

Estimating GNSS satellite antenna phase center offsets and variations simultaneously with flatness constraints

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1. Introduction and background

Accurate information on antenna phase center offsets (PCOs) and phase variations (PVs) of global navigation satellite system (GNSS) satellites is indispensable for high-precision geodetic applications. In the absence of consistent pre-flight calibrations, satellite antenna PCOs and PVs are commonly estimated based on observations of a global network constraining the scale to the International Terrestrial Reference Frame (ITRF) derived from Satellite Laser Ranging (SLR) and Very Long Baseline Interferometry (VLBI).

Given the high correlations between satellite PVs and constant phase biases, a zero-mean condition can be used to separate both effects in an unambiguous manner. Satellite PVs are also indistinguishable from PCOs and are thus usually estimated sequentially in two steps fixing either PCOs or PVs to given a priori values. However, possible differences related to the order of these two steps have not received proper attention so far. In view of the complexity and ambiguity of the two-step method, a lean and generic method is developed to estimate satellite PCOs and PVs **simultaneously** in a **single** processing step.

2. Antenna model

GNSS satellite orbits are determined with respect to the center of mass (CoM). GNSS measurements, on the other hand, refer to the phase center (PC) of the transmitting antenna, which is not a physical point. The total phase center correction

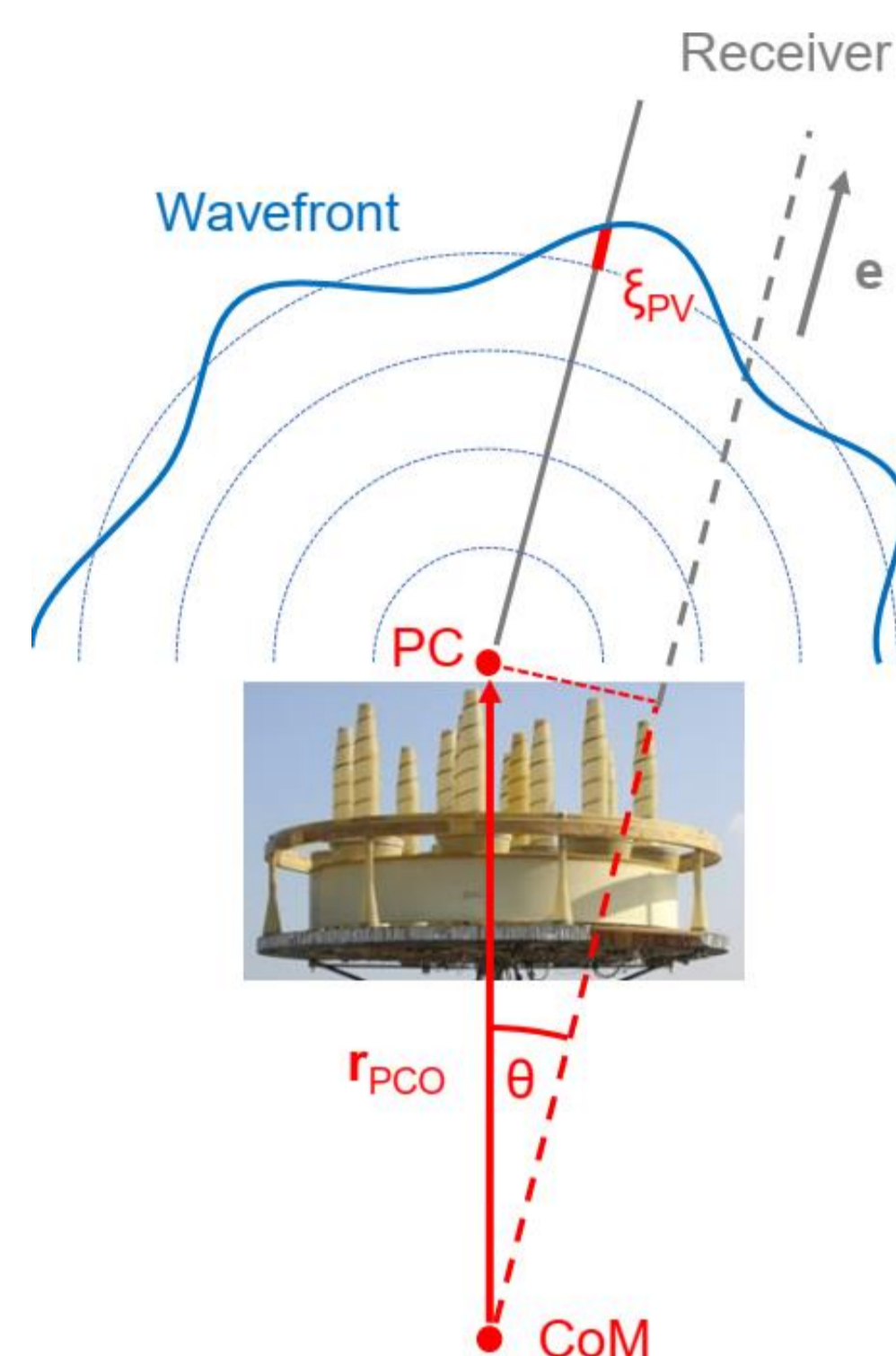


Fig.1 Satellite antenna PCO and PVs

to the geometric distance is partitioned into a PCO contribution r_{PCO} and the supplementary direction dependent phase variation ξ_{PV} , where e denotes the satellite-to-receiver unit vector.

$$\xi_{PCO/PV} = -e^T \cdot r_{PCO} + \xi_{PV} \quad (1)$$

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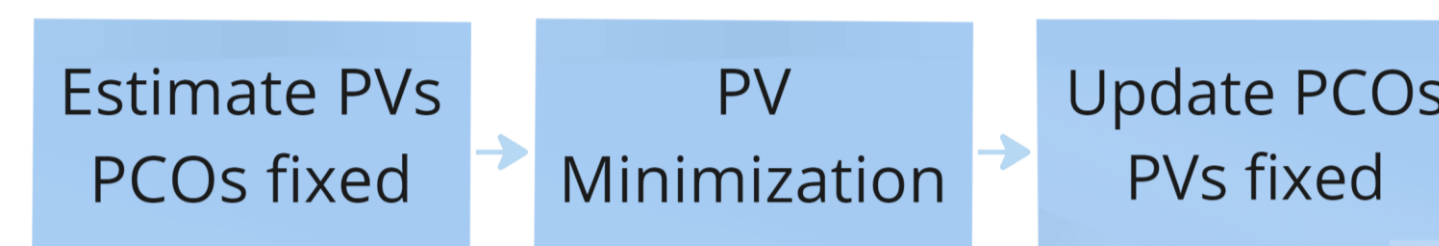
Satellite PVs (ξ_{PV}) are indistinguishable from satellite PCOs and constant phase biases Δb . A change in PCO can thus be compensated by corresponding changes in PVs:

$$\xi_{PV} = -\sin(A) \sin(\theta) \Delta X_{PCO} - \cos(A) \sin(\theta) \Delta Y_{PCO} - \cos(\theta) \Delta Z_{PCO} + \Delta b + \hat{\xi}_{PV} \quad (2)$$

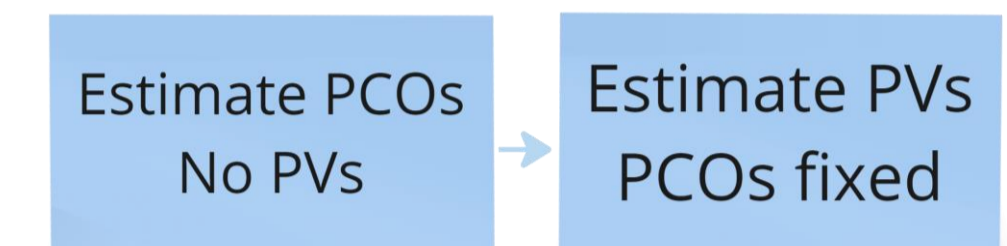
Here, A denotes the azimuth, θ represents the boresight angle, and $\hat{\xi}_{PV}$ denotes the zero-mean and flat PVs. In this work for GNSS satellites, azimuth-dependent satellite PVs are not considered. Thus, only ΔZ_{PCO} is relevant. A piecewise-linear polygon in boresight angle with a sampling of one degree is applied to describe the satellite phase variations.

3. Methodology

(1) Two-step method (PVs - PCOs)



(2) Two-step method (PCOs - PVs)



In (1), block-specific satellite PVs y are first estimated with PCOs fixed to a priori values applying a zero-mean condition with unit weight P of each boresight angle. The flat satellite PVs \hat{y} are then computed by subtracting a cosine representation of ΔZ_{PCO} .

$$\hat{y} = y + \Delta Z_{PCO} \cdot B \quad (3)$$

Here, $B = [-\cos(0) \dots -\cos(\theta_{max})]^T$, $y = [\xi_{PV}(0) \dots \xi_{PV}(\theta_{max})]^T$, $\Delta Z_{PCO} = (B^T P B)^{-1} B^T P y$.

In (2), satellite PCOs are first computed with no PVs, which implicitly assumes that the neglected PVs are flat, weighting individual boresight angle intervals in proportion to the respective number of observations. This corresponds to the condition

$$\Delta Z_{PCO} = (B^T P' B)^{-1} B^T P' y = 0 \quad (4)$$

where the weighting matrix P' reflects the number of observations in each boresight angle.

(3) One-step method

By neglecting the azimuth dependency of satellite PVs, Eq. (2) can be written as

$$\xi_{PV}(0 \dots \theta_{max}) = A x + \hat{\xi}_{PV}(0 \dots \theta_{max}) \quad (5)$$

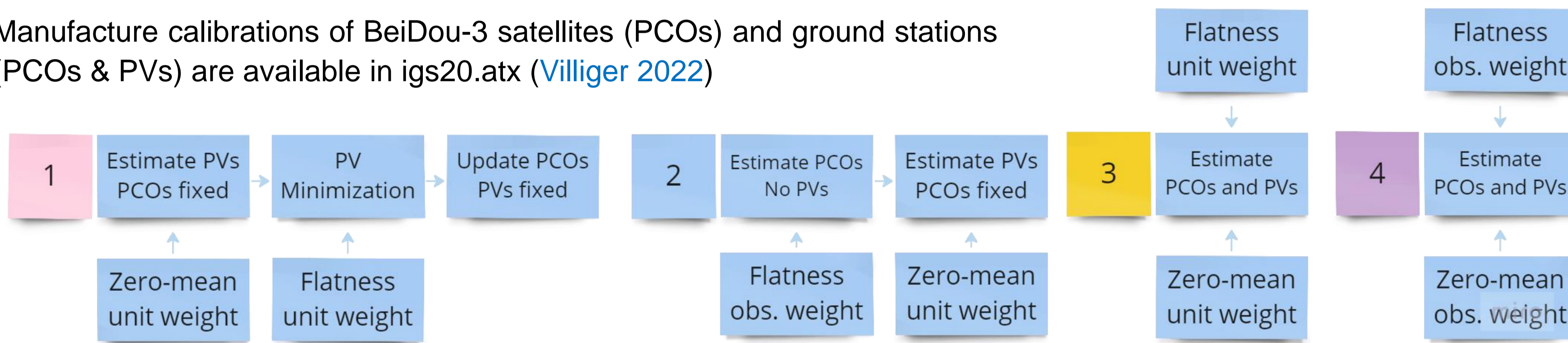
with $A = \begin{bmatrix} -\cos(0) & \dots & -\cos(\theta_{max}) \\ 1 & & 1 \end{bmatrix}^T$, $x = [\Delta Z_{PCO} \Delta b]^T$. As a standard least squares adjustment problem, x can in principle be solved as $x = (A^T P A)^{-1} A^T P \xi_{PV}$ by having $\hat{\xi}_{PV}^T \hat{\xi}_{PV} = \min$.

Zero-mean and flatness conditions are then equated to $(A^T P A)^{-1} A^T P \xi_{PV} = 0$, which can be used as additional fictitious observations to estimate PCOs and PVs simultaneously in a single step.

4. Experiments

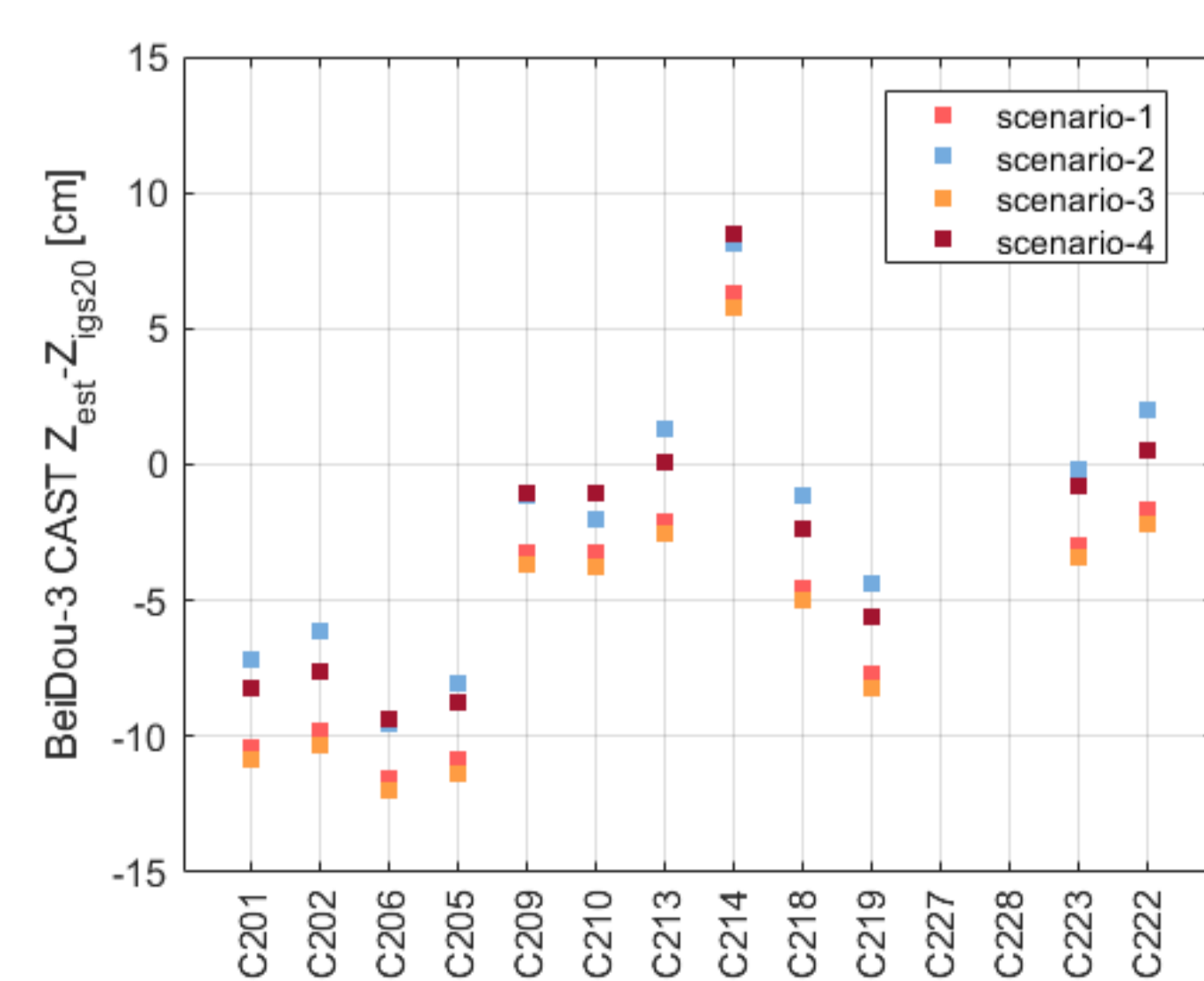
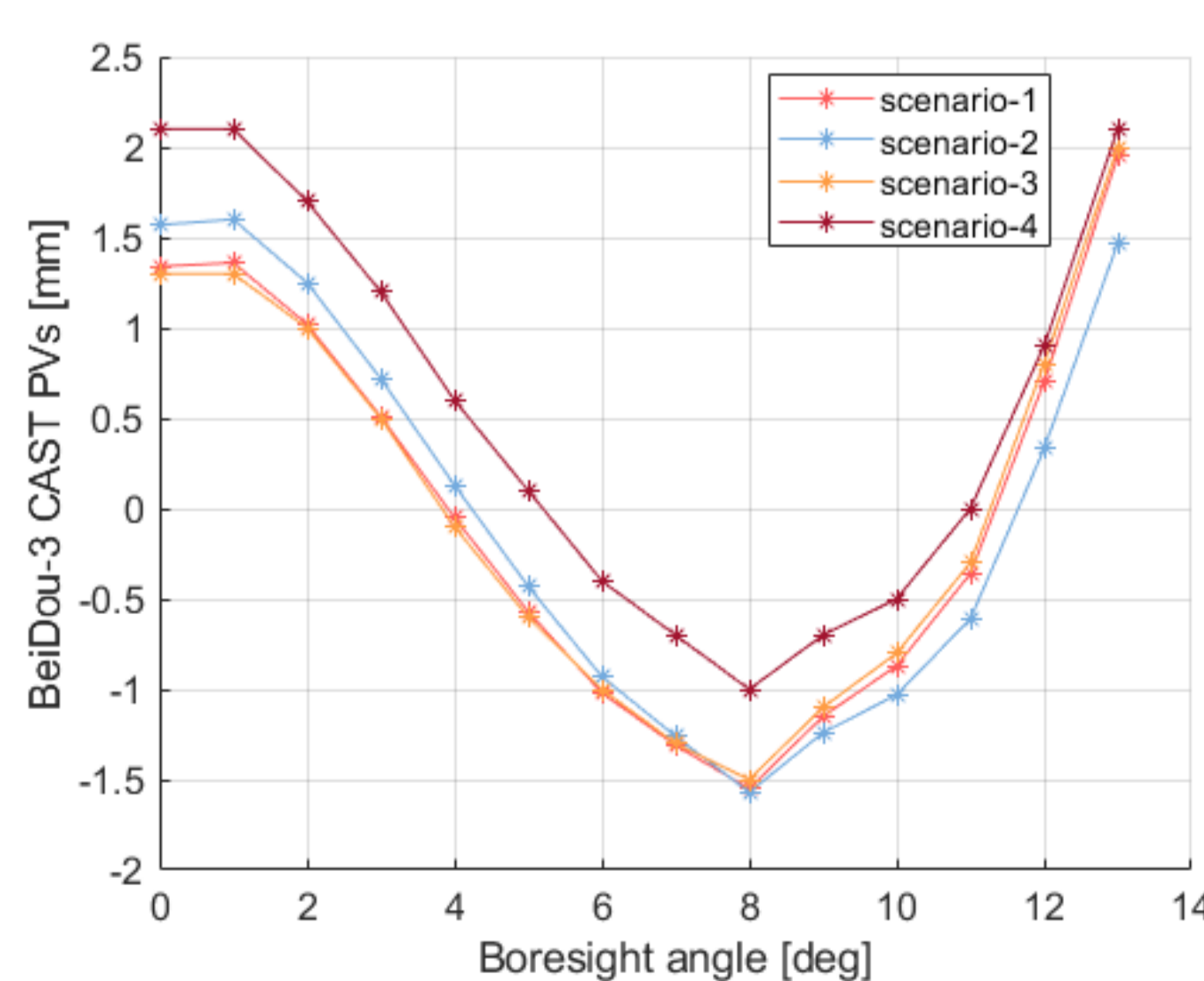
In view of significant offset (+4.3 mm at epoch 2015.0) and drift (+0.11 mm/year) between Galileo-based scale and the ITRF2020 scale derived from SLR and VLBI techniques (Rebischung et al. 2024), the unknown GPS and GLONASS PCOs were estimated and the available calibrations for Galileo and GPS BLOCK IIIA satellites were adjusted to the ITRF scale. Due to the limited full-service time span, BeiDou-3 was not included in the IGS repro3. Therefore, PCO and PV estimations of BeiDou-3 satellites are taken as an example in this work to show the differences between different two-step and one-step methods.

Manufacture calibrations of BeiDou-3 satellites (PCOs) and ground stations (PCOs & PVs) are available in igs20.atx (Villiger 2022)



Settings	Value
Network	130 stations, day 001 to 365 of 2023
Observations	GPS+Galileo+BeiDou-3 (B1C/B2a)
Satellite PCO	Satellite-specific
Satellite PV	Block-type-specific
Processing scenarios	
Scenario-1	Two-step (PVs - PCOs)
Scenario-2	Two-step (PCOs - PVs)
Scenario-3	One-step with unit weight P
Scenario-4	One-step with obs.dep. weight P'

5. Results



The left figure shows the estimated PVs of BeiDou-3 medium Earth orbit (MEO) satellites manufactured by China Academy of Space Technology (CAST). PV results from scenario-1 and 3 are nearly identical. Differences between scenario-1 and 2 can be 0.5 mm due to the use of different weights P and P' in the flatness condition. Scenario-4 results in a shift of about 1 mm compared to results from scenario-1 due to the use of P' in the zero-mean condition.

The right figure shows the corresponding Z-PCO estimations from different scenarios. The differences between results from scenario-1 and 4 are about 3 to 4 cm.

6. Conclusions

Due to the high correlations between satellite PCOs and PVs, individual effects are commonly estimated sequentially in two steps. Results for BeiDou-3 MEO satellites show differences of about 1 mm in PV and 30 mm in PCO when first estimating PCOs compared to results from first estimating PVs. The reason is that first estimating PCOs with no PVs minimizes the root-sum-square observation residuals and is thus equivalent to a flatness constraint weighting individual boresight angle intervals in proportion to the respective number of observations.

The developed one-step method is mathematically the same as the two-step method if the same conditions are used, but it is very lean and computes consistent satellite PCOs/PVs in a more simple way. For precise positioning applications, there is no difference by consistently using satellite products (i.e., PCOs, PVs, phase biases) computed from either of the methods.

Reference

- Rebischung, P., Altamimi, Z., Métivier, L. et al. (2024) Analysis of the IGS contribution to ITRF2020. J Geod 98, 49. [10.1007/s00190-024-01870-1](https://doi.org/10.1007/s00190-024-01870-1)
- Villiger A (2022) [IGS-ACS-1570] Update igs20_2221.atx including BLOCK IIIA x/y-pco update and BEIDOU, QZSS and IRNSS