Observation equation and analysis strategies

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Tour de l'IGS 3rd Stop: GNSS processing based on IGS products
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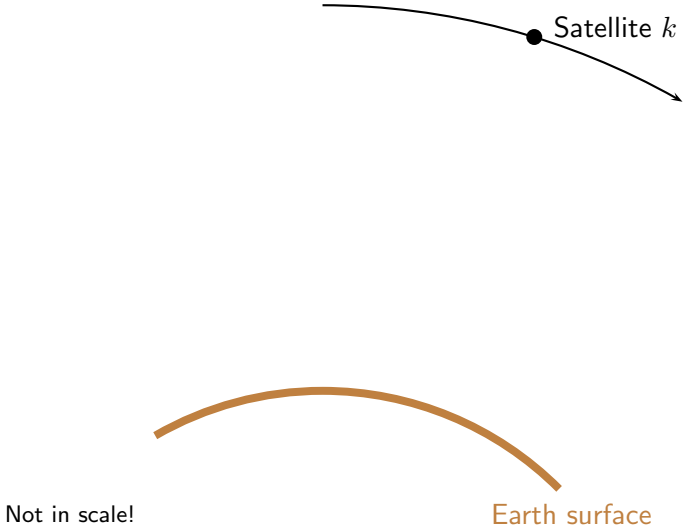
Overview

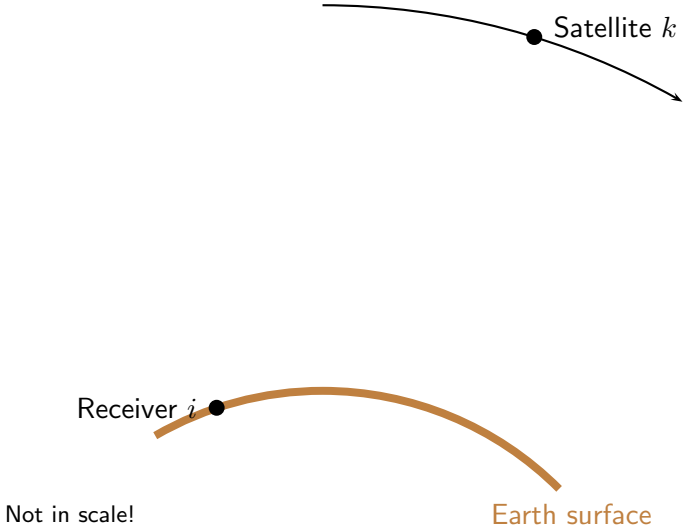
Introducing the observation equation

Principles of GNSS-Data Analysis

Introducing the observation equation



Introducing the observation equation



Introducing the observation equation

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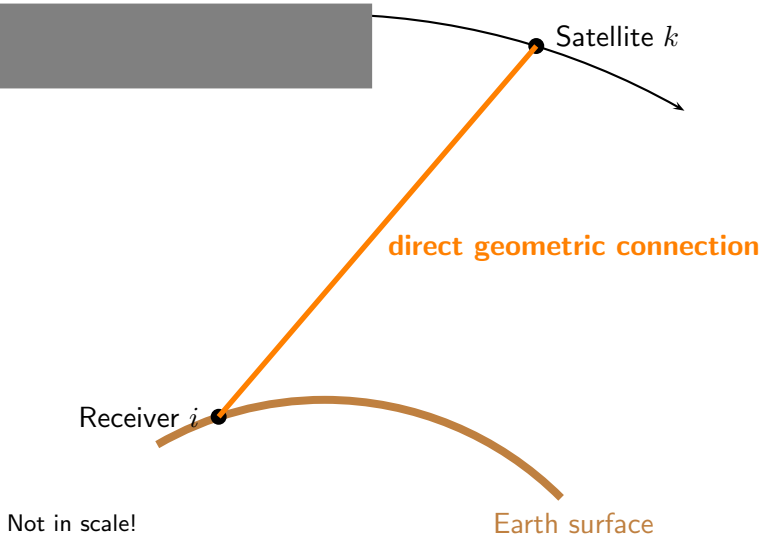
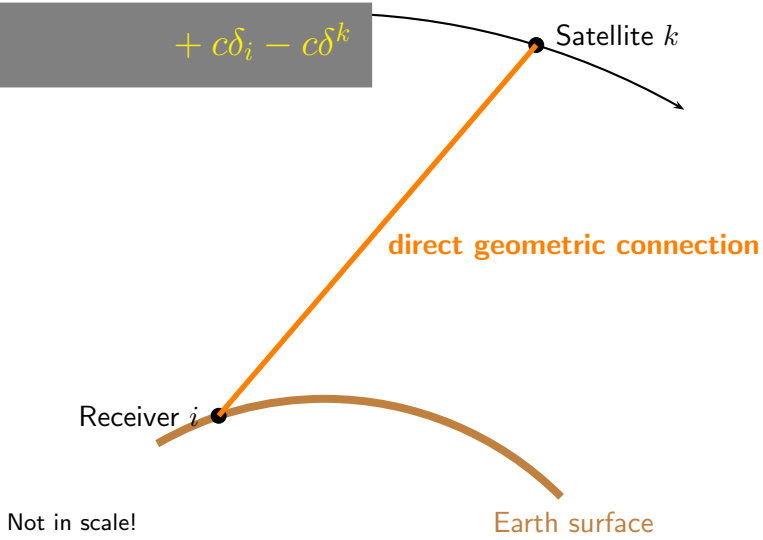


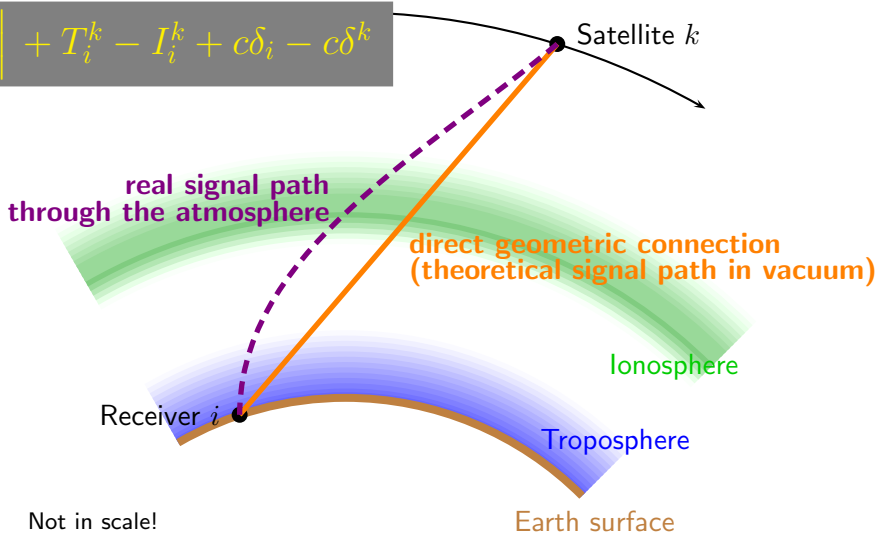
Introducing the observation equation

$$P_i^k = \left|\vec{x^k} - \vec{x_i}\right| + c\delta_i - c\delta^k$$



Introducing the observation equation

$$P_i^k = \left|\vec{x^k} - \vec{x_i}\right| + T_i^k - I_i^k + c\delta_i - c\delta^k$$



GNSS Observation Equations

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P_i^k, L_i^k	code/phase observation of station i to satellite k
$\vec{x}_i, \vec{x}^k$	position vector of station i and satellite k, respectively
T_i^k	signal delay in the troposphere
I_i^k	signal delay in the ionosphere
δ_i, δ^k	clock correction of the receiver at the station i, and transmitter of satellite k with respect to GPS time
c	speed of light
N_i^k	phase ambiguity (one and the same for one pass)
λ	wavelength of the carrier phase

Remarks on GNSS Observation Equations

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- All terms of the observation equation are (more or less) time dependent.

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- Since we are creating the vector difference between the satellite and receiver position both need to be in the same reference frame realization.
- Datum definition for regional or local applications is necessary anyhow – Sonia Costa: Processing a regional/continental dataset

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  Arturo Villiger: Antenna calibrations

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- It is beneficial to recover the integer nature of the ambiguity term N_i^k.

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  Michael Coleman & Urs Hugentobler: Clock parameters
  Stefan Schaer: Bias and ambiguity resolution

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- The ionosphere is for GNSS signals a dispersive medium why the influence can be canceled by forming a linear combination from dual-frequency signals (and correcting for higher order terms).

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- The delay in the troposphere needs a proper modelling.
  Johannes Böhm: Troposphere modelling

GNSS Network Solution

We have the GNSS observations of several stations to some satellites:

$$L_i^k = \left|\vec{x^k} - \vec{x_i}\right| + T_i^k + c\delta_i - c\delta^k + \lambda N_i^k \qquad L_i^l = \left|\vec{x^l} - \vec{x_i}\right| + T_i^l + c\delta_i - c\delta^l + \lambda N_i^l \qquad \ldots$$

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... we may form differences between observations to cancel out the clock parameters:

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$$L_{ij}^{kl} = \left|\vec{x^k} - \vec{x_i}\right| - \left|\vec{x^k} - \vec{x_j}\right| - \left|\vec{x^l} - \vec{x_i}\right| + \left|\vec{x^l} - \vec{x_j}\right| + T_{ij}^{kl} + \lambda N_{ij}^{kl}$$

GNSS Network Solution

Conclusions:

- A consequent creation of (artificial) double-difference observations is equivalent to pre-eliminating the clock parameters on normal equation level.

GNSS Network Solution

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- A consequent creation of (artificial) double-difference observations is equivalent to pre-eliminating the clock parameters on normal equation level.
- When using the same original observations, we obtain the same estimates for the geometry-relatd parameters on zero-, single- or double difference level (given that all existing correlations are considered).

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- Effects that cancel out when differencing the observations are absorbed by the satellite clock parameters in the zero-difference approach.
- The ambiguity resolution is directly possible only on double-difference level (otherwise some bias parameters are needed).

PPP – Precise Point Positioning

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GNSS observation equations for a large number of stations

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Supporting graphic

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- That's why the observation equations for a PPP processing have to be fully consistent to the related network solution.
  Any inconsistency will degrade your PPP results.

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- PPP-based station coordinates join the datum realization of the original network solution.

THANK YOU

for your attention

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