

# Impact of solar radiation pressure mis-modeling on **GNSS** satellite orbit determination

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## Introduction

The accuracy of GNSS orbits is mainly associated with the orbit modeling, in which the solar radiation pressure (SRP) is the largest non-gravitational force. In general, the SRP effect can be handled with three types of models: (1) the analytical model (Fliegel et al. 1992), (2) semi-analytical model (Rodriguez-Solano et al. 2012) and (3) empirical model (Beulter et al. 1994; Arnold et al. 2015).

The Empirical CODE Orbit Model (ECOM) is widely used as the empirical model to take care of the SRP effect in the International GNSS Service (IGS) community. The first version of the ECOM model, called ECOM1 in this work, was optimised for a cubic-like satellite, such as GPS, but may not be suitable for more elongated satellite designs, such as GLONASS and GALILEO. Thus, a modified version of the ECOM1 model, namely ECOM2, is developed for the elongate satellites and the major change in ECOM2 was the introduction of even-order periodical perturbations in the satellite-sun direction.



Figure 2: The estimation of D0 (ECOM1: blue and ECOM2: red) as a function of  $\beta$  angle for GPS, GLONASS, GALILEO and BDS constellations over 2015



## **Orbit fitting**

$$\mathbf{r}(t) = \mathbf{r_0}(t) + \sum_{i=1}^{n} \frac{\partial \mathbf{r_0}(t)}{\partial Z_i} \cdot (Z_i - Z_{0,i})$$

where  $\mathbf{r}(t)$  denotes the actual orbit from the GNSS precise ephemeris at certain epoch  $\mathbf{t}$ ; the Z denotes the unknown parameter vector,  $Z_0$  is the initial values of the unknown parameters and the index *i* denotes the number of the unknown parameters;  $r_0(t)$  denotes the initial orbit;  $\partial \mathbf{r_0}(t)/\partial Z_i$  is obtained from the solution of the so-called variational equations.

## **ECOM1 and ECOM2 SRP models**

$$\mathbf{e}_{\mathbf{D}} = \frac{\mathbf{r}_{\mathbf{SUN}} - \mathbf{r}_{\mathbf{SAT}}}{|\mathbf{r}_{\mathbf{SUN}} - \mathbf{r}_{\mathbf{SAT}}|}, \ \mathbf{e}_{\mathbf{Y}} = -\frac{\mathbf{e}_{\mathbf{r}} \times \mathbf{e}_{\mathbf{D}}}{|\mathbf{e}_{\mathbf{r}} \times \mathbf{e}_{\mathbf{D}}|} \text{ and } \mathbf{e}_{\mathbf{B}} = \mathbf{e}_{\mathbf{D}} \times \mathbf{e}_{\mathbf{Y}}$$

where  $\mathbf{e}_{\mathbf{r}}$  is the unit vector associated with a geocentric satellite position vector  $\mathbf{r}_{SAT}$ ;  $\mathbf{r}_{SUN}$  is the geocentric position vector of the Sun;  $e_{\rm D}$  points to the Sun direction from the satellite;  $e_{\rm Y}$ is parallel to the rotation axis of solar panel and is always perpendicular to the  $e_{\rm D}$  axis, and the  $e_B$  axis is given by the right-hand rule of  $e_D$  and  $e_V$  axes, as shown in Figure 1.



Figure 3 (a)-(c) shows the orbit residuals of the selected GNSS satellites with the ECOM1 and ECOM2 modeling. For satellites G23, R07 and E19 (G: GPS, R: GLONASS and E: GALILEO), we discover that the orbit residuals from ECOM1 present the periodical variations in the radial (R), along-track (T), and cross-track (N) directions but those from ECOM2 do not. The periodical variations of the orbit residuals are associated with the inconsistence in D0 between ECOM1 and ECOM2. Fig. 3 (d) shows the difference of the reconstructed acceleration between ECOM1 and ECOM2 for the satellite E19 at  $\beta$ =30°. We can see the variations of the acceleration differences are similar to the periodical variations of the orbit residuals. This is due to the fact that the satellite positions in the GNSS ephemeris are resulted from the orbit determination and are then regarded as the pseudo observations in the orbit fitting. In another word, when ECOM1 model is used to fit the satellite positions resulted from ECOM2 model, the error caused by the inconsistent SRP models may occur. The error caused by the inconsistent SRP model is detected by the SLR residuals, which is shown in Fig. 4.



**Figure 1**: Definition of the ECOM-based model and the Sun-fixed frame

#### ECOM1 SRP model

 $D(\Delta u) = D_0 + D_C \cdot \cos\Delta u + D_S \cdot \sin\Delta u$  $Y(\Delta u) = Y_0 + Y_C \cdot \cos \Delta u + Y_S \cdot \sin \Delta u$ 

 $B(\Delta u) = B_0 + B_C \cdot \cos \Delta u + B_S \cdot \sin \Delta u$ 

### ECOM2 SRP model

 $D(\Delta u) = D_0 + \sum_{i=1}^{n_D} \{D_{2i,C} \cos 2i\Delta u + D_{2i,S} \sin 2i\Delta u\}$  $Y(\Delta u) = Y_0$  $B(\Delta u) = B_0 + \sum_{i=1}^{n_B} \{B_{2i-1,C}\cos(2i-1)\Delta u + B_{2i-1,S}\sin(2i-1)\Delta u\}$ 

## **SRP-caused acceleration**

Figure 3: The orbit residuals from ECOM1 (left) and from ECOM2 (right) in the R, T and N over 2015: (a) G23, (b) R07, (c) E19 and (d) the differences between the accelerations caused by ECOM1 and ECOM2 models for the E19 satellite with a  $\beta$  of 30°



Figure 4: The SLR residuals of (a) GLONASS and (b) GALILEO for the reference orbit fitted by the ECOM1 model over 201

| Table 1: Statistic information of<br>SLR orbit validations for<br>GLONASS and GALILEO<br>satellites (unit: cm) |       | GLONASS      |      | GALILEO |      |
|--|-------|--------------|------|---------|------|
|  |       | Mean         | STD  | Mean    | STD  |
|  | ECOM1 | <b>-0.66</b> | 3.37 | 4.50    | 5.40 |
|  | ECOM2 | 1.06         | 3.15 | 3.82    | 3.66 |

Figure 2 shows the estimation of D0 as a function of  $\beta$  angle for GPS, GLONASS, GALILEO, BDS satellites over 2015. Here the BDS GEO is excluded. The order of magnitude of the SRP-caused acceleration is approximate to 100 nm/s<sup>2</sup>. For GPS, the three groups of D0 are referred to different Block types and no significant difference in D0 between ECOM1 and ECOM2 can be found. Only the accelerations from ECOM2 in the orbit eclipse slightly disagree with those from ECOM1. However, this is not the case for GLONASS and GALILEO. For the GLONASS, the D0 from ECOM1 agrees with that from ECOM2 only when  $|\beta| > 45^{\circ}$  but starts to disagree at  $|\beta| \approx 45^{\circ}$ . The inconsistence in D0 is likely caused by the exchanges of the illuminated cross-section areas, which are differently handled in ECOM1 and ECOM2. For the high  $|\beta| = 90^{\circ}$ , the satellite X side is kept illuminated, leading to the consistence in D0 acceleration between ECOM1 and ECOM2. Conversely, when the D0 starts to be inconsistent between the both, the satellite gradually changes the illuminated areas.



## Conclusions

- The estimation of D0 from ECOM1 starts to be inconsistent with that from ECOM2 at  $|\beta| \approx 45^{\circ}$  and  $\approx 55^{\circ}$  for GLONASS and GALILEO, respectively. This suggests that (1) the different  $\beta$  boundaries are mainly resulted from the different areas of the satellite body X and Z sides for GLONASS and GALILEO; (2) the inconsistence in the DO estimation is caused by the changes of the illuminated cross-section areas, which are differently handled in ECOM1 and ECOM2 cases.
- The large orbit residuals are found at  $|\beta| \leq 45^{\circ}$  and  $\leq 55^{\circ}$  for GLONASS and GALILEO, and perform the periodical variations, which are similar to the distribution of the SLR residuals.

#### References

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