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# Solar radiation pressure model for BeiDou GEO satellites

Jing Guo[1,2], Chen Wang[1], Qile Zhao[1], Zhenhong Li[2], Jingnan Liu[1]

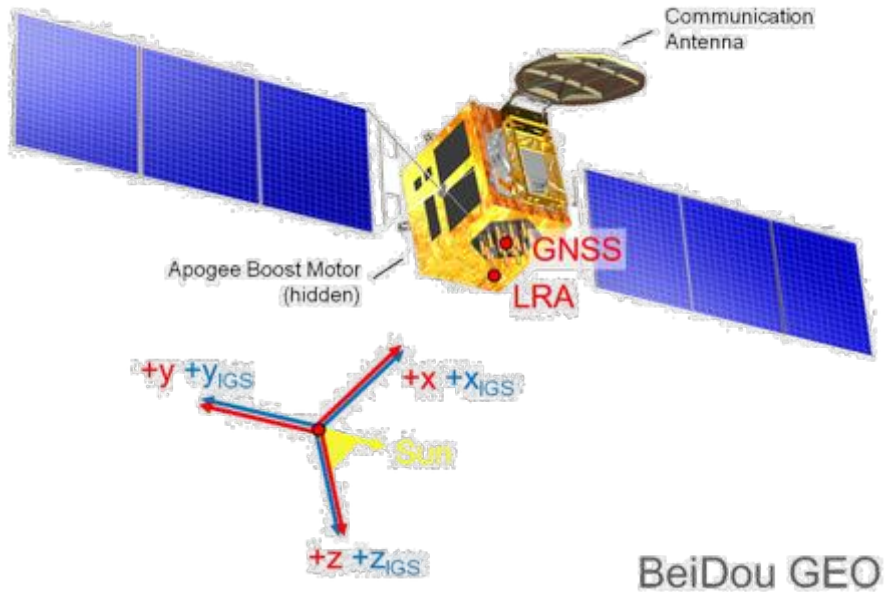
(Email: jingguo@whu.edu.cn; jing.guo@ncl.ac.uk)

1) GNSS Research Center, Wuhan University, China
2) School of Civil Engineering and Geosciences, Newcastle University, UK

# BeiDou GEO satellites

| SVN | PRN | Longitude | Launch date |
|---|---|---|---|
| G1 | C01 | 140.0° E | Jan 17, 2010 |
| G2 | ----- | drifting | Apr 15, 2009 |
| G3 | C03 | 110.5° E | Jun 2, 2010 |
| G4 | C04 | 160.0° E | Nov 1, 2011 |
| G5 | C05 | 58.75° E | Feb 25, 2012 |
| G6 | C02 | 80.00° E | Oct 25, 2012 |

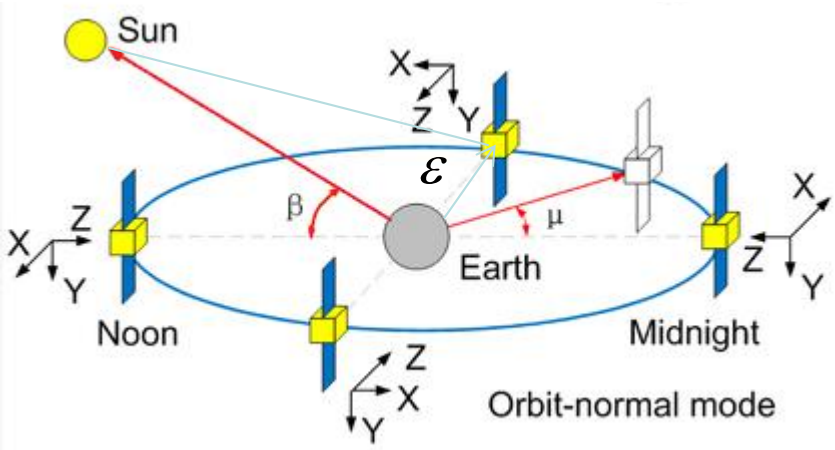
| Panel | Area | α | ρ | δ |
|---|---|---|---|---|
| ±X | 3.748 | 0.350 | 0.650 | 0.0 |
| ±Y | 4.400 | 0.114 | 0.856 | 0.0 |
| ±Z | 3.440 | 0.350 | 0.650 | 0.0 |
| SP | 11.352 | 0.720 | 0.280 | 0.0 |
| CA | 3.000? | 0.350? | 0.650? | 0.0 |

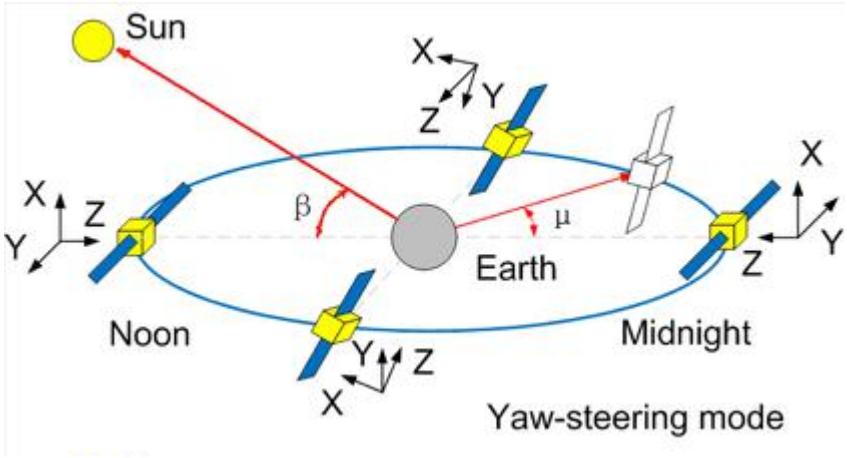



(Montenbruck et al, 2017, ASR)

SP: solar panel
CA: Communication antenna

# BeiDou GEO attitude: orbit normal




$$\mathbf{e}_{X,ON} = \mathbf{e}_{Y,ON} \times \mathbf{e}_{Z,ON} \qquad \mathbf{e}_{Y,ON} = -\frac{\mathbf{r}\times\mathbf{v}}{\|\mathbf{r}\times\mathbf{v}\|} \qquad \mathbf{e}_{Z,ON} = -\frac{\mathbf{r}}{\|\mathbf{r}\|}$$

$$\mathbf{e}_{X,YS} = \mathbf{e}_{Y,YS} \times \mathbf{e}_{Z,YS} \qquad \mathbf{e}_{Y,YS} = \frac{\mathbf{e}_{\odot}\times\mathbf{r}}{\|\mathbf{e}_{\odot}\times\mathbf{r}\|} \qquad \mathbf{e}_{Z,YS} = -\frac{\mathbf{r}}{\|\mathbf{r}\|}$$

DYB$_{ON}$

$$\mathbf{e}_{D,ON} = \mathbf{e}_{Y,ON} \times \mathbf{e}_{B,ON} \qquad \mathbf{e}_{Y,ON} = -\frac{\mathbf{r}\times\mathbf{v}}{\|\mathbf{r}\times\mathbf{v}\|} \qquad \mathbf{e}_{B,ON} = \mathbf{e}_{\odot}\times\mathbf{e}_{Y,ON}$$

DYB$_{YS}$

$$\mathbf{e}_{D,YS} = \mathbf{e}_{\odot} \qquad \mathbf{e}_{Y,YS} = \frac{\mathbf{e}_{\odot}\times\mathbf{r}}{\|\mathbf{e}_{\odot}\times\mathbf{r}\|} \qquad \mathbf{e}_{B,YS} = \mathbf{e}_{D,YS} \times \mathbf{e}_{Y,YS}$$

$$\cos\varepsilon = \cos\beta\cos\mu$$

# BeiDou GEO POD: limitation

- Tracking geometry
  - almost static w.r.t ground stations
  - LEO or MEO onboard data
- Multipath
- Elevation-and-satellite dependent bias in pseudorange
- Maneuvers due to orbit-keeping
- Solar radiation pressure
  - Liu et al. (2015) $D_0$, $Y_0$, $Y_C$, $B_0$, $D_S$, $B_S$
  - Tan et al. (2016) IGGBSPM

# Solar radiation pressure models

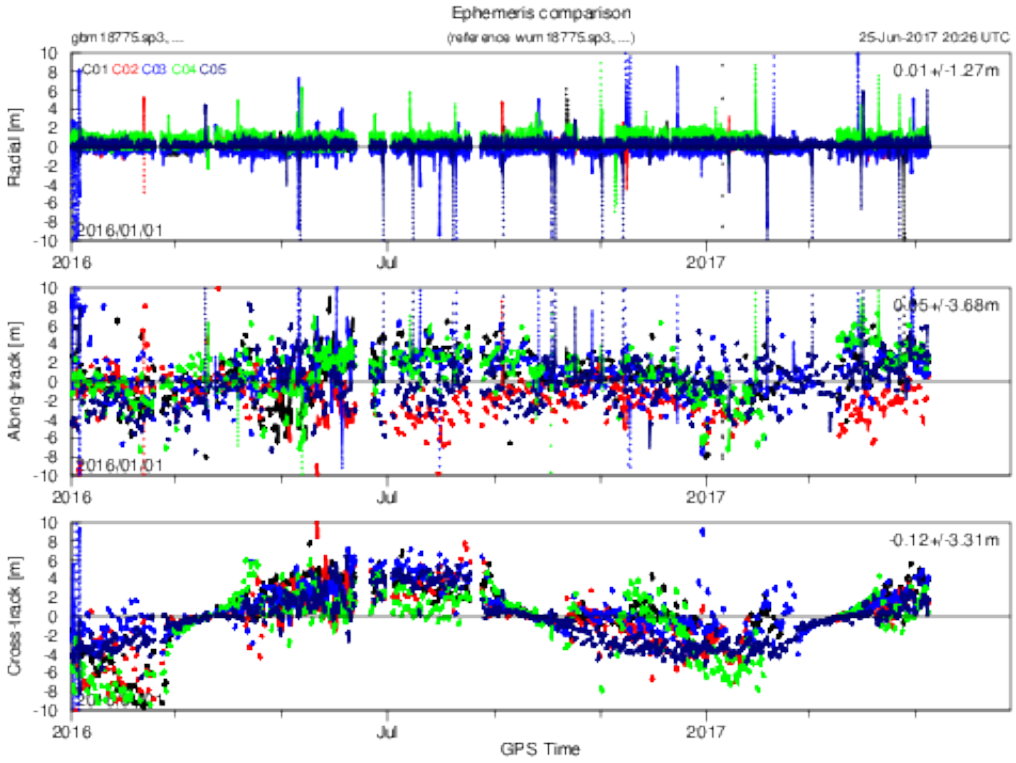
- Extended CODE Orbit Model (ECOM)
- Adjustable box-wing model (ABW)
- DLR (Montenbruck et al, 2017)

$$
\begin{aligned}
a_{\overline{D}} &= \boldsymbol{a}_{\mathrm{ON}}^{T} \boldsymbol{e}_{\overline{D}} \\
&= -a_{zx}^{x\delta} \cdot \left( (|\cos\mu| + |\sin\mu|) \cdot \cos^2\beta + \frac{2}{3}\cos\beta \right) \\
&\quad - \Delta a_{zx}^{x\delta} \cdot \left( (|\cos\mu| - |\sin\mu|) \cdot \cos^2\beta + \frac{2}{3}\cos(2\mu) \cdot \cos\beta \right) \\
&\quad - 2a_{zx}^{\rho} \cdot \left( |\cos\mu|^3 + |\sin\mu|^3 \right) \cdot \cos^2\beta \\
&\quad - 2\Delta a_{zx}^{\rho} \cdot \left( |\cos\mu|^3 - |\sin\mu|^3 \right) \cdot \cos^2\beta \\
&\quad - \Delta a_{z}^{x\delta} \cdot \left( \cos\mu \cdot \cos^2\beta + \frac{2}{3}|\cos\mu| \cos\mu \cdot \cos\beta \right) \\
&\quad - 2\Delta a_{z}^{\rho} \cdot \cos^3\mu \cdot \cos^2\beta \\
&\quad - \Delta a_{x}^{x\delta} \cdot \left( \sin\mu \cos^2\beta + \frac{2}{3}|\sin\mu| \sin\mu \cos\beta \right) \\
&\quad - 2\Delta a_{x}^{\rho} \cdot \sin^3\mu \cdot \cos^2\beta - a_{y}^{x\delta} \cdot \left| \frac{1}{2}\sin(2\beta) \right| \\
&\quad + \Delta a_{y}^{x\delta} \cdot \frac{1}{2}\sin(2\beta) - \left( a_{\mathrm{sp}}^{x\delta} + 2a_{\mathrm{sp}}^{\rho} \right) \cdot \cos^2\beta - \frac{2}{3} a_{\mathrm{sp}}^{\delta} \cdot \cos\beta,
\end{aligned}
$$

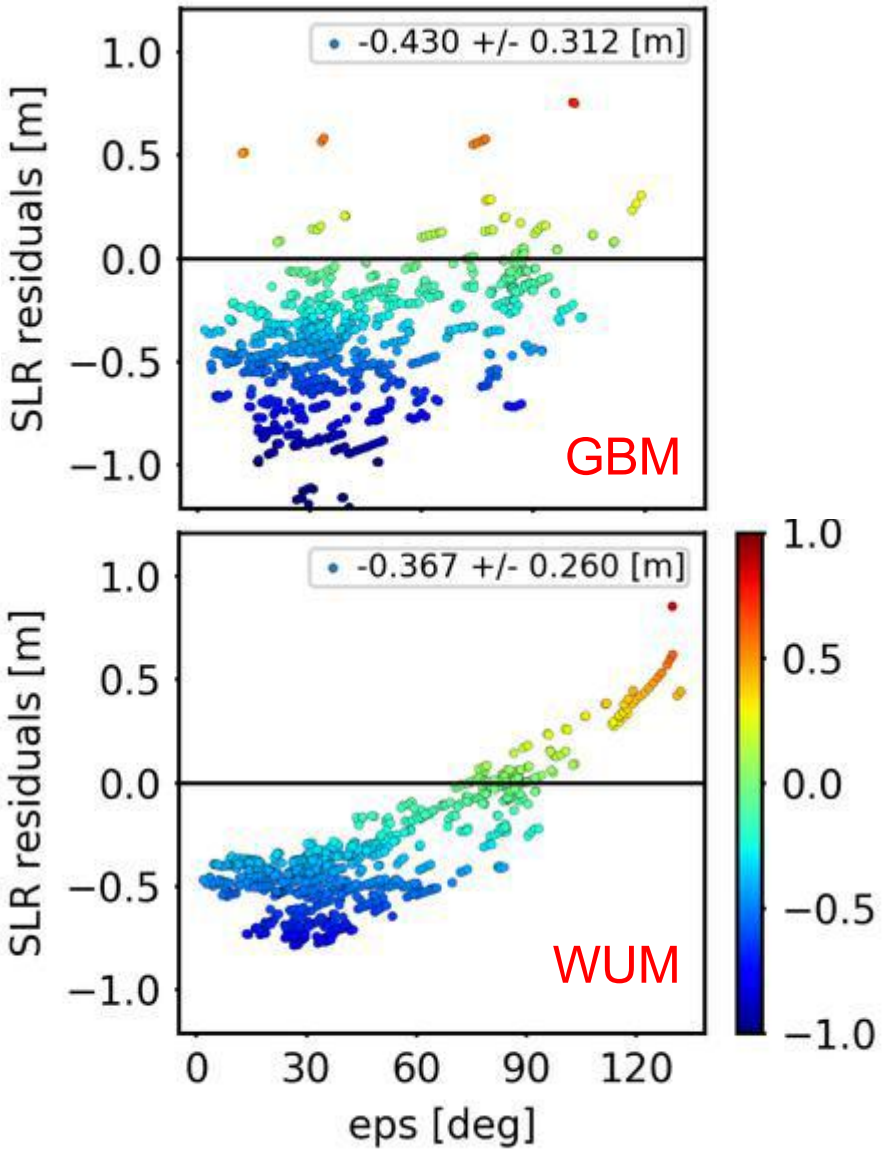
$$
\begin{aligned}
a_{\overline{Y}} &= \boldsymbol{a}_{\mathrm{ON}}^{T} \boldsymbol{e}_{\overline{Y}} \\
&= +a_{zx}^{x\delta} \cdot (|\cos\mu| + |\sin\mu|) \cdot \frac{1}{2}\sin(2\beta) \\
&\quad + \Delta a_{zx}^{x\delta} \cdot (|\cos\mu| - |\sin\mu|) \cdot \frac{1}{2}\sin(2\beta) \\
&\quad + \Delta a_{z}^{x\delta} \cdot \cos\mu \cdot \frac{1}{2}\sin(2\beta) \\
&\quad + \Delta a_{x}^{x\delta} \cdot \sin\mu \cdot \frac{1}{2}\sin(2\beta) \\
&\quad + a_{y}^{x\delta} \cdot \left( |\sin\beta| + \frac{2}{3} \right) \cdot \sin\beta \\
&\quad - \Delta a_{y}^{x\delta} \cdot \left( \sin^2\beta + \frac{2}{3}|\sin\beta| \right) \\
&\quad + 2a_{y}^{\rho} \cdot |\sin\beta| \sin\beta \\
&\quad - 2\Delta a_{y}^{\rho} \cdot \sin^2\beta + a_{\mathrm{sp}}^{x\delta} \cdot \frac{1}{2}\sin(2\beta),
\end{aligned}
$$

$$
\begin{aligned}
a_{\overline{B}} &= \boldsymbol{a}_{\mathrm{ON}}^{T} \boldsymbol{e}_{\overline{B}} \\
&= -\frac{4}{3}\Delta a_{zx}^{x\delta} \cdot \cos\mu \sin\mu \cdot \cos\beta \\
&\quad - 2a_{zx}^{\rho} \cdot (|\cos\mu| - |\sin\mu|) \cos\mu \sin\mu \cdot \cos^2\beta \\
&\quad - 2\Delta a_{zx}^{\rho} \cdot (|\cos\mu| + |\sin\mu|) \cos\mu \sin\mu \cdot \cos^2\beta \\
&\quad - \frac{2}{3}\Delta a_{z}^{x\delta} \cdot |\cos\mu| \sin\mu \cdot \cos\beta \\
&\quad - 2\Delta a_{z}^{\rho} \cdot \cos^2\mu \sin\mu \cdot \cos^2\beta \\
&\quad + \frac{2}{3}\Delta a_{x}^{x\delta} \cdot |\sin\mu| \cos\mu \cdot \cos\beta \\
&\quad + 2\Delta a_{x}^{\rho} \cdot \sin^2\mu \cos\mu \cdot \cos^2\beta.
\end{aligned}
$$

# ECOM model

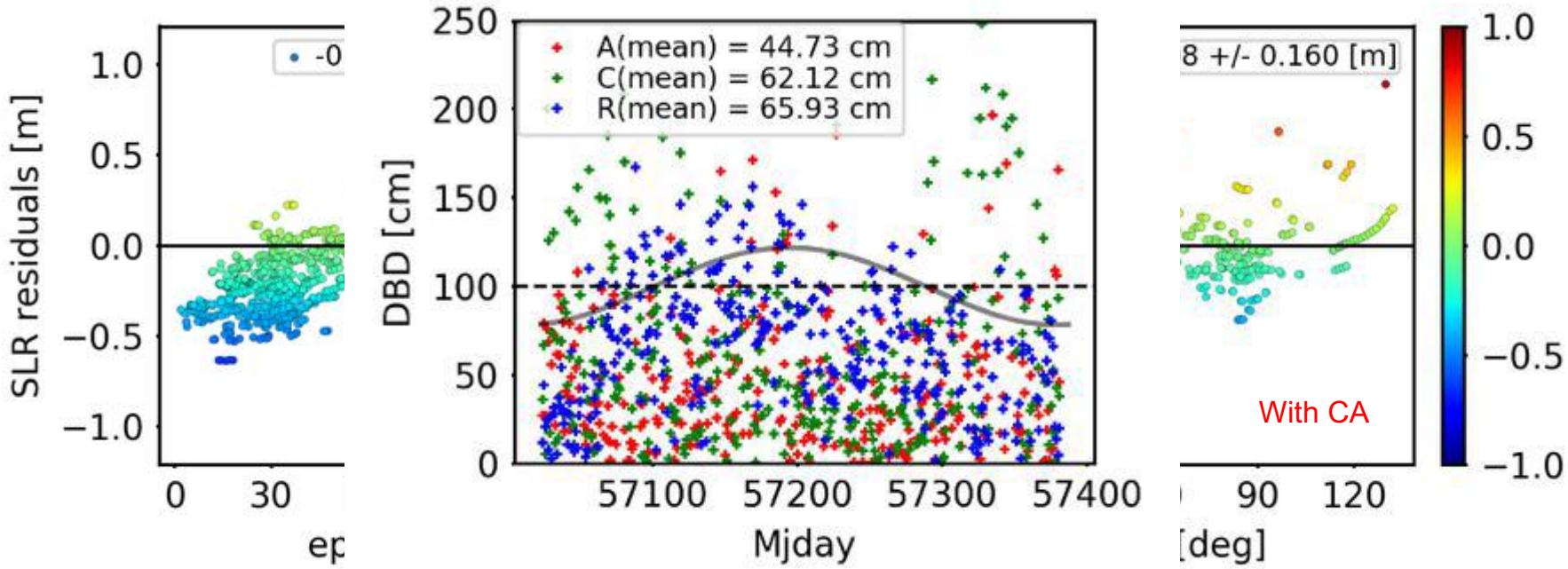



http://mgex.igs.org/analysis/ephcmp_BDG.php



# ABW model

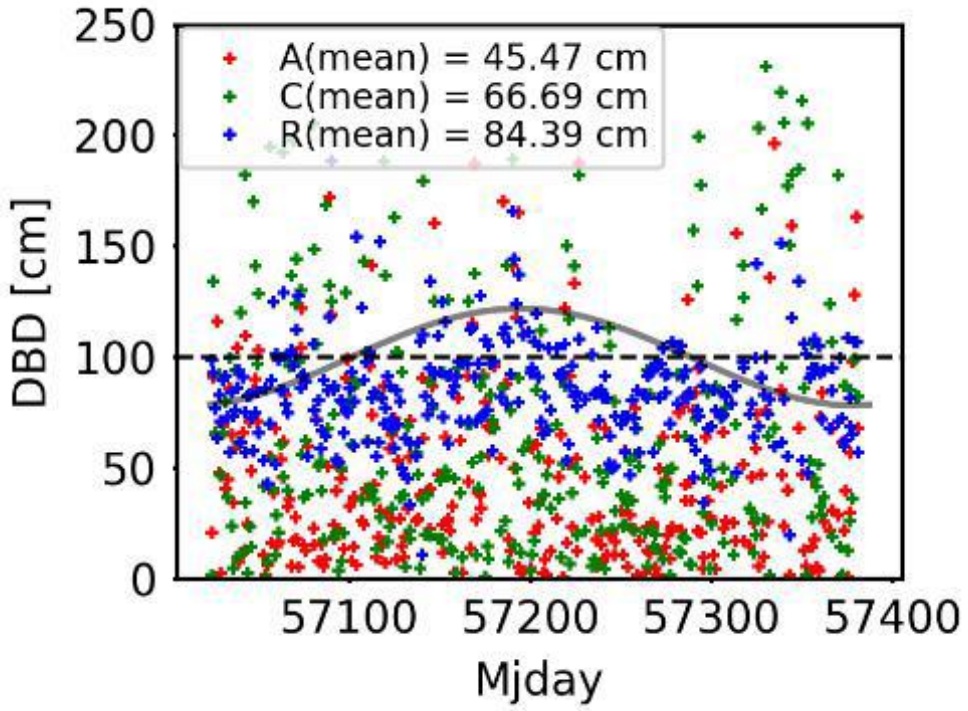
- Orbit normal mode used, SP points to $\mathbf{e}_{D,ON}$
- The absorption plus diffuse reflection (αδ) as well as the specular reflection (ρ) for all illuminated satellite bus surfaces were adjusted



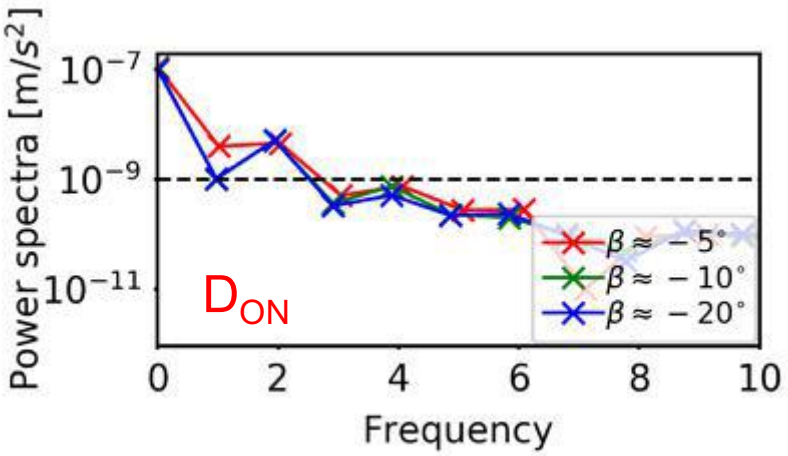
# DLR model

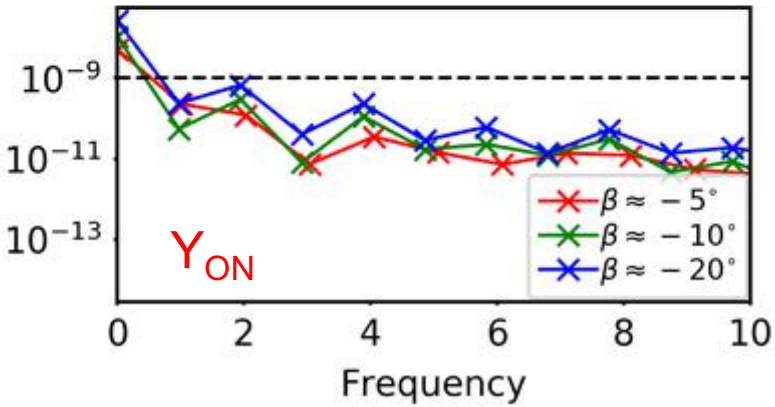
- Adjustment of the parameters with reconstructed accelerations from ABW model
- A prior model in $DYB_{ON}$ + ECOM ($DYB_{ON}$)

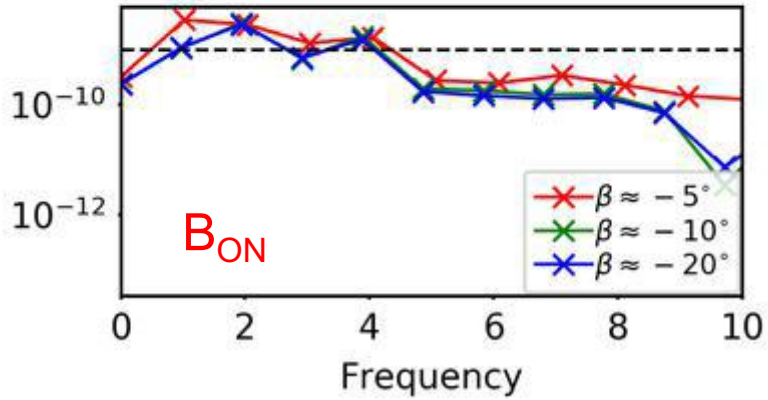
| Parameter | Value [nm/s²] |
|---|---|
| $a_{zx}^{\alpha\delta}$ | |
| $a_{zx}^{\rho}$ | 16.3 |
| $\Delta a_{zx}^{\alpha\delta}$ | 5.5 |
| $a_{y}^{\alpha\delta}$ | 7.7 |
| $a_{y}^{\rho}$ | 10.9 |
| $a_{sp}^{\alpha\delta}$ | 8.6 |
| $a_{y}^{\rho}$ | 49.1 |
| | 11.2 |



# Amplitude spectra of accelerations



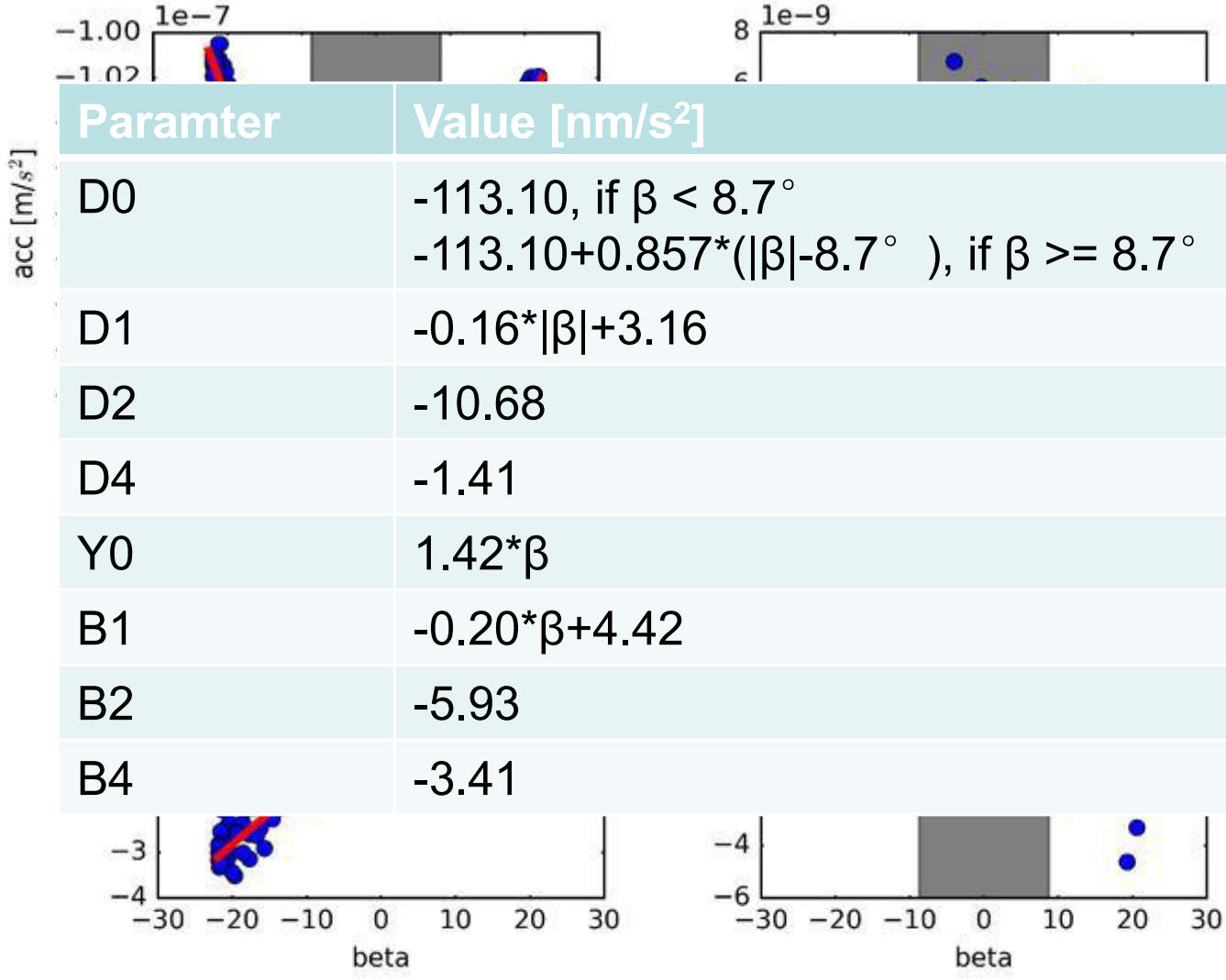




$$D_{ON} = D_0 + D_1 \cos\mu + D_2 \cos 2\mu + D_4 \cos 4\mu$$

$$Y_{ON} = Y_0$$

$$B_{ON} = B_1 \sin\mu + B_2 \sin 2\mu + B_4 \sin 4\mu$$

# Model parameters

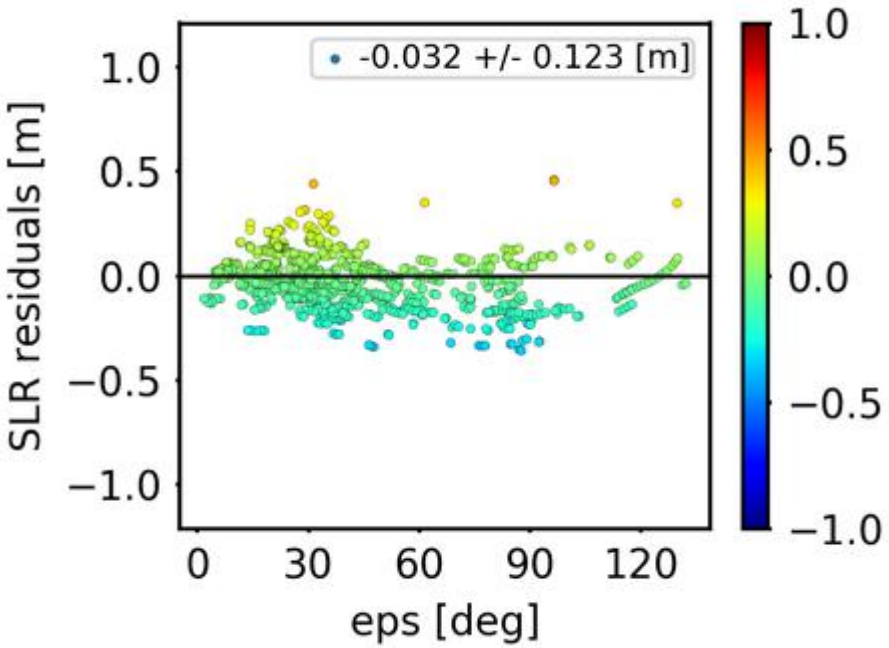


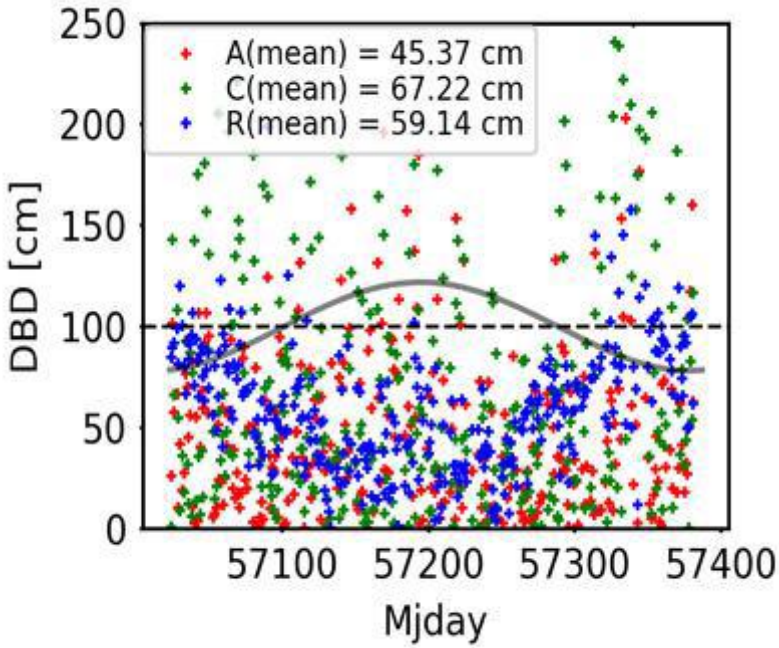

| Paramter | Value [nm/s²] |
|---|---|
| D0 | -113.10, if β < 8.7°<br>-113.10+0.857*(\|β\|-8.7° ), if β >= 8.7° |
| D1 | -0.16*\|β\|+3.16 |
| D2 | -10.68 |
| D4 | -1.41 |
| Y0 | 1.42*β |
| B1 | -0.20*β+4.42 |
| B2 | -5.93 |
| B4 | -3.41 |

β is in degree

# Model validation

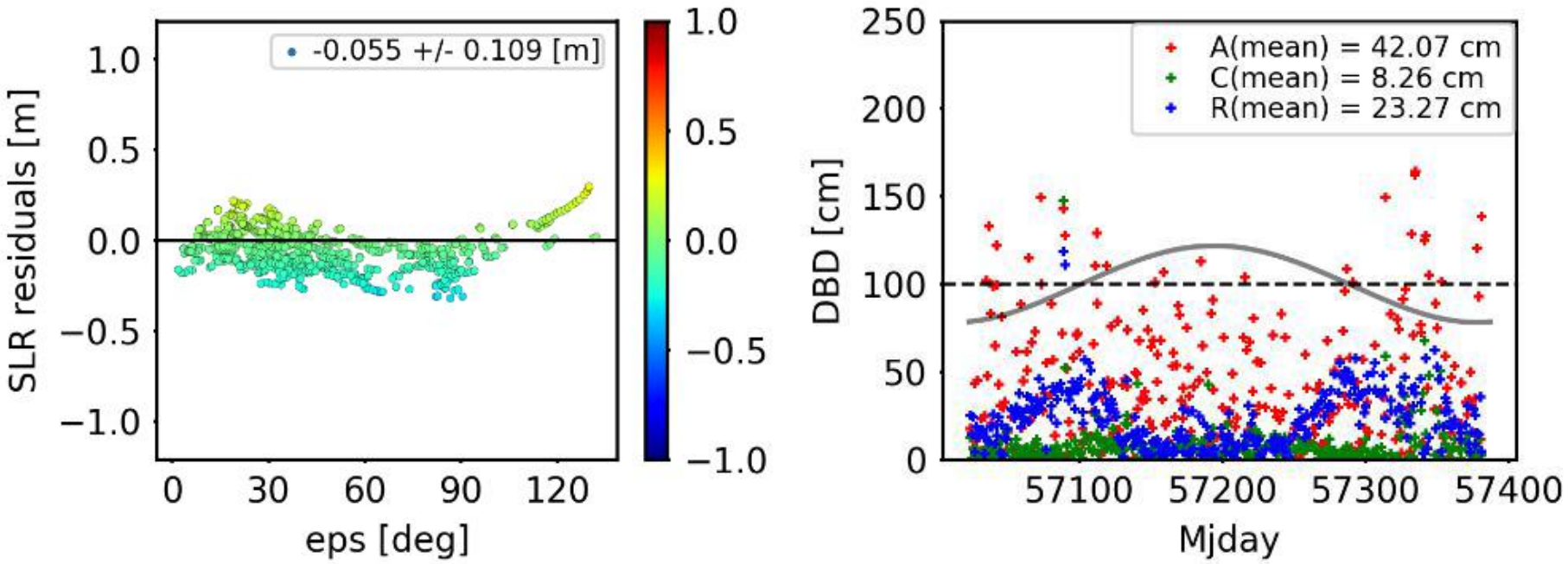
A prior model in $DYB_{ON}$ + ECOM ($DYB_{ON}$)



# Model validation

A prior model in $DYB_{ON}$ + ECOM ($DYB_{YS}$)

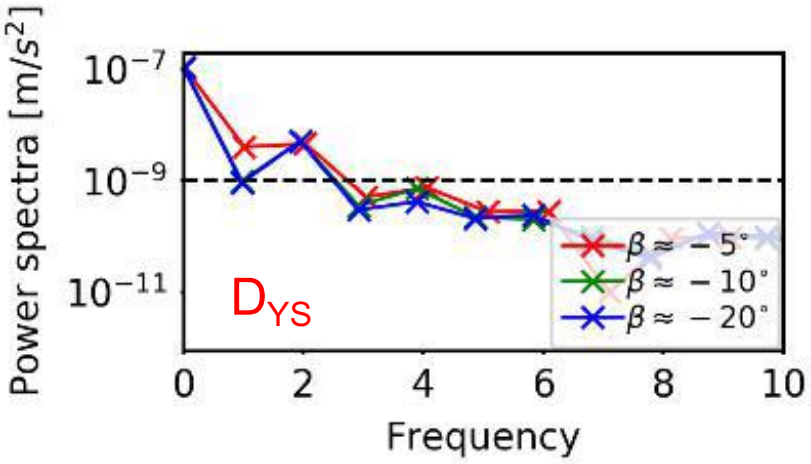


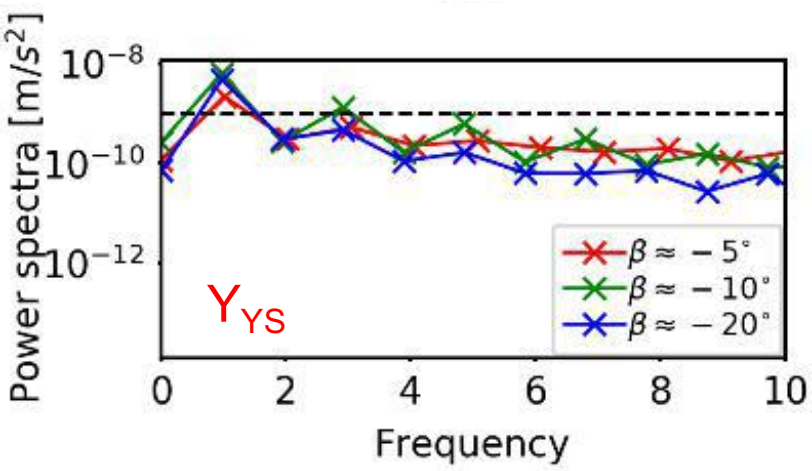
# Summary

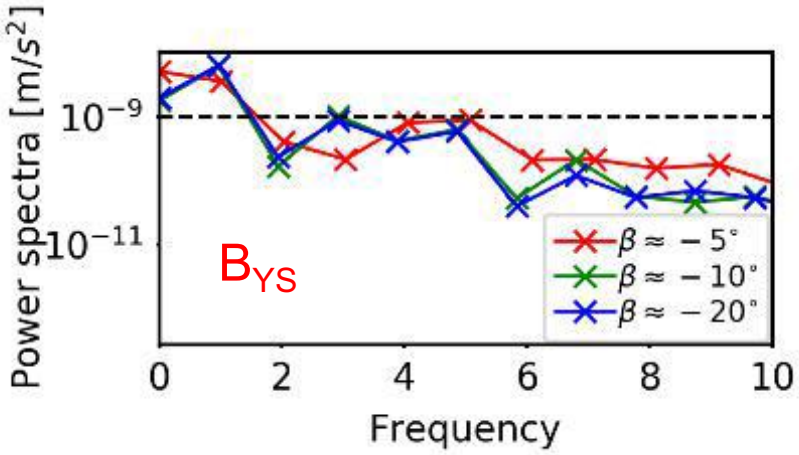
- Large bias and ε angle dependent error are in SLR residuals for BeiDou GEO orbits determined by ECOM model
- These systematic errors are mainly caused by unmodeled perturbation from CA
- The a prior derived by empirical fitting approach has similar performance as the analytical DLR model
- DBD is not good when the $DYB_{ON}$ is used for ECOM, but can be improved by using the $DYB_{YS}$ orientation


# Thanks !

# Amplitude spectra of accelerations



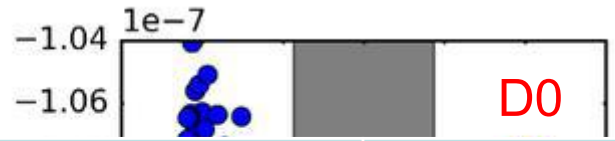



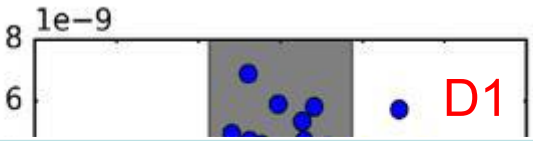

$$D_{YS} = D_0 + D_1 \cos\varepsilon + D_2 \cos 2\varepsilon + D_4 \cos 4\varepsilon$$

$$Y_{YS} = Y_1 \sin\mu$$

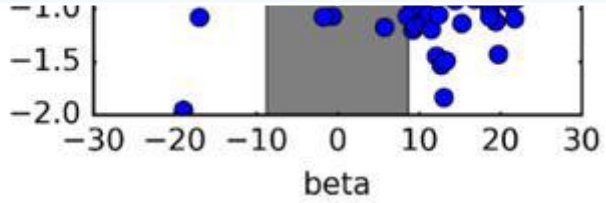
$$B_{YS} = B_0 + B_1 \cos\varepsilon + B_3 \cos 3\varepsilon$$

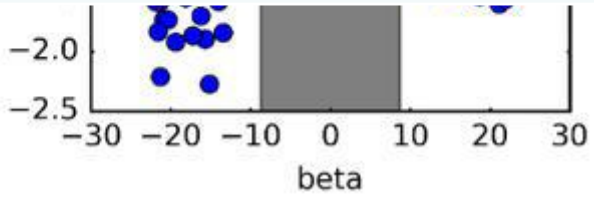
# Model parameters




| Paramter | Values [nm/s²] [β in degree] |
|---|---|
| D0 | -113.0, if β < 8.7°<br>-113.0+0.465*(\|β\|-8.7° ), if β >= 8.7° |
| D1 | -0.199*\|β\|+3.70 |
| D2 | -11.69 |
| D4 | -1.458 |
| Y1 | -0.386*β-0.142 |
| B0 | 1.27 |
| B1 | -0.448*β-4.95 |
| B3 | -1.25 |



# Model validation

A prior model in $DYB_{YS}$ + ECOM ($DYB_{YS}$)