axes are tight (usually less than one second of arc) and certain approximations are used to simplify rotation matrices; these simplified matrices are a common feature of the Bursa–Wolf and the Molodensky-Badekas transformations. The transformation of GNSS survey coordinates into the geodetic local system which constituting the basis of the mapping system in force in every country. These local datums in which the results of geodetic works must be expressed, requires knowledge of the passage parameters with the use of a suitable both two and three dimensions models. The transformation is usually presented either under the form of a spatial similarity (Bursa-Wolf ...) either from of a geographic formalism known as the Molodensky model or a bi dimensional 2D affine regression.

## Objectives and Methodology

The undertaken methodology adopted for the reparation was based on the following points:

- Identification of any discrepancies in scale and orientation between the two networks, Identification and validation of the coordinates of points, the GPS, terrestrial and astronomical data, levelling measurements,…),
- Affining a methodology of adjustment, combining conventional and GNSS data by writing a dedicated software,
- Adjustment of combined measures GNSS & Classical for the realization of 3D geodetic networks.

For steering the National NS-59 Datum reparation task, our choice was on using the Vanicek model instead of the mapping system in the adjustment, Establishing a New Algerian Triangulation Network (NTA) and adopting a methodology of combination of classical and GNSS techniques by following the different steps:

- ❑ Step 1: Identification and validation of measurements based networks (available in INCT) and Preliminary process of classical data,
- ❑ Step 2: Adoption of a specific adjustment to the Algerian territory (adjustment of large networks incorporating minimal constraints),
- ❑ Step 3: Appraisal (scale and orientation) 2D national network by integrating GPS data.
- ❑ Step 4: New geodetic Algerian network adjustment using the model of free network adjustment with minimal constraints and including the same ellipsoid (North Sahara - Clarke 1880), the same measurements performed by IGN and INCT, the same map projection (UTM), but the origin of the network will be the fundamental point of Voirol site (Algiers)

In the following assumptions taken im consideration to resolve the transformation problem for North Algerian network, the Bursa-Wolf model was less suitable (see the Table 1,2), Molodesky-Badekas gave better results than Bursa because it use the centered reduced coordinates related to the centroids for the reasons cited in the Molodensky Paragraph hereafter.

## ▪ Transformation contest using Molodensky-Badekas, Bursa-Wolf and Vaníček models

**Estimating the NS59 cartesian coordinates**

**Vanicek Model:**

In order to assess the six transformation parameters, we need obviously the set of double points whose coordinates are known in both systems. To obtain the new NS59 cartesian coordinates ($X_{nns59}$ $Y_{nns59}$ $Z_{nns59}$), we apply the following equation:

$$Q_{NS59} = Q_{WGS} - (A_i * P)$$

The vector of the unknown transformation parameters to be determined is *P*. In this study, the least squares approach defined by Equation above, was used to estimate the unknown parameters.

$$Q_{NS} - Q_{WGS} = -M_i R - T$$

$$P = (A^T C_{\Delta Q}{}^{-1} A)^{-1} A^T C_{\Delta Q}{}^{-1} \Delta Q$$

**With : Variance covariance Matrix of the Transformation parameters**

$$C_{\Delta Q} = C_{Q_{WGS}} + C_{Q_{NS59}}$$

**Molodensky Model**

**Use of centroids**

We postulate that the use of a centroid for coordinates transformations was popular for two reasons:

- A centroid reduced the magnitude of coordinates applied in a least squares solution and so reduced the effect of round-off caused either by (a) human geodetic first generations computers or by the limited floating-point precision of the 16- and 32-bit processors available in computers during the 1970s through to the late 20th century when 64-bit processors improved, but did not entirely resolve, the round-off issue,
- A centroid produces statistics for the derived parameters that appear 'better' (i.e., smaller in magnitude) than a method without a centroid due to an arbitrary pretransformation of the source coordinates that is not properly reflected in the functional and stochastic models.

$$\vec{X}_0 \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix} - \begin{bmatrix} \frac{\sum_{i=1}^{n} P_i X_i}{P_i} \\ \frac{\sum_{i=1}^{n} P_i Y_i}{P_i} \\ \frac{\sum_{i=1}^{n} P_i Z_i}{P_i} \end{bmatrix} \quad \text{System n°1}$$

$$\vec{x}_0 \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} - \begin{bmatrix} \frac{\sum_{i=1}^{n} P_i x_i}{P_i} \\ \frac{\sum_{i=1}^{n} P_i y_i}{P_i} \\ \frac{\sum_{i=1}^{n} P_i z_i}{P_i} \end{bmatrix} \quad \text{System n°2}$$

## Results

**Zone of study and Source of WGS points coordinates**

For the validation purpose of this study, a set of twelve benchmarks (points that coordinates are known in both systems) from zero order North Algerian GNSS observed network is used (Figure below). Indeed the results obtained by implementing the 3D Moldensky-Badekas and Bursa-Wolf models, show how much the two methods are almost similar,

on the other hand, Vanicek when used following the three assumptions seems to be better and differs slightly in parameters accuracy (Table 2).

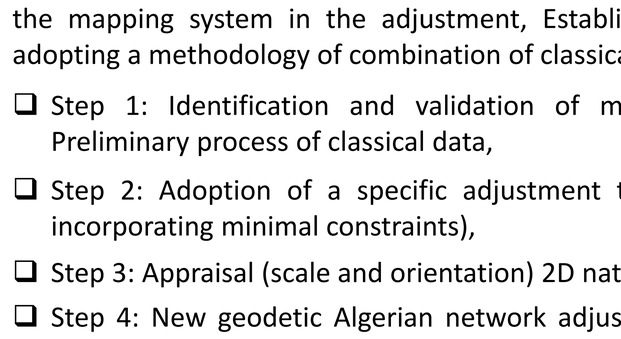


**Table 1.** Computed transformation parameters and their accuracies compared with Algerian parameters (INCT, Institut National de Cartography et de Teledetection) ; case of both Formalisms (MB, BW, Vanicek ).

| Parameters | Bursa-Wolf | Molodensky-Badekas | Vaníček | INCT (Parameters) |
|---|---|---|---|---|
| $T_X(m)$ | 76.0382 ± 0.0256 | 51.815 ± 0.0167 | 181.433 ± 0.145 | 209.363 |
| $T_Y(m)$ | -46.427 ± 0.02470 | -62 ± 0.01210 | -41.523 ± 1.662 | 87.817 |
| $T_Z(m)$ | -114.334 ± 0.0312 | -106 ± 0.03070 | -398.365 ± 0.95 | -404.619 |
| $\delta k(ppm)$ | 1.4344 ± 0.0261 | 1.634 ± 2.16E-03 | - | 1.4547220 |
| $\varepsilon_X(sec)$ | -0.12494 ± 0.066 | -1.40041 ± 0.0482 | -1.42E-06 ± 0.78E-06 | -0.00461215 |
| $\varepsilon_Y(sec)$ | -0.4534 ± 0.1067 | 2.50843 ± 0.130 | -3.25E-05 ± 2.17E-07 | -3.47842207 |
| $\varepsilon_Z(sec)$ | 0.0998 ± 0.0668 | -0.046567 ± 0.050 | 3.65E-06 ± 1.02E-07 | -0.58048472 |

**Table 2.** Shift in coordinates of the network transformed points in Easting and Northing components

| | Vaníček | | Molodensky-Badekas | | Bursa-Wolf | | MRE's | |
|---|---|---|---|---|---|---|---|---|
| Point ID | ΔE | ΔN | ΔE | ΔN | ΔE | ΔN | ΔE | ΔN |
| 1 | 0.15 | 0.75 | 0.38 | 0.82 | 0.41 | 0.78 | -0.24 | 0.38 |
| 2 | 0.31 | 0.51 | 1.02 | 0.64 | -0.88 | 0.67 | -1.24 | 0.58 |
| 3 | 0.61 | -0.72 | 0.96 | -0.81 | 0.91 | 0.83 | 0.54 | 0.52 |
| 4 | 0.12 | -0.15 | 0.35 | 0.25 | 0.44 | 1.02 | -0.71 | 0.46 |
| 5 | 0.25 | 0.14 | -1.20 | 0.65 | 0.96 | 0.55 | 0.47 | -0.28 |
| 6 | -0.14 | -0.44 | 0.96 | -0.34 | -1.23 | 0.52 | 0.67 | 1.15 |
| 7 | -0.36 | -0.24 | 0.39 | 1.24 | 0.84 | 0.76 | 1.22 | 0.82 |
| 8 | -0.58 | 0.78 | -0.48 | 1.02 | -0.61 | 0.91 | -0.43 | 0.63 |
| 9 | -0.29 | -0.25 | 0.32 | -0.65 | -0.39 | 0.56 | 1.32 | 0.68 |
| 10 | 0.63 | 0.53 | 0.74 | 1.53 | 0.85 | 1.22 | -0.53 | 0.18 |
| 11 | 1.06 | 1.05 | 1.10 | 0.95 | 1.25 | -0.93 | -0.36 | 1.05 |
| 12 | 0.62 | 0.55 | 0.58 | 0.64 | 0.78 | 0.75 | 1.01 | 0.53 |

## Conclusion

Overall analysis of Figure1 and 2 indicates that the residuals in Northings were higher than the Eastings. These residuals incurred by the Vanicek Modelclearly depict the limitation in most mathematical models that they could only produce results approximating the existing data. Although the height component was not applied in this transformation, the residuals obtained suggest that the classical models (Molodensky , Bursa Wolf) could not completely absorb the horizontal coordinates distortions mainly contributed by the astro-geodetic network system. Although the use of quietely qccurqte WGS84 coordinates issues from GNSS survey, these factors however, have contributed to the inability of the FV model to notice its potential of providing higher (sub-metre or even sub-centimetre) accuracy even though it is a rigorous Model. That is why it is is imperativ to use more Multi GNSS and Multi frequencies positioning Task double coordinates in order to improve the quality of data processed for Transformation parameters assessment.