Position Paper
GNSS Orbit Model Considerations for IGS Repro3

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with moral support from
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Abstract

As discussed and initiated at the IGS workshop in Wuhan, China in November 2018 we have made an analysis of the IGS final orbit products to investigate what kind of systematic effects are visible between the different AC orbits. This investigation should lead to some recommendations to the ACs for the modeling of the orbits for the third reprocessing and of course also the IGS routine processing. Besides comparing the different IGS solutions the investigations also made use of solutions simulating certain modeling approaches using a homogeneous data set and the same software so that the differences between the solutions are purely caused by the difference introduced in the processing. These test solutions have assisted in understanding which models work for which satellites. Based on all this we have come up with a set of conclusions and a limited set of recommendation which are meant as base for discussion at the workshop.

1 Disclaimer

My original intention was to have this paper ready two weeks before the workshop so we could have some time for discussion and iteration. In particular with the co-authors I had planned/selected for this work. Unfortunately I did not find enough time to come up with a draft and consequently my co-authors did not have to time to contribute to this work. Nevertheless, I have kept them in the author list as without the efforts done at CODE and JPL we would not have the diversity in the products which were direly needed for the investigations presented here. So the inclusion of Rolf and Ant is to be seen as a general recognition of the very significant work that has taken place at CODE and JPL in the field or GNSS orbit modeling.

As you will notice when reading this paper this is still very much a draft version. Also there are many many more figures which are not included in the paper. At the workshop I will circle an USB stick with the complete material of the investigations so that everybody can get all the info I have and in particular study the results of your own AC!

2 Introduction

As discussed and initiated at the IGS workshop in Wuhan, China in November 2018 we have made an analysis of the IGS final orbit products to investigate what kind of systematic effects are visible between the different AC orbits. This investigation should lead to some recommendations to the ACs for the modeling of the orbits for the third reprocessing and or course also the IGS routine processing. In this position paper we present two in depth investigations into the IGS products. The first investigation using recent data (2017/2018) focusing on the IIF satellites. Results of this were distributed in December 2018 by e-mail. A second investigations was made using data from the year 2014 to also review the modeling of the GPS block II, IIA, and IIR satellites. The results of both investigations are presented in this position paper and will be presented at the IGS AC Workshop on April 15-17 in Potsdam, Germany. Besides comparing the different IGS solutions the investigations also made use of solutions simulating certain modeling approaches using a homogeneous data set and the same software so that the differences between the solutions are purely caused by the difference introduced in the processing. These test solutions have assisted in understanding which models work for which satellites. Based on all this we have come up with a set of conclusions and a limited set of recommendation which are purely meant as base for discussion at the workshop.

2.1 Background info

In this position paper:

- If we talk about ECOM approach this is basically the 5-parameter approach: D0, Y0, B0, BC, BS. However, some ACs also estimate the full set of 9 parameters but in that case with some constraints on some of the parameters. Some ACs also allow for small velocity changes in addition to the ECOM model (e.g. COD and GFZ)
- If we talk about ECOM2 approach this is the new 7-parameter approach: D0, DC2, DS2, Y0, B0, BC, BS
- I may not have been consistent in using the term JPL model or GSPM model. Both terms refer to the same model. Only for the 2017 tests we were not ready with our JPL GSPM implementation in NAPEOS and thus used only the terms similar to the ROCK model to "simulate" the JPL model. With the ROCK model terms we mean all the cosine

and sine terms in the X and Z direction as function of the α angle (angle between Earth-Satellite and the Satellite-Sun vectors).

Some citations I should have used in this paper:

- ECOM and ECOM2 citations [3, 7, 1]
- GSPM model citation [2]
- NAPEOS software citation [8]
- IGS citation [4], to be updated to the latest IGS citation.

AC orbit model basics

To be able to interpret the results it is important to understand the orbit models employed by the different ACs. Before we discuss the observed differences I would like to point out that we have a couple of different orbit modeling approaches of the different IGS ACs, namely:

- COD: ECOM2 approach (but in 2014 still ECOM)
- EMR and JPL: JPL GSPM model, also using the same software
- ESA: Box-wing model
- GFZ, MIT, NGS, SIO: ECOM model (with small differences like "pulses" and all 9 instead of 5 parameters). Also MIT and SIO are using the same software but the results seem to be significantly different
- GRGS: 1 scale of solar pressure force (scaling of box-wing model?), 1 Y-bias, once per revolution terms in the two directions perpendicular to Y. An additional set of 3 empirical accelerations is estimated for each satellite crossing the Earth shadow

Here I must say that the GRGS model description (taken from their web site and quoted reference paper) poses several questions Perpendicular to Y-axis? Can be many but most likely is either X- and Z-axis (body-fixed) or D- and B-axis (Sun-Sat related), which is it?
Solar scale. What is scaled here? The box-wing model? thought that was turned off? If not what value do you use there?

Solution basics

All solutions covered either the full year of 2014 or 2017 and used exactly the same input data by doing the RINEX pre-processing step only for the first solution and (re)using the same "raw" data for all consecutive solutions. So each solution is completely independent except that the starting data is the same. Between the solutions only the orbit model changes. I.e. box-wing model as apriori versus the JPL GSPM model as apriori, or no model as apriori. Only in the case of ECOM2 the estimated parameters are changed to include the DC2 and DS2 terms.

Plot basics

The orbit difference plots you will find in this paper all use the X-axis, Y-axis and color coding scheme and also the range of the scales is identical for all plots. Typically these are three dimensional plots where third dimension, the orbit differences, are color coded. The X-axis is always the argument of latitude of the satellite w.r.t. the Sun ($\mu - \mu_{Sun}$) which is the argument used in the cosine and sine terms of the ECOM models. On the Y-axis is always the elevation of the Sun above the orbital plane, the β angle. The orbit differences are computed for a fine grid of boxes computing the average differences in each box and the size of the differences is then color coded. For all plots the same scale was used of +-30mm.

Kurtosis explanation

When performing integer ambiguity resolution we always make a histogram of the fraction parts of the unresolved double difference ambiguities. If the float solution was accurate it may be expected that this histogram represents a nice normal distribution. From a normal distribution one can compute the so-called "kurtosis" which is a measure of the steepness of the normal distribution with a higher value of the kurtosis representing a steeper normal distribution which means that more of the fractional parts are closer to zero. So if, for whatever reason, our float solution improves in quality, it is reasonable to expect the kurtosis to improve as well. In particular since in our case the kurtosis is computed based on all the double difference ambiguities on all the baselines up to 6000 km. Even if the ambiguity resolution process will typically prefer, and thus select, shorter baselines for the actual ambiguity resolution we do evaluate all baselines. And the longer the baseline the more sensitive it is to the orbital errors. An old GPS "rule of thumb" says that:

$$dx = dR \cdot \frac{L}{4\ R} \tag{1}$$

Where dx is the baseline component error, dR the orbit error, and R and L the orbit height and the baseline length respectively. The factor 4 is an empirical parameter which was derived from experience and quoted by several authors. If we apply this to our case where we are looking at orbit differences, and hence orbit errors, at the 50 mm level we may expect effects of around 4 mm on the 6000 km baselines we consider in our integer ambiguity resolution. With the virtual wavelength of the ionosphere free linear ambiguities being merely 100 mm this is about 8% of half a cycle, which is the maximum size of our fractional parts. It is therefore clear that orbit differences at the tens of mm's should lead to changes in the kurtosis of the normal distribution of our float ambiguities. We are therefore convinced that an improvement of the orbit should manifest itself in a higher value of the kurtosis.

3 IGS Orbit Differences for 2018

As discussed at the IGS workshop in Wuhan, China in November 2018 we have made an analysis of the IGS final orbit products to investigate what kind of systematic effects are visible between the different AC orbits. For this purpose we used the IGS final orbits from GPS week 1970 (October 8, 2017) until 2024 (October 27, 2018), a little bit more then a year. First we compared all the orbits to the IGS final orbits. But since the IGS orbits are a nice "average" of the orbits the systematic differences between the AC orbits show up a bit "smoothed". So we did a second and comparison taking the JPL final orbits as reference. And since we have two, rather different, satellite types we made these comparisons once for the IIR and once for the IIF satellites. In these comparisons we have looked in particular at the radial and cross-track differences. The along-track differences are much less interesting as they do not show much systematic differences.

In Table 1 and 2 you find some simple statistics (units are mm) of these comparisons which are not all that interesting, just provided for completeness. In these statistics there is one "extra" group for "All GPS" satellites as besides 19 IIR and 12 IIF satellites there were still 2 IIA satellites active in this period. In making these statistics, and also when making the plots, a mean is subtracted for every AC and a 5*Sigma outlier criteria is used to remove outliers (number of outliers removed are given in the #Rej column).

There is one thing which the table shows very clearly and that it that the orbits of the IIF satellites show a significantly lower sigma than the orbits of the IIR satellites. This indicate that we may expect to see larger systematic differences for the IIR satellites than for the IIF satellites. A somewhat unexpected but pleasant surprise as it means we have less issues with the IIF then expected!

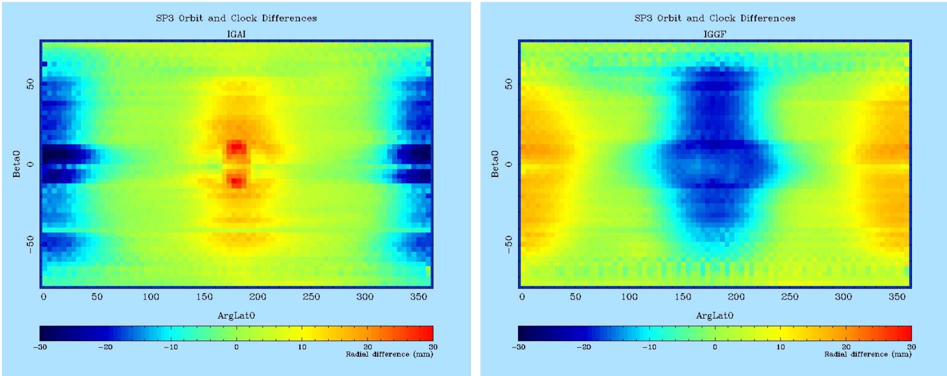
More interesting are the plots of the orbit differences. The full set of plots of these comparisons are available digitally on request (will have an USB stick for sharing at the IGS AC WS in Potsdam) but they were distributed by e-mail already. The plots show for each AC the radial and cross-track differences as a function of the argument of latitude w.r.t. the Sun (on the X-axis) and the elevation of the Sun above the orbital plane, the beta-angle (on the Y-axis). The differences are computed for a fine grid of boxes

Table 1: AC Orbit Comparisons Statistics (mm) over the time-frame of GPS week 1970 to 2024) versus IGS Final Orbits

	Radial				**Cross-Track**				
igs-	#obs	#rej	Sigma	Mean	#obs	#rej	Sigma	Mean	
igr	1145137	1282	12.3	-0.0	1145950	469	10.0	0.1	All GPS (33)
cof	1145137	1282	12.3	-0.0	1145950	469	10.0	0.1	
emr	1132464	2640	16.2	3.8	1134481	623	13.3	1.3	
esa	1136526	296	10.7	-0.4	1136724	98	10.6	-0.8	
gfz	1138616	322	11.0	2.1	1138694	244	11.9	-0.7	
grg	1133353	1559	16.2	-5.0	1134578	334	14.4	1.6	
jpl	1136021	427	12.3	5.1	1136049	399	11.5	1.3	
mit	1137498	1350	12.7	-1.1	1138791	57	13.1	-0.0	
ngs	1136280	456	9.0	0.2	1136377	359	11.4	0.4	
sio	1130542	146	15.6	-2.5	1130589	99	11.8	-0.8	
igr	675047	595	4.4	0.6	675460	182	5.7	-0.9	GPS IIR (19)
cof	675326	320	13.4	0.8	675533	113	10.5	0.2	
emr	671737	1319	18.2	3.0	672991	65	14.5	1.2	
esa	674036	76	11.1	-1.3	674112	0	10.7	-1.2	
gfz	673778	40	13.0	4.1	673815	3	13.5	-0.7	
grg	671873	511	17.0	-4.2	672323	61	14.8	1.6	
jpl	672924	36	13.6	4.5	672874	86	11.9	1.2	
mit	673227	885	15.4	-1.1	674112	0	14.4	0.2	
ngs	672546	30	10.4	1.5	672576	0	12.2	0.8	
sio	669883	5	18.8	-6.6	669848	40	11.9	-1.2	
igr	442949	560	3.6	-1.4	443309	200	4.7	-0.8	GPS IIF (12)
cof	442691	818	10.5	-1.3	443297	212	9.1	-0.0	
emr	439211	1237	12.8	5.3	439896	552	11.2	1.3	
esa	442879	151	10.1	0.8	443002	28	10.3	-0.2	
gfz	442601	247	7.4	-0.6	442680	168	8.9	-0.8	
grg	440401	1007	15.3	-6.4	441135	273	13.8	1.3	
jpl	439981	275	9.9	6.1	440065	191	10.9	1.2	
mit	442479	465	7.0	-1.0	442887	57	10.9	-0.3	
ngs	441945	423	6.3	-1.4	442017	351	10.1	0.1	
sio	439443	141	8.9	3.2	439525	59	11.6	-0.2	

Table 2: AC Orbit Comparisons Statistics (mm) over the time-frame of GPS week 1970 to 2024) versus JPL Final Orbits

	Radial				**Cross-Track**				
jpl-	#obs	#rej	Sigma	Mean	#obs	#rej	Sigma	Mean	
igs	1136021	427	12.3	-5.1	1136049	399	11.5	-1.3	All GPS (33)
cof	1135666	782	14.7	-5.2	1136251	197	14.7	-1.1	
emr	1126603	5141	11.1	-1.2	1131018	726	12.7	0.0	
esa	1132222	386	14.0	-5.6	1132334	274	13.8	-2.2	
gfz	1134599	313	20.0	-3.0	1134665	247	19.2	-2.1	
grg	1130792	856	17.9	-10.2	1131420	228	17.4	0.2	
mit	1134362	358	21.6	-6.3	1134648	72	20.5	-1.3	
ngs	1133395	269	18.2	-4.8	1133378	286	18.1	-0.8	
sio	1127522	190	23.8	-7.7	1127594	118	17.1	-2.1	
igs	672924	36	13.6	-4.5	672874	86	11.9	-1.2	GPS IIR (19)
cof	672535	425	13.5	-3.7	672884	76	14.6	-1.0	
emr	669281	2911	11.9	-1.4	672011	181	13.3	-0.0	
esa	672795	69	13.3	-5.9	672788	76	12.8	-2.5	
gfz	672649	23	23.6	-0.4	672660	12	21.5	-2.0	
grg	671284	428	16.2	-8.8	671675	37	17.1	0.3	
mit	672706	158	26.2	-5.8	672851	13	22.5	-1.0	
ngs	672072	24	21.5	-3.0	672081	15	19.6	-0.4	
sio	669312	0	28.8	-11.1	669286	26	17.4	-2.4	
igs	439981	275	9.9	-6.1	440065	191	10.9	-1.2	GPS IIF (12)
cof	439903	353	16.3	-7.5	440135	121	14.9	-1.3	
emr	436053	2091	10.0	-0.7	437604	540	11.9	0.0	
esa	440068	188	14.6	-5.3	440182	74	15.1	-1.5	
gfz	439954	206	12.8	-6.8	440060	100	15.1	-2.1	
grg	438594	414	20.3	-12.6	438817	191	17.7	0.0	
mit	440056	200	12.1	-7.3	440197	59	17.0	-1.6	
ngs	439732	236	12.0	-7.6	439697	271	15.6	-1.1	
sio	437186	190	13.5	-2.9	437288	88	16.4	-1.4	



(a) Group 1: IGS vs COD

(b) Group 2: IGS vs GFZ

Figure 1: GPS IIR Radial Orbit Differences (mm)

computing the average difference in each box and the size of the differences is then color coded. For all plots the same scale was used of +-30mm. There is a number of interesting things we can see and learn from these plots.

3.1 GPS Block IIR Orbit Differences

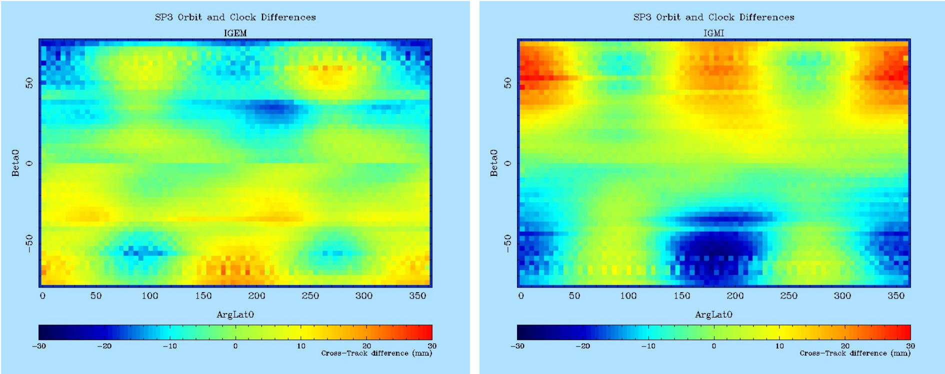
From the IGS radial orbit difference plots we can clearly distinguish two different groups of results

- Group 1: COD EMR ESA JPL
- Group 2: GFZ MIT NGS SIO
- Results of GRG do not seem to fit with any of the groups (but closer to group 1).

The JPL radial orbit difference plots confirm this as we basically find:

- Group 1: COD EMR ESA differences versus JPL very flat
- Group 2: GFZ MIT NGS SIO very clear systematic differences. IGS also showing similar but smaller pattern.

In the IGS cross-track difference plots we can see the same two groups as for the radial differences. Except that the SIO results show very limited systematics versus IGS. In the JPL plots we can see the same two groups as for the radial differences. Both the IGS and JPL differences show a similar for the GFZ MIT NGS SIO group but the differences versus JPL are significantly larger. Figures 1 and 2 give an example of the two observed orbit differences versus the IGS orbits.



(a) Group 1: IGS vs EMR (b) Group 2: IGS vs MIT

Figure 2: GPS IIR Cross-track Orbit Differences (mm)

3.2 GPS Block IIF Orbit Differences

From the IGS radial orbit difference plots we can observe only one clear group: GFZ MIT NGS SIO. Furthermore EMR and JPL show very similar systematics. This is most likely caused by the fact that they are using the same orbit model, the JPL GSPM13 model, and the same software. The results from COD, ESA, and GRG seem to be rather different from each other and also from all the others. This is not really surprising as all three use significantly different orbit models. Interestingly this did not show up very clearly in the IIR results but it does show up in the IIF results.

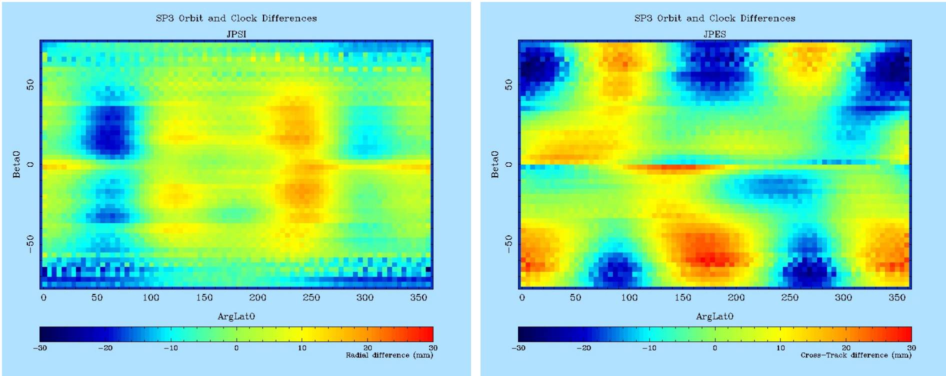
The JPL radial orbit difference plots confirm this as we basically again can identify one group: GFZ MIT NGS SIO IGS. In this comparison it also becomes clear that JPL and EMR are in very good agreement but different from the group. COD, ESA, and GRG are again very different but some similarity between COD and GRG.

In the IGS cross-track orbit difference plots the differences are all very small except for the ESA results. In the JPL plots we can see a small but potentially similar pattern for all, except EMR, but most pronounced for ESA. Figures 3 shows an example of the radial differences for the group and the cross-track differences of the ESA results.

3.3 Evaluation of the results

The block IIR results are completely in line with what may be expected based on the assumption that COD, EMR, ESA and JPL do some kind of enhanced RPR modelling whilst the others do not, with:

- COD: ECOM-2 approach
- ESA: box-wing model



(a) Group 1: JPL vs SIO radial

(b) JPL vs ESA cross-track

Figure 3: GPS IIF Orbit Differences (mm)

- JPL and EMR: using JPL GSPM model
- Others: no model, mostly 5 parameter ECOM with "pulses" or 9 parameter ECOM with constraints

COD, EMR, ESA, and JPL have selected their enhanced modeling based on finding that it improved their results. Given that the three very different approaches seem to lead to rather similar results it seems prudent to advice all other ACs to follow suit and implement one of the three approaches for the block IIR satellites.

However, the block IIF results are not in line with what was to be expected. Firstly, and very positively but also a bit surprisingly, it seems the agreement between the ACs is better for the IIF then for the IIR. The lower sigma values of the orbit comparisons shows this quite clearly. But, the very different results between COD, ESA, and JPL (+EMR) do not make a strong case for a IIF model, or at least not for the approaches of COD and ESA. ESA has been mentioning to have issues with modeling the IIF satellites and these results clearly show this. Nevertheless it is surprising to see that for the IIR satellites the ECOM2, box-wing, and the JPL model give rather similar results whereas for the IIF satellites there are significant differences. So from these comparisons it is a bit hard to make a clear recommendation for the IIF satellites for repro3 at present. But clearly the radiation pressure mis-modelling is less of an issue for the IIF satellites then it is for the IIR satellites. So the ECOM approach seems to be valid for the IIF satellites.

3.4 IIF Investigation

To understand the differences observed for the GPS IIF satellites a bit better five tests were done using the ESA NAPEOS software and using as base setup the ESA IGS analysis strategy. The full year of 2017 was used and both GPS and GLONASS. With these five tests we have tried to mimic the COD, ESA, and JPL processing setup with one and the same software so we can evaluate if the differences we observed are indeed coming from the orbit model differences or if they are related to other software and/or processing differences. So the five test we did were:

1. efa (BW): Same as the ESA IGS routine solution

2. efb (BW+ECOM2): As test 1 (efa) but added D2 terms (cosine and sine) of ECOM2 model for all satellites

3. efc (ECOM): As test 1 (efa) but for IIF turned off the box-wing model (by setting the areas to zero). Should be similar to the solutions of the group GFZ MIT NGS SIO

4. efd (ECOM2): As test 3 (efc) but but added D2 terms (cosine and sine) of ECOM2 model for all satellites. Should be similar to the COD solutions

5. efe (JPL ROCK): As test 1 (efa) but replaced box/wing model for IIF with the (partial) JPL model. As JPL model was not yet fully implemented we used the part that is similar to the ROCK models. Turned off Earth Albedo and IR for all GPS (assuming the IIF JPL model contains this implicitly, but this was a wrong assumption). This should give results similar to JPL and EMR.

From the differences between test 1 and test 2 (efa vs efb) as well as from the differences between test 3 and test 4 (efc vs efd) we should learn the effect of the D2 terms. And the differences between test 1 and test 3 (efa vs efc) as well as the differences between test 2 and test 4 (efb vs efd) should show the effect of the box-wing model. Finally test 5 compared to test 1 (efa vs efe) and test 3 (efc vs efe) should show the merit of the JPL model (although only partially implemented).

To determine the quality of the solutions we looked at two quality indicators:

1. Orbit overlap, a single midnight epoch overlap (smaller is better)

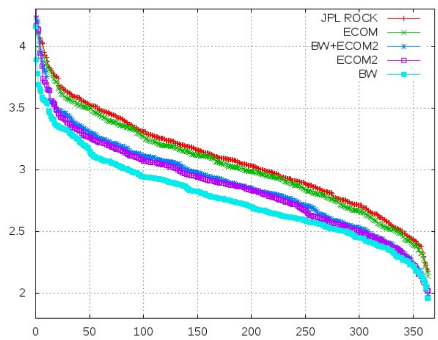
2. Kurtosis, i.e., the steepness of the normal distribution of the fractional parts of the ambiguities before fixing (bigger is better)

The orbit overlap statistics are given in Table 3 and the kurtosis results are shown in Figure 4

So lets look at test 1 and test 2 (efa vs efb). In the orbit overlaps D2 clearly helps for the IIF satellites except in the radial direction. For the IIR the differences are very small and seem insignificant. The GLONASS

Table 3: Orbit Overlap Statistics (mm) over 2017

	GPS-IIR			GPS-IIF			GLONASS		
	#obs	RMS	Mean	#obs	RMS	Mean	#obs	RMS	Mean
efa (BW): Base solution									
RAD	6883	23.00	-1.65	4325	19.89	-2.44	8153	26.45	1.05
ALO	6884	27.56	-2.27	4330	27.62	-0.68	8156	87.06	-17.03
CRO	6887	20.87	0.64	4343	21.23	0.86	8158	45.41	-0.89
3D	6884	41.56	37.94	4330	40.44	36.19	8155	101.48	85.66
efb (BW+ECOM2): Base solution + D2 terms									
RAD	6883	23.35	-0.93	4319	20.36	-1.36	8149	30.63	1.73
ALO	6884	27.75	-2.12	4325	26.90	-0.10	8156	86.76	-16.53
CRO	6887	20.02	1.07	4342	18.00	1.17	8158	50.23	-0.91
3D	6884	41.45	37.82	4324	38.73	34.48	8157	105.22	88.79
efc (ECOM): Base solution but box-wing for IIF turned off									
RAD	6883	22.78	-1.79	4325	19.04	-0.53	8153	26.31	1.04
ALO	6884	27.47	-2.30	4328	26.99	0.69	8156	87.56	-18.84
CRO	6887	20.59	0.56	4343	18.18	0.88	8158	45.34	-1.02
3D	6884	41.23	37.59	4329	38.09	33.87	8155	101.84	85.92
efd (ECOM2): As efc + D2 terms (ECOM2 solution									
RAD	6883	23.45	-1.11	4322	20.42	-0.94	8149	30.64	1.64
ALO	6884	27.70	-2.08	4327	26.96	0.32	8157	87.11	-17.96
CRO	6887	20.12	1.07	4341	18.26	0.95	8158	50.34	-0.85
3D	6884	41.52	37.86	4324	38.65	34.44	8157	105.50	89.11
efe (JPL ROCK): Base solution but JPL model for the IIF									
RAD	6883	20.18	-1.89	4325	18.91	-0.83	8153	26.18	1.13
ALO	6883	25.69	-1.29	4327	26.45	1.24	8156	87.38	-22.44
CRO	6887	19.26	0.55	4342	17.59	1.72	8158	44.69	-1.10
3D	6884	38.01	34.47	4327	37.22	33.24	8155	101.37	85.37



(a) Kurtosis (bigger is better)

Figure 4: Kurtosis as quality indicator of different solutions for 2018

results get noticeably worse. The D2 terms also clearly give better kurtosis statistics and is thus the clear winner. So conclusion from this is that the D2 terms (ECOM2) have a positive effect here in particular for IIF.

Now lets look if test 3 and test 4 (efc vs efd) confirm the positive effect of D2. The main difference between this comparison and the previous one is that now no a priori orbit model for the IIF satellites is used as the box-wing model is turned off. In the orbit overlaps most statistics get slightly worse. And also in the kurtosis the solution without D2 terms is the clear winner. So the conclusion from this test is that the D2 terms do not help. In fact it seems they rather deteriorate the results, in particularly GLONASS but also a bit GPS. This seems to be in contradiction to the previous test results (efa vs efb)!? But we should look at the effect of the box-wing model as well to get the full picture.

So lets compare test 1 and test3 (efa vs efc) as this should show the effect of the box-wing model on the IIF satellites. In the orbit overlaps the efc solution clearly wins for the IIF. For the IIR and GLONASS the differences are very small and hardly significant, as to be expected as nothing changed for these satellites. In the kurtosis the efc solution also wins. So clearly turning off the box-wing model for the IIF satellites has a positive effect. This should be confirmed by comparing test 2 and test 4 (efb vs efd). For the orbit overlaps it is hard to pick the winner between these two. But in the kurtosis solution efb beats the efd solution.

Based on the above results it seems that the box-wing model, as used at ESA, introduces a wrong signal which can, at least partially, be absorbed by the D2 terms of the ECOM model. However, in reality the D2 terms are hardly needed to model the solar radiation on the IIF satellites. So when comparing solutions efa and efb we see a positive effect of the D2 parameters but when we turn off the box-wing model we do not see a positive effect of the D2 parameters (efc vs efd). So from these four solution the best solution actually turns out to be efc where for the IIF we turn of the box-wing model and estimate no D2 parameters. This means we are getting the "best" results using the ECOM approach for the IIF satellites. This is also in line with what we found comparing the IGS orbits. Basically the orbits without any apriori model compared better to each other and to the JPL and EMR solutions then both the ECOM2 based solutions of COD as well as the box-wing based solutions of ESA. Seems both ECOM2 and the box-wing model do not work very well for the IIF satellites.

This motivated us to do a test also using the JPL model. Unfortunately at this time our implementation of the JPL model was not fully ready so we only used the parts which are similar to the ROCK model from the JPL model. So we will not get the full performance of the model but we expect to get the most significant part of the model. This solution should perform at least as good as the efc solution but hopefully even a bit better. So we should now compare our efc solution, the best solution thus far using no apriori model for the IIF, with the efe solution using the JPL model for the IIF. We see some clear improvements in the overlaps when using the JPL model. Surprisingly the largest improvements are for the IIR satellites? The reason for this must be in the Earth Albedo and Infrared modeling. As we assumed these effects are implicitly included in the JPL model we turned this model off. Clearly this must have improved the IIR modeling. But we also see improvements for the IIF satellites. Furthermore, the kurtosis of the efe solution outperforms that off all other solutions. This can be seen very clearly in the Figure 4. So the JPL model, contrary to the ECOM2 and the box-wing model, seems to model the IIF satellites properly and this with even just a partial implementation of the model. However, the efe test also shows that we may have some issues with our Earth Albedo and/or Infrared modeling. The Infrared modeling is the most suspect as we have basically no material properties for this spectrum of the radiation. But also the lack of information regarding the back-side of the solar panels may have a significant effect both for Albedo and Infrared as these are very large surfaces.

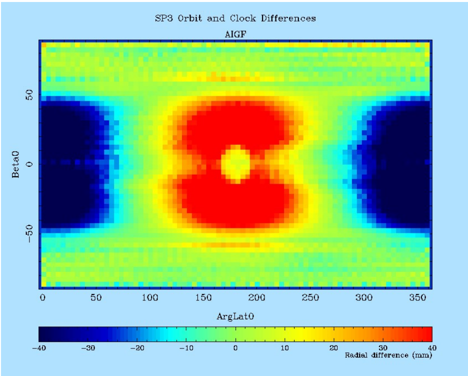
3.5 GLONASS

We do not want to spend a lot of time on GLONASS. But since several ACs include this in their processing and in the reprocessing it should be on our "radars" as well. The IGS orbit comparisons showed very significant differences between the ACs. In particular the COD results seem to be a bit off. The results in the previous section also indicated that the ECOM2 may not be working very well for GLONASS, so that may be an explanation. The ECOM results from GFZ seem to be closer to the box-wing results from ESA. So like for the IIF it seems that the ECOM2 does not work very well for GLONASS. Figure 5 shows the relative large systematic differences of the COD orbits when compared to both GFZ and ESA. But also the GFZ and ESA orbits have noticeable differences but clearly smaller then what is observed vs COD.

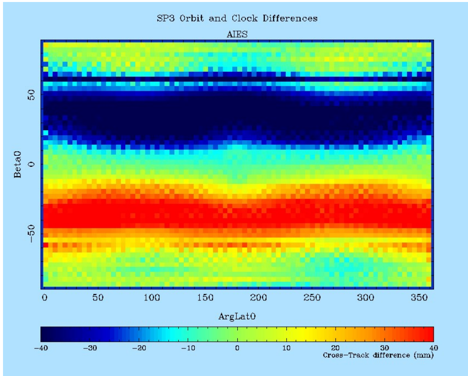
4 IGS Orbit Differences for 2014

In the 2017 analysis there were only two, very old, GPS block IIA satellites and no block II satellites. Since in the reprocessing they play an important role it was decided to analyze an older year where there are enough II/IIA satellites. We decided for the year 2014 where we have

- 7 block II/IIA satellites
- 18 block IIR/IIRM satellites
- 7 block IIF satellites



(a) COD vs GFZ Radial



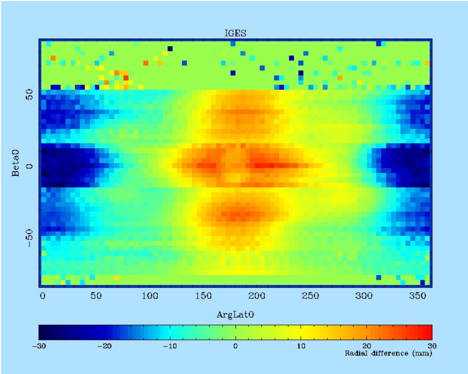
(b) COD vs ESA Cross-track

Figure 5: GLONASS Orbit Differences (mm)

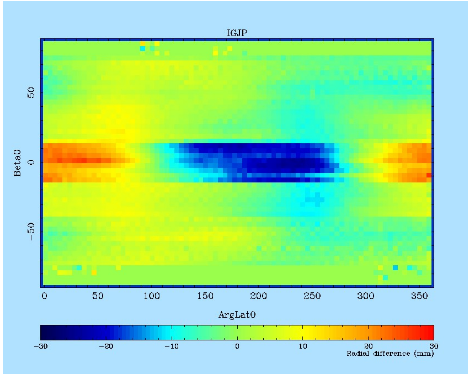
When analyzing the results, and in particular when comparing the IGS orbits, we did not always get a clear picture. Problem here is that 2014 was the end of the second reprocessing. So most repro2 solutions only cover a part of the 2014 if at all. Consequently, also the routine solutions at some point switched from the "old" processing of the AC to the new repro2 processing setup. So this is interfering with finding a clean and homogeneous reference solution and makes interpreting the results sometimes a bit difficult. It was therefore decided to not include the table of these differences because they are just not very meaningful due to sometimes very significant changes over the year in the solution characteristics.

4.1 GPS Block II/IIA satellites

Although being different satellites we have "lumped" the block II/IIA satellites into one group. It should be noted that the JPL GSPM model does make a clear difference between them. In the radial orbit differences some interesting features are observable. Firstly, the JPL model shows a very clear signature in the eclipse period ($|\beta| < 14^0$), see Figure 6. Secondly the SIO results seem to be a bit off both for the routine series (SIO) as well as for the reprocessed series (SI2). The BW model also shows a clear pattern, see Figure 6. In the cross-track direction the results of GRG are clearly different. The BW and JPL models give similar results and ECOM is not far off. So in conclusion for the block II/IIA satellites there are noticable differences between the three approaches BW, JPL, and ECOM. In the cross-track direction it is clear that the ECOM approach gives significantly different results than the BW and JPL models.



(a) IG2 vs ES2 Radial



(b) IG2 vs JPL Radial

Figure 6: Block II/IIA Orbit Differences (mm)

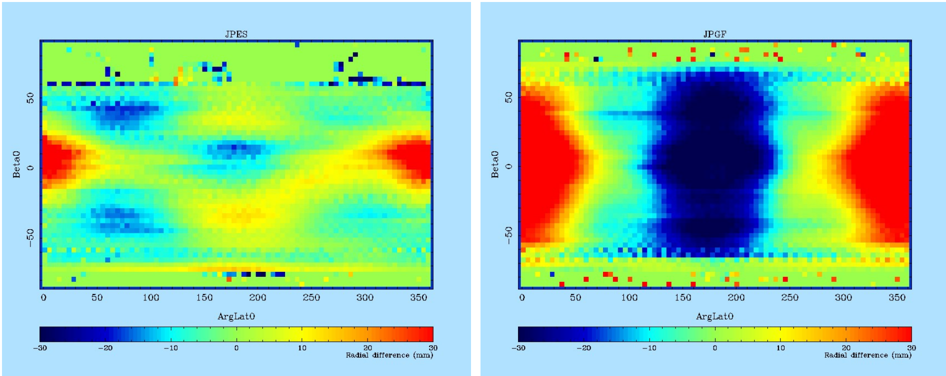
4.2 GPS Block IIR satellites

The IIR radial component shows the most significant differences of all comparisons. In the cross-track direction we also see very significant differences. The JPL and the BW approach seem to agree reasonably well but the differences are significant. The huge differences observed here, see Figure 7, indicate that there is a very significant model error in at least one of the approaches. Given that the two fastly different approaches of ESA and JPL have a good agreement and that in 2017 also the COD ECOM2 approach agrees with the ESA and JPL results it seems reasonable to assume that the ECOM approach is the one that is failing for the IIR satellites. This could be the prime reason for the significant draconic signals in the IGS time series. But given that from repro2 also the ESA and JPL timeseries still had such effects it is very likely that even those models still have significant room for improvement.

4.3 GPS Block IIF satellites

The IIF satellites were already studied in detail with the 2017/2018 data. The results of 2014 show a very similar picture as what we got before. Interesting is that the IGS comparisons show a clear difference between the JPL and JP2 results indicating that the IIF model was updated for the repro2 processing. This is logical given that the IIF were fairly new in 2014 (and was confirmed by Ant). The results confirm that the ESA box-wing model is not working well for the IIF satellites.

So if in the orbit comparisons we ignore the (old) JPL results and the ESA and ES2 results the agreement between all the ACs in the radial direction is fairly good. The reaffirms that the ECOM approach works fine for the IIF as as we concluded earlier. In the cross-track direction some



(a) JPL vs ES2 Radial

(b) JPL vs GF2 Radial

Figure 7: Block IIR Orbit Differences (mm)

signal may be observed for the ECOM results but in general the agreement is reasonable.

4.4 Evaluation of Different Orbit Model Approaches

To figure out which model(s) are working the best we generated four different test solutions to evaluate the different models that are currently being used in the IGS, which resulted in the following solutions:

- BW (egj): Bow-wing solution using a "tuned" box-wing model for the IIF satellites (ESA like solution)
- GSPM (egm): Full implementation of JPL GSPM orbit model (JPL and EMR like solution)
- ECOM (egn): ECOM, 5 parameter approach (COD, GFZ, MIT, NGS like solution)
- ECOM2 (ego): ECOM2, 7 estimated parameters (Current COD solution but not in 2014 also not for CO2)

All solutions covered the full year of 2014 and used exactly the same input data as described in the introduction. So each solution is completely independent except that the starting data is the same. Between the solutions only the orbit model changes. I.e. box-wing model as a priori versus the JPL model as a priori, or no model as a priori. Only in the case of ECOM2 the estimated parameters are changed to include the DC2 and DS2 terms.

As the 2017 analysis had shown that our box-wing model was not working very well we tuned the IIF box-wing model, by changing the material properties, to give a force profile which is much more similar to the JPL model for the IIF satellites. This should improve the BW model performance for the IIF satellites.

To evaluate the quality of these four different solutions, and with that evaluate the approaches they represent, we looked at the following quality indicators:

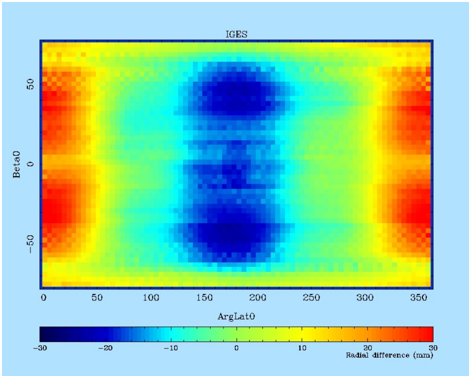
- Orbit comparison to IG2, COD, ESA, and JPL
- Orbit overlap, our single midnight epoch overlap (smaller is better)
- Kurtosis, i.e., the steepness of the normal distribution of the fractional parts of the ambiguities before fixing (bigger is better)

In the orbit comparison and overlap computations for these test solutions we noticed that we have a small bug in the our GSPM implementation. Most likely for the cases where the β angle of the satellite orbit goes through zero. The sign flip which happens in such a case should be ignored in the computation of the force which we overlooked in our implementation in the ESA NAPEOS software. So this may have negatively impacted the results a bit. However, as you will see the performance of the GSPM solutions is very good so the impact has not been all that significant. Most likely because this only happens for satellites in deep eclipse for which we have some modeling issues anyway.

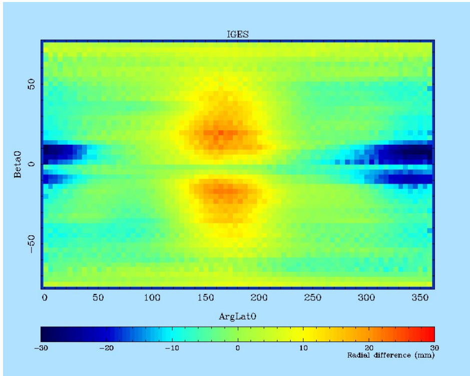
In the orbit comparisons little could be seen except that as expected the BW solution agreed best with the ESA solution, the GSPM solution agreed best with the JPL solution, and the ECOM solution agreed best with both IG2 and COD. As there were no ECOM2 based solution in 2014 this solutions did not look to perform all that good. The positive thing about this is that it shows that our test solutions do indeed resemble the solutions they are supposed to resemble. This is even clearer if we look at the orbit difference plots (not included in this position paper, but available separately).

In the orbit overlaps the differences between the solutions were surprisingly low. Here the GSPM solution most likely suffered from the small bug in our implementation. But even the differences between the ECOM and ECOM2 and between the ECOM and BW solution were surprisingly small. Especially considering the large differences that are visible in the orbit comparisons, e.g. in Figure 8 which shows the differences for the ECOM and ECOM2 solutions compared to the IG2 solution which are as large as 60 mm. The fact that this gets hardly reflected in the orbit overlap statistics is most likely due to the almost perfect 12 hour repeat period of the GPS satellites. So the orbit error also repeats perfectly from day to day giving rise to very small orbit overlap differences.

So from the three quality indicators only the kurtosis analysis gave significant differences. Of course the orbit differences themselves are very interesting and informative but they do not show which solution is better. We



(a) IG2 vs ECOM Radial



(b) IG2 vs ECOM2 Radial

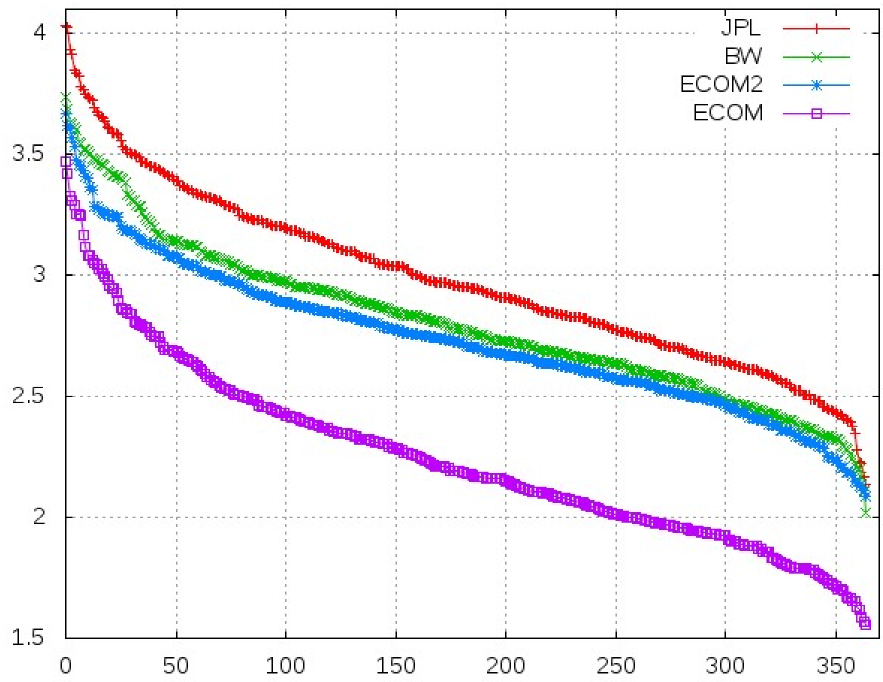
Figure 8: Block IIR Orbit Differences (mm)

believe the kurtosis does show which solution is better as we explained in the introduction of this paper. In Figure 9 we plotted the kurtosis numbers we obtained for our four different test solutions for all the 365 days of 2014. To make the differences a bit easier to see the values are not plotted as function of time but sorted by size which gives a very clear picture of the obtained results. The figure clearly demonstrates the the JPL GSPM model outperforms the three other solutions. The significant differences are somewhat surprising although in our 2018 block IIF investigation, Figure 4, we had already seen that the (partial) JPL model did outperform the ECOM2 approach and the box-wing model. So seeing the same here is not a complete surprise. For the IIF satellites the ECOM model did perform surprisingly well. In the IGS orbit comparisons over 2014 we have clearly seen that the ECOM model is performing very different for the II/IIA satellites in the cross-track direction and shows particularly large differences for the IIR satellites in the radial direction. The poor performance of the ECOM model in the kurtosis analysis is now a very strong indicator that the ECOM model is the one which is under performing.

On a side note I would like to mention that the “good old” ROCK T30 model for IIR does improve things significantly.

5 Recommendations

If one looks at the presented orbit differences it becomes very obvious why all our IGS time series are full with signals with draconic periods as presented many times over the last decade, see e.g. [5]! To reduce those signals we must improve the orbit models! The somewhat disturbing fact here is that despite the very good performance of the JPL GSPM model even the JPL time series still show significant draconic periods. So most likely also in



(a) Kurtosis (bigger is better)

Figure 9: Kurtosis as quality indicator of different solutions for 2014

repro3 we will not get rid of these artifacts in our solutions. But hopefully we can get a better agreement between the different ACs then we currently have and with that hopefully also some real accuracy improvements. In any case these results clearly show that the largest error source in our IGS products today are in our GNSS orbits. And time spend on improving our understanding of the GNSS orbits is most likely time that is very well spend!

As we have distinctively different satellites it turns out that certain approaches work well on one type of satellite do not on an other. E.g. ECOM2 is clearly failing for the block IIF satellites but works well for the II/IIA and IIR satellites. So we, the IGS ACs, will not be able to avoid doing different things for the different satellite block types.

From the work we have done in the scope of this paper we have learned the following:

- The JPL GSPM model works very well
- The ECOM approach can no longer be considered adequate for modeling the block II/IIA and the block IIR satellites. It does, however, work well for the block IIF satellites
- The ECOM2 approach does not work very well for the block IIF satellites nor (most likely) for the GLONASS satellites
- The IGS/ESA box-wing model is not working properly for the IIF satellites, the newly "tuned" model seems to perform OK
- The SIO AC has to improve its handling of the IIR satellites, in particular the radial component
- The GRGS AC has to improve its handling of the II/IIA and the IIF satellites

Based on the results obtained in the scope of this paper we make the following recommendations:

1. The JPL GSPM model may be used for all GPS satellites
2. The ECOM2 approach may be used for the block II/IIA and block IIR satellites but not for IIF. Most likely not very well suited for GLONASS either
3. The IGS/ESA box-wing model may be used for all GPS satellites and for GLONASS (with tuned values for IIF)
4. Much more research effort has to be put into the satellite orbit model in order to reduce, if not eliminate, the spurious draconic terms in the different IGS products.

Although a lot of effort has been put into this work much more remains to be done as it is clear that the orbit errors the dominating error source in our GNSS products. Some further items to be investigate are:

- Is there is significant difference between the block II and IIA satellites. The values in the JPL GSPM model do seem to indicate this

- How good is our Earth Albedo modeling (EA). I assume that the mean effect (scale change) is reasonably accurate. However, we have no reliable material properties for the back side of the solar panels. This is may leads to significant modeling errors.

- How good is our Earth Infra-red modeling (IR). Here we have no reliable values for any of the surfaces. This may lead to significant modeling errors. In our IIF investigations where we tried to use the satellite clocks (the IIF satellites have good clocks) as a quality indicator for the radial orbit errors. In these test we found that turning of EA and IR did in fact improve the clocks, i.e. improved the radial orbit component. More work needs to be done in this direction.

- We have not done much with nor for GLONASS nor is there a JPL GSPM model for GLONASS. Some efforts in this domain are certainly warrented.

6 Appendix: ESA box-wing model

Below in the Table 4 we have given the values that are used at ESA for the box-wing model where:

- area: Specifies the surface area (m^2) of the indicated satellite section (e.g. the +X side, Solar Panel (SP) front)
- vis_sr: Material property for specular reflection of visible light
- vis_dr: Material property for diffuse reflection of visible light
- vis_ar: Material property for absorption of visible light

Note that we use an IGS-specific axis convention which differs from manufacturer specifications for certain satellite types. See the Antex format description for the axis convention. So the +X-Side is the +X direction of the IGS-specific axis convention. Note that the JPL GSPM model follows the satellite specific axis conventions.

The values for II/IIA and the IIR satellites are based on the work of [6]. It also gives the old IIF values we adopted at ESA and the newly tuned values based on the work presented here. The Infra-red properties that we use, are the IGS adopted values as proposed by Carlos Rodriques, which are 0.1, 0.1, and 0.8 for specular-reflection, diffuse-reflection, and absorption respectively. We use this for all surfaces.

Notice furthermore that due to the attitude law of the GPS satellties only the +X axis (IGS definition) will see the Sun. So the values for the -X axis are not really needed except for Earth Albedo and Infra Red radiation as well as during attitude maneuvers which happen during the eclipse season.

Table 4: Optical Properties and area sizes as used in the ESA Box-wing model

Property	II/IIA	IIR	IIF	old IIF	GLONASS
+X Side	2.719	4.110	5.72	5.72	4.2
vis_sr	0.100	0.000	0.200	0.112	0.094
vis_dr	0.400	0.055	0.400	0.448	0.335
vis_ar	0.500	0.945	0.400	0.440	0.571
-X Side	2.719	4.110	5.72	5.72	4.2
vis_sr	0.100	0.000	0.200	0.112	0.094
vis_dr	0.400	0.055	0.400	0.448	0.335
vis_ar	0.500	0.945	0.400	0.440	0.571
+Z Side	2.881	4.25	5.40	5.40	1.66
vis_sr	0.112	0.000	0.700	0.000	0.246
vis_dr	0.448	0.060	0.300	0.400	0.381
vis_ar	0.440	0.940	0.000	0.600	0.374
-Z Side	2.881	4.25	5.40	5.40	1.66
vis_sr	0.083	0.000	0.700	0.000	0.328
vis_dr	0.335	0.060	0.300	0.400	0.331
vis_ar	0.582	0.940	0.000	0.600	0.341
SP front	11.851	13.92	22.25	22.25	30.85
vis_sr	0.197	0.249	0.196	0.196	0.196
vis_dr	0.057	0.044	0.035	0.035	0.035
vis_ar	0.746	0.707	0.770	0.770	0.770
SP back	11.851	13.92	22.25	22.25	30.85
vis_sr	0.197	0.249	0.196	0.196	0.238
vis_dr	0.057	0.044	0.035	0.035	0.042
vis_ar	0.746	0.707	0.770	0.770	0.720

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