Fully populated Variance Covariance Matrices and the float ambiguity solution - Real Time Kinematic applications

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Motivation and Introduction

► When an incorrect or approximated stochastic model is used in least-squares adjustment, the solution is biased. Following quantities are impacted:

- The estimates (position, float ambiguities)
- The test statistics (overall model test, outlier tests)
- The precision

► Thus, the reliability of the solution is weaker

► The integration of **fully populated variance covariance matrix** (VCM) of the observations in the least-squares adjustment impacts the **float ambiguities solution** for RTK like applications when used without additional corrections

► This holds particularly true for very short sessions of observations when ambiguities cannot be fixed to integer with enough confidence (i.e. baseline length >20km)

► The recently introduced Mátern model (Kermarrec and Schön 2017) is used to study the impact of the stochastic model on the solution with a focus on the 3Drms and the aposteriori variance factor

Covariance model for GPS phase measurements

The proposed function is an extended and simplified form of the phase covariance for modeling turbulent tropopheric refractivities fluctuations (Kermarrec and Schön 2014)

$$C^{i,j}(t,t+\tau)=\frac{\rho\delta}{\sin(El_i(t))\sin(El_j(t+\tau))}(\alpha\tau)^{\nu}K_{\nu}(\alpha\tau),$$

Matern covariance function
Correlation length: $1/\alpha$
Smoothness ν
Determination:
Maximum Likelihood Estimation
Fixed apriori: atmospheric correlations
$[\alpha,\nu]=[0.01,1]$

Elevation dependent weighting
$El_i(t)$: elevation of satellite i at epoch t
δ>0: scaling factor (variance 1 for satellite at 90°El)
ρ: correlations between different satellites

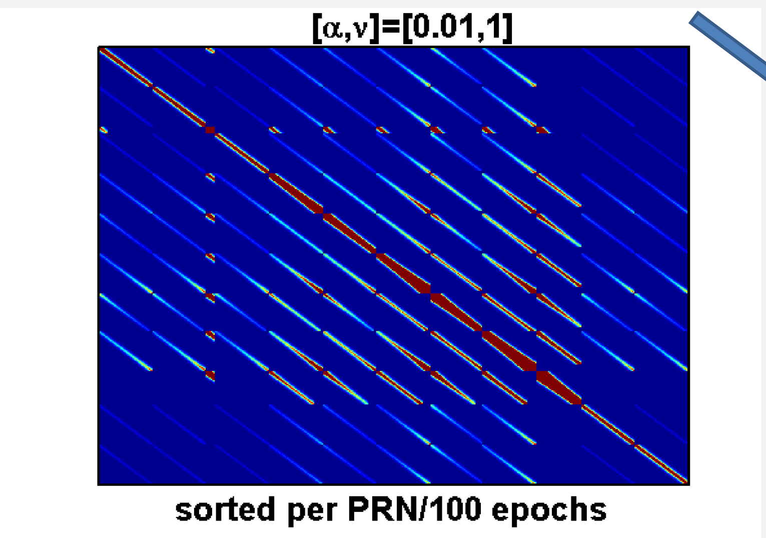

Fig.1 *Example for the VCM,* $[\alpha,\nu]=[0.01,1]$

Wrong stochastic model: bias analysis

An approximated VCM $\hat{\mathbf{W}}$ results in a bias in the least-squares solution $\hat{\mathbf{x}}_0$. We call **A** the design matrix, $\mathbf{W}_0$ the true VCM, and $\hat{\mathbf{x}}$ the solution under $\hat{\mathbf{W}}$.
$\hat{\mathbf{x}}^1_{\mathbf{amb,fix}}$: first float ambiguity solution, $\hat{\mathbf{x}}^2_{\mathbf{amb,fix}}$ second one, μ_R the threshold for the ratio test (usually 0.5)
Following quantities are for instance impacted by an incorrect stochastic model:

The estimates Position, float ambiguity vector	Aposteriori variance factor	Ratio test
$\hat{\mathbf{x}}=\hat{\mathbf{x}}_0-(\mathbf{A}^T\hat{\mathbf{W}}^{-1}\mathbf{A})^{-1}\mathbf{A}^T\Delta\mathbf{P}\mathbf{v}_0=\hat{\mathbf{x}}_0+\Delta\mathbf{x},$ $\mathbf{P}=\mathbf{P}_0+\Delta\mathbf{P},\ \mathbf{P}_0=\mathbf{W}_0^{-1},\ \mathbf{P}=\hat{\mathbf{W}}^{-1}$	$E(\hat{\sigma}^2_{\hat{W}})=\sigma^2+..$ $tr\{(\mathbf{I}+\hat{\mathbf{W}}^{-1}\mathbf{A}(\mathbf{A}^T\hat{\mathbf{W}}^{-1}\mathbf{A})^{-1}\mathbf{A}^T)\hat{\mathbf{W}}^{-1}\Delta\mathbf{W}\}\frac{\sigma^2}{n-u}$	$R=\frac{\lVert\hat{\mathbf{x}}^1_{\mathbf{amb,fix}}-\hat{\mathbf{x}}_{\mathbf{amb,float}}\rVert_{\mathbf{Q_A}}}{\lVert\hat{\mathbf{x}}^2_{\mathbf{amb,fix}}-\hat{\mathbf{x}}_{\mathbf{amb,float}}\rVert_{\mathbf{Q_A}}}=\frac{d_1}{d_2}\le\mu_R$

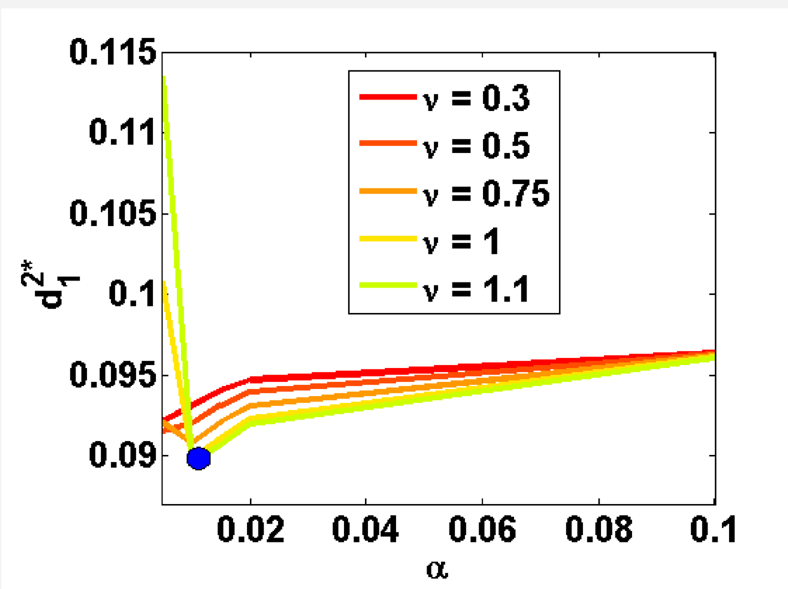

Fig.2 Euclidian distance float-fixed ambiguities (Monte Carlo simulations with known ambiguity vector).
$d_1^{2*}=\lVert\hat{\mathbf{x}}^1_{\mathbf{A,fix}}-\hat{\mathbf{x}}_{\mathbf{A,float}}\rVert_{ID}$
Matern parameters are varied around $[\alpha,\nu]_0=[0.01,1]$. Correlations are vanishing as α grows.

► Distance float-fixed has a minimum when correlations are correctly considered (minimum bias)
► If unknown, the correlation length should not be underestimated.
► Neglecting correlations leads to a higher distance (higher ratio test value simultaneously).

References

Kermarrec G, Schön S (2014) On the Mátern covariance family: a proposal for modeling temporal correlations based on turbulence theory. Journal of Geodesy 88:1061-1079
Kermarrec G, Schön S (2017) A priori fully populated covariance matrices in least-squares adjustment – case study: GPS relative positioning. Journal of Geodesy 91(5):465-484
Blewitt G (1998) GPS Data Processing Methodology: From Theory to Applications. In Teunissen PJG and Kleusberg A (Eds.) GPS for Geodesy (pp231-270). 2nd ed. Springer-Verlag Berlin Heidelberg New York

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Methodology: taking correlations into account

Description of the observations:

EPN network
Short baseline: KRAW-KRA1
Long baseline: KRAW-CRTM
1Hz observations, unpreprocessed, no additional corrections
RTK positioning (double differences)
Coordinates are known in advance (IGS long term)
$\sigma_0=2-4mm$

Methodology:
Position and ambiguities are computed simultaneously in a least-squares adjustment using the Lambda method. Validation is made with the ratio test with 0.5 as threshold
Different VCM are used:
► *FULLY model*: fully populated VCM with $[\alpha,\nu]=[0.01,1]$
► *ELEV model*: cosine diagonal VCM (heteroscedasticity)
► *ID model*: homoscedasticity

All matrices are scaled to 1 for a satellite at 90° elevation, no noise matrix is added
The maximum batch length (number of epochs per satellite) is varied from 10 to 400s
The mean of the given quantities over 30 batches is computed:
► 3Drms (global indicator)
► aposteriori variance factor (correctness of the solution)

Whitening effect of the VCM on the observations

Double differenced observations $\mathbf{l}$ are whitened with ELEV and FULLY double differenced VCM, i.e. $\sqrt{\mathbf{W}_{\mathbf{DD}}}\,\mathbf{l}$. Example for KRAW-CRTM, batch length 1500s

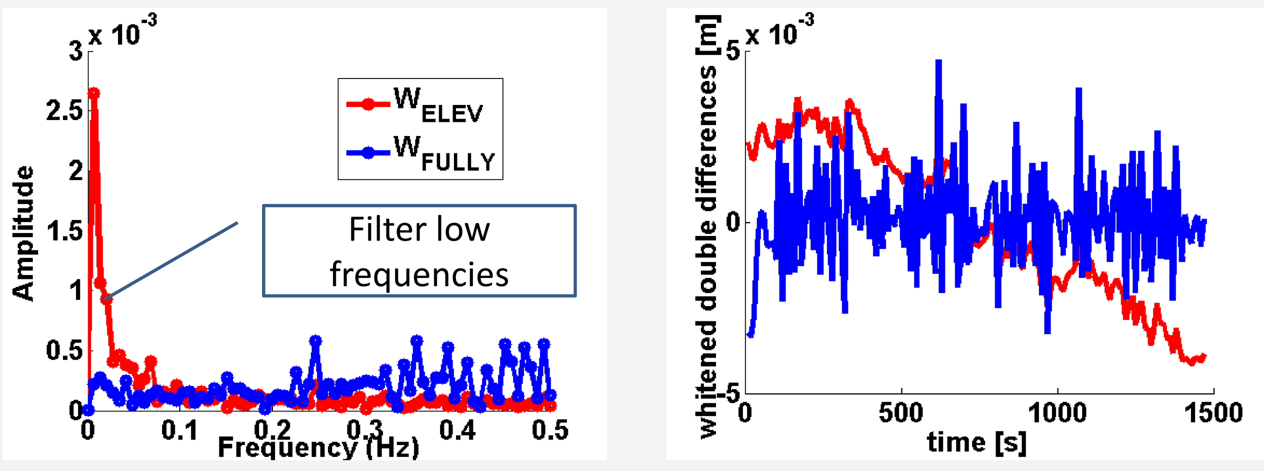


Fig.3 right: *Whitened time series* $[\alpha,\nu]=[0.01,1]$ ***left:*** *Corresponding Fourier Analysis (Amplitude)*

► More homogeneous frequencies repartition with FULLY model, the drift and mean are corrected compared with ELEV model.

RTK case study: results

Case long baseline

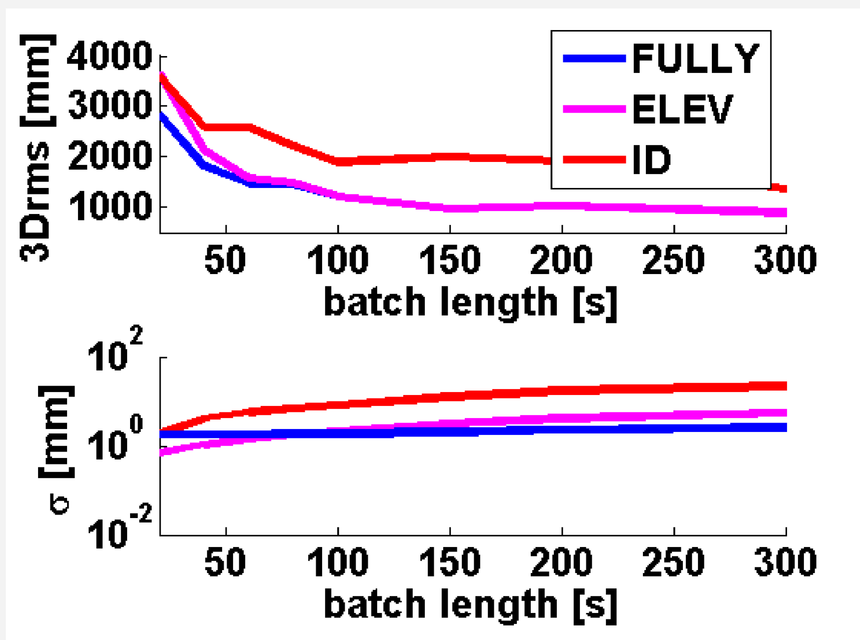
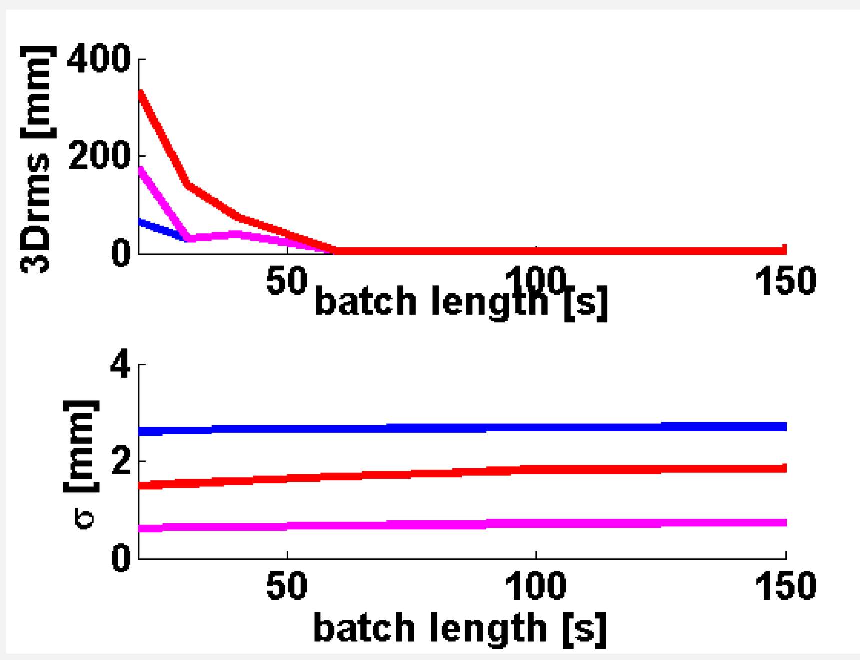


Fig.4 *Case study KRAW-CRTM: top 3Drms, bottom* $\hat{\sigma}$

►Ambiguities are let float depending on the ratio test
►Batch length increases: differences between the models less important, solution is more robust to changes of the stochastic model (FULLY or ELEV). The ID model represents a crude approximation.
►The 3Drms is improved by up to 80 cm for batches <20s, the aposteriori variance factor is not underestimated and stays constant for all batch lengths
►The Up component is improved by 60 cm with respect to the ELEV model for batch length 50 s, 20 cm for batches 100 s

Case short baseline



►Ambiguities are fixed from batch length 60 epochs: difference between FULLY and ELEV model is at the submm level
►The aposteriori variance factor is less biased under the FULLY model
►For batch length 20s, the RMS of the Up component is 30 cm smaller with FULLY than with ELEV

Fig.5 *Case study KRAW-KRA1:* ***top*** *3Drms* ***bottom*** *aposteriori variance factor*

Conclusions

► Correlations should not be neglected for a less biased solution. The bias of the least-squares solution is sensible to underestimation of the correlation length
► The proposed model allows a description of the elevation dependent GPS phase correlations
► The impact of correlations is more important for short batches of observations (<100 epochs), particularly when the ambiguities cannot be fixed with enough confidence
► FULLY models whitened correctly the observations
► The aposteriori variance factor and the precision are more reliable

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