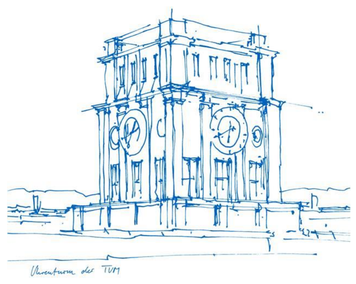

Least Squares Cubic Splines for the Polar Motion Estimations

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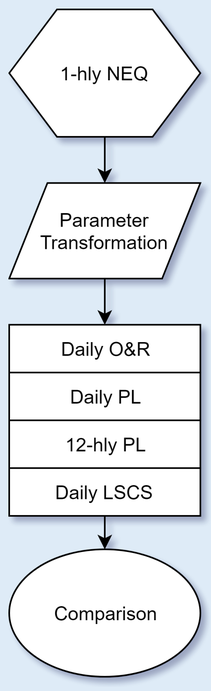
Overview

Polar motion (PM) parameters are estimated by the IGS ACs using linear models—with or without continuity constraints between boundaries, and the conflict of whether to choose the piecewise linear model or the offset and rate model remains unsolved.

In this presentation, an alternative non-linear model—least squares cubic splines (LSCS) aiming to increase the performance of PM representation is studied and realized using several years of GPS normal equations (NEQ) based on hourly piecewise linear ERPs. Daily LSCS PM parameters are reconstructed at NEQ level using parameter transformation, and the corresponding results are compared and analyzed.

Procedure



Models

Models	Offset and Rate	Piecewise Linear		Least Squares Cubic Splines
Types (abbreviation)	Daily O&R	Daily PL	12h PL	LSCS
Parameters	Daily offset and rate	Daily offset	12-hly offset	Daily offset, 2nd derivative at day boundary and Lagrange multiplier
Number of parameters (in n days)	2n	n+1	2n+1	n+3

Table 1: *model types and coresponding number of parameters in n days*

Estimated Pole Parameters

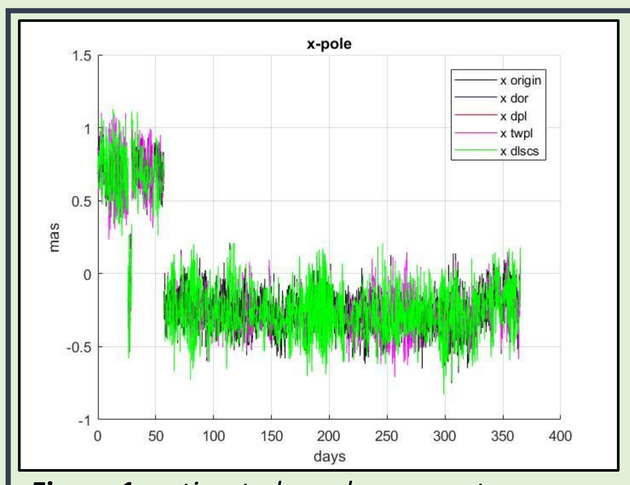


Figure 1: *estimated x-pole parameters 1-day arc*

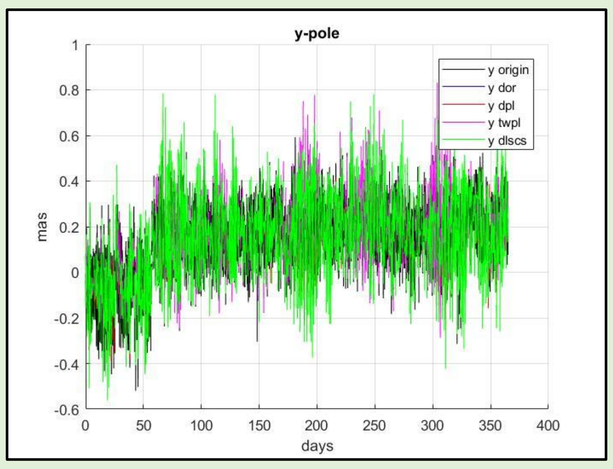


Figure 2: *estimated y-pole parameters 1-day arc*

Hourly parameters are generated based on the hourly NEQ of 365 days (from Jan. 1st to Dec. 31st 2010). Parameters other than pole parameters are pre-eliminated, and pseudo-observations according to J. Hefty et al. (1999) are added to prevent retrograde diurnal terms. The arc-length is one day. Here only the corrections to a priori poles are displayed. Jumps in the x-pole and y-pole around day 57 can be observed, however the reason for this is not yet clear. It may due to reference frame change or the change of the a priori poles.

Transformation matrices between the hourly pole parameters and daily O&R, daily PL, 12 hourly PL and daily LSCS parameters are derived using least squares. Then parameter transformation is done and corresponding pole parameters are generated.

In 1-day-arc, the daily PL parameters have relatively concentrated values, and cubic splines parameters spread out more.

Notice that in a one-day arc, the daily O&R and daily PL have exactly the same value.

Stacking

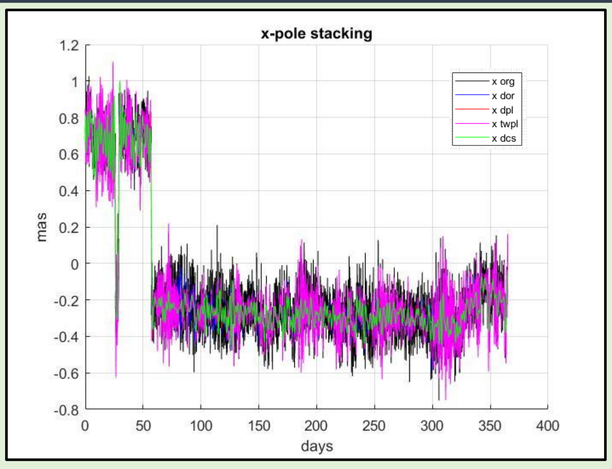


Figure 3: *estimated x-pole parameters after stacking*

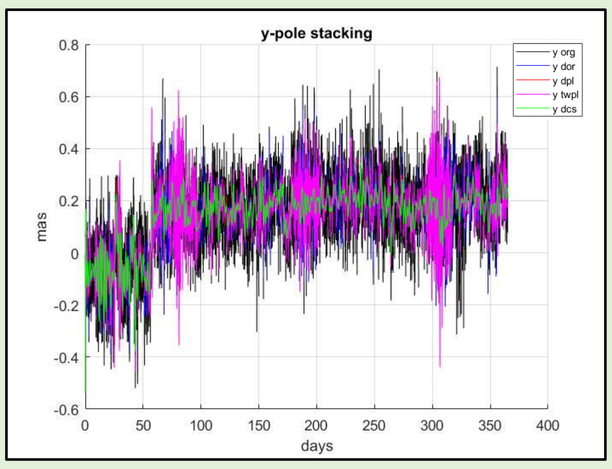


Figure 4: *estimated y-pole parameters after stacking*

Stacking of the 1-day-arc NEQs to a 365-days-arc are then done to generate the daily PL and 12 hourly PL with continuous boundaries. While daily O&R remains unchanged.

As for the daily LSCS, Lagrange multipliers are also added to ensure the continuous of 1st derivatives.

We can see after the staking, cubic splines parameters are more restricted to the center and 12 hourly PL ones are more spreading out.

PSD Comparison

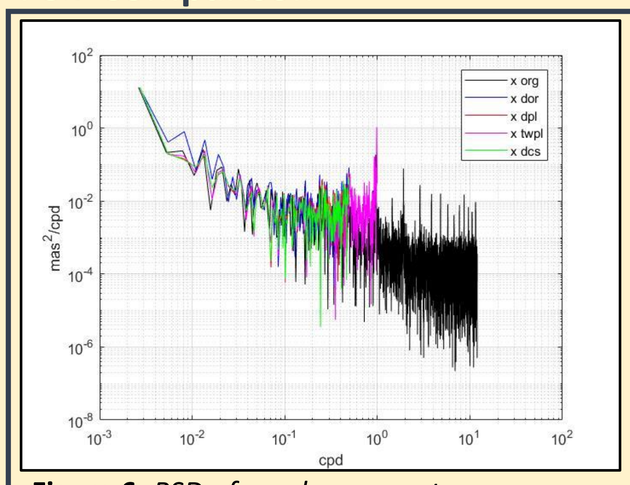


Figure 6: *PSD of x-pole parameters after stacking*

Figure 7: *PSD of x-pole parameters after stacking*

PSD of the hourly PL, daily O&R, daily PL, 12-hourly PL and daily LSCS are generated and plotted in the loglog scale. For daily results, the Nyquist frequency is 0.5 cpd, for 12 hourly ones 1 cpd and for hourly ones 12 cpd. Here the knots are selected at 0 UTC and thus no smoothing effect in sight. Later knots at 12 UTC will be looked into as well for the smoothing effect.

We can see for x-pole, all models agree with each other and with the hourly PL well, daily O&R seems to possess a bit higher values than others. While for y-pole, obvious difference of the daily O&R model can be viewed—not only the values are much higher, the pattern looks distinct. Reason for the dissimilar of x-pole and y-pole could caused by the jump around day 50 in x-pole, which blur the performance of continuous models especially cubic splines.

Since there is unexpected jump in day 57, the PSD only consider later dates (day 58 to day 365) are also generated, from which similar pattern can be observed.

Conclusion and Outline

In this poster, daily offset and rate model, daily piecewise linear model, 12 hourly piecewise linear model and daily least squares cubic splines are realized based on hourly NEQ of one year. For piecewise linear and cubic splines models, those 1-day-arc parameters are then combined using stacking and Lagrange multipliers to generate typical continuous arcs. Afterwards, a comparison of PSD with selected knots (0 UTC for daily parameters and 0&12 UTC for 12 hourly parameters) are accomplished.

According to the PSD, we can conclude that the polar motion has a similar behavior as power law noise which fits previous assumptions (J. Ray's previous studies), however, with a different way of selecting knots, daily offset and rate model seems to have a poor performance.

Further studies are needed to confirm that, which will include but not limited to,

1. Smoothing effect in the PSD of selected daily knots at 12 UTC,
2. Studies based on residuals (residual plots and hypothesis test)
3. Studies based on observations (pseudorange and carrier phase) instead of NEQs

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