Clock Models and Interpolation for Precise Point Positioning

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Tour de l'IGS – Stop III
Virtual Presentation

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Presentation Overview

The IGS's clock products are useful for performing Precise Point Positioning (PPP). We will demonstrate the accuracy that can be retained using IGS clocks from several constellations at epochs between available solution points.

Clock Models and Predictions // Mike C

- Models and parameters used for clock characterization.
- URE computation as result of clock noise processes.
- Resultant URE distributions for various vehicles/constellations.

Precise Point Positioning // Urs

- Substitutions of clock types in Rinex.
- Precision attained with interpolated clocks in PPP.

Clock Solutions

How are the IGS Clock Products generated ?

- IGS Analysis centers (ACs) solve for a network of clocks including some IGS tracking station clocks and all satellite clocks.

- Generally, a clock solution for θ_i is derived from equations relating a satellite and receiver position (orbit and clock offset) in

$$\delta t \;=\; c^{-1}\left|\left|\mathbf{p}_s - \mathbf{p}_r\right|\right| + \left(\theta_s - \theta_r\right) + \delta t_{\text{trop}} + \delta t_{\text{rel}} + \ldots$$

- The models and techniques vary among the ACs (a benefit to the IGS!) Each analysis center will produce a clock solution set $\{\theta_i\}$ at rates of

Product	Solution Rate Rate	Combined Precision (STD)
Final Satellite	30 Second	∼20 ps
Final Stations	5 Second	∼20 ps
Rapid	5 Minute	∼25 ps

- The IGS clock combination weights the contributing clock solutions from ACs to obtain the published products at the precision levels noted in this table.

Discrete Clock Solutions

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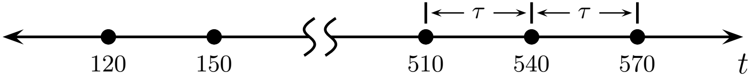

Timeline for a τ =30 second solution.

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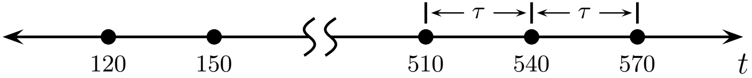

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Or, if we'd like to use clock products in more real time setting, where the latest published clock solutions are at some time in the past?

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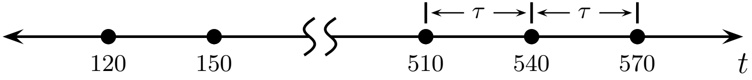

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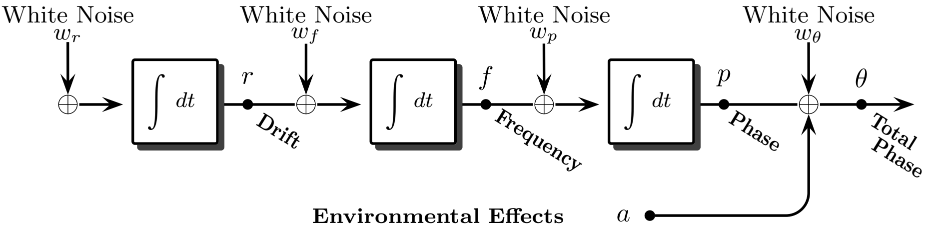
Before we handle these questions, we should quantify how clocks behave ...

Clock States and Noise Processes



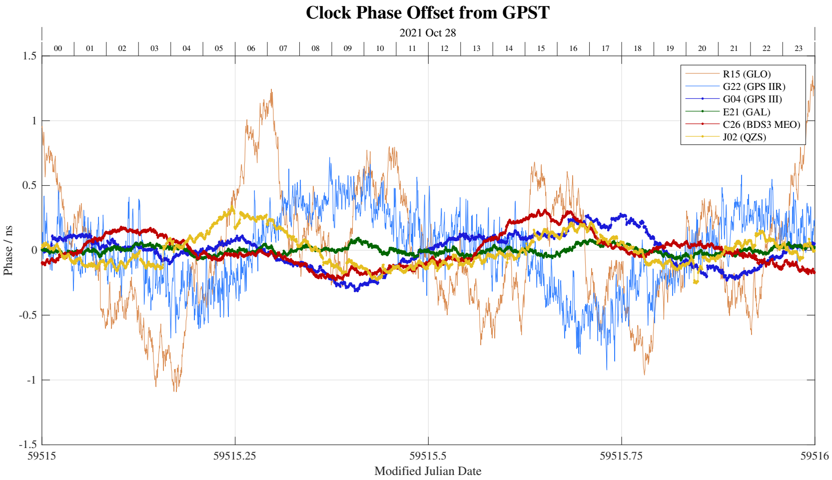
Clock State Vectors

States and white noise processes can be assembled into vectors.

$$\mathbf{x}(t) = \begin{bmatrix} \theta(t) \\ p(t) \\ f(t) \\ r(t) \\ a(t) \end{bmatrix} \begin{matrix} \text{Total Phase} \\ \text{Phase} \\ \text{Frequency} \\ \text{Drift} \\ \text{Environment} \end{matrix} \qquad \begin{matrix} \text{White Phase Noise} \\ \text{Random Walk Phase Noise} \\ \text{Random Walk Frequency Noise} \\ \text{Random Walk Drift Noise} \\ \text{White noise on Env state} \end{matrix} \begin{bmatrix} w_\theta(t) \\ w_p(t) \\ w_f(t) \\ w_r(t) \\ w_a(t) \end{bmatrix} = \mathbf{w}(t)$$

Clock Solution Sample

Clock solutions obtained from CODE (Center for Orbit Determination in Europe) MGEX contribution. Sample is from 28 October 2021.



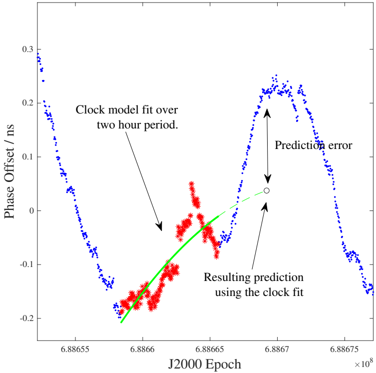
Interpolation vs. Prediction

Prediction

Clock states are propagated from one epoch forward are governed by:

$$\mathbf{x}(t_{k+1}) \;=\; \Phi(\tau, t_k)\mathbf{x}(t_k) \;+\; \int_{T_k} \Phi(t - t_k, t_k)\mathbf{w}(t)\, dt$$

where T_k is the interval from t_k to t_{k+1} and $\Phi(\tau, t_k)$ is the state transition matrix.



Interpolation

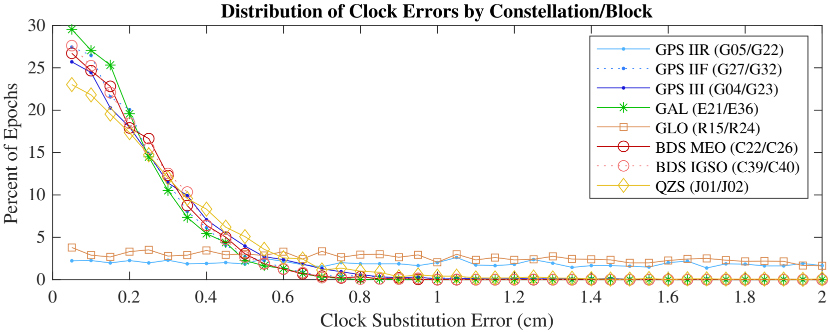
Interpolating clocks is much more accurate as a best fit (even linear) over minutes or even seconds minimizes error. In this case, errors between the true clock interpolated are

$$e(t) \;=\; \left|\theta(t) - L_k(t)\right|$$

for $t \in [t_k, t_{k+1}]$ and where

$\theta(t_k) = L_k(t_k)$ and $\theta(t_{k+1}) = L_k(t_{k+1})$.

Interpolation Errors – 5 Minute Interpolation Intervals

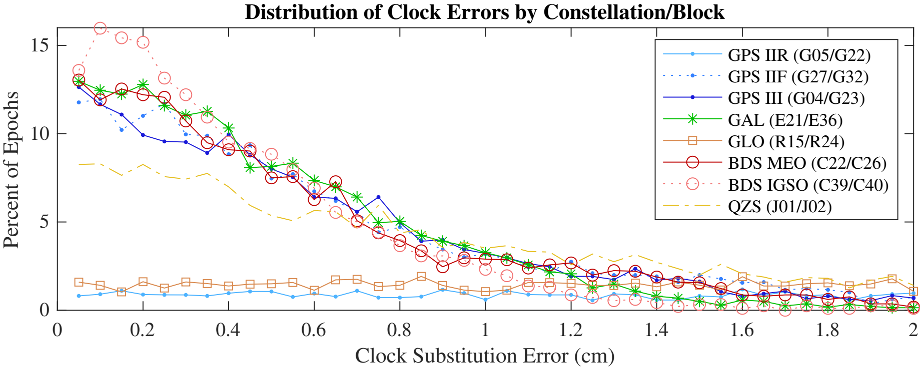


- Clock interpolation errors for various clock types that are onboard our GNSS vehicles.
- Recent satellites have either passive masers or Rb frequency standards so that clock stability is excellent.
- For a five minute interpolation, errors are due (mostly) to short term noise.

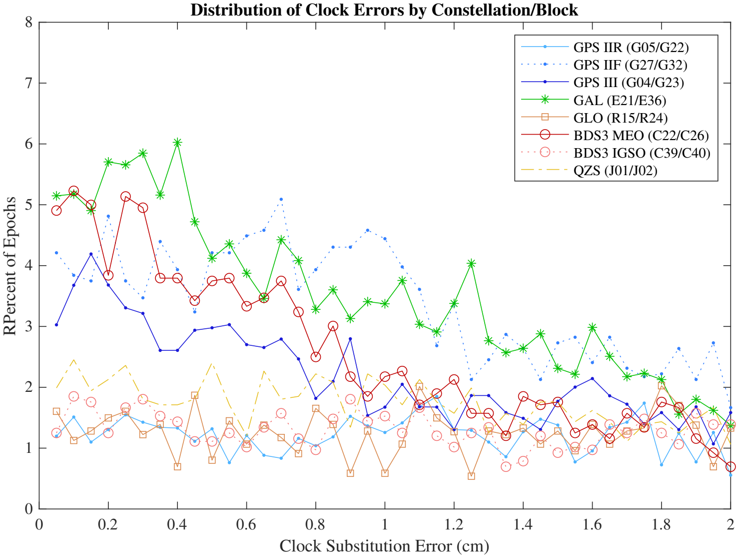
Based on CODE (Center for Orbit Determination in Europe) MGEX clock solution from 28 October 2021.

Constellation or Block	Clock Interp. Error / cm 68%	95%	99%
GPS IIR	3.7450	8.1716	13.3565
GLO	2.4975	5.7056	9.1416
Others	0.2556	0.5457	1.3381

Interpolation Errors – 1 Hour Interpolation Intervals

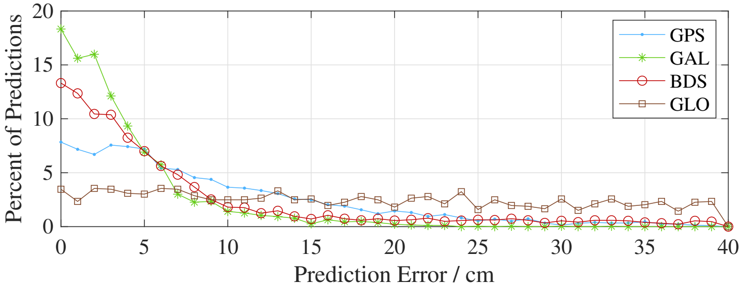


Interpolation Errors – 6 Hour Interpolation Intervals



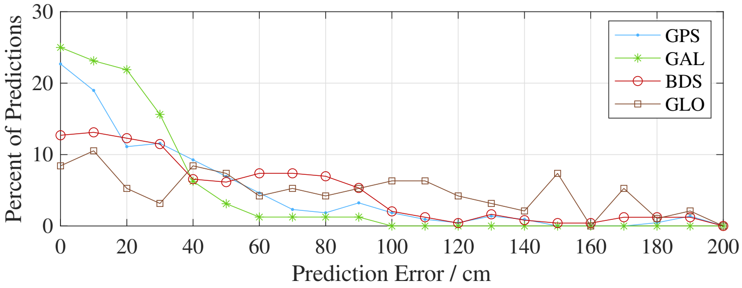
Prediction Errors – 1 Hour

- These histograms depict the errors in a propagating the clock solution and comparing to the known solution.
- Each prediction is generated by using a polynomial fit over a 2 hour window and propagating 1 hour further to epoch t_*.
- Error computed by comparing prediction to actual clock solution at t_*.



Prediction Errors – 12 Hours

- These histograms depict the errors in a propagating the clock solution and comparing to the known solution.
- Each prediction is generated by using a polynomial fit over a 2 hour window and propagating 12 hour further to epoch t_*.
- Error computed by comparing prediction to actual clock solution at t_*.



Prediction Errors – 24 Hours

- These histograms depict the errors in a propagating the clock solution and comparing to the known solution.
- Each prediction is generated by using a polynomial fit over a 2 hour window and propagating 1 day further to epoch t_*.
- Error computed by comparing prediction to actual clock solution at t_*.