Monitoring the variation of covariance matrices of line-of-sight BDS triple frequency signals

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GNSS functional and statistic models

Geometry-based observational models:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rho + \begin{bmatrix} \frac{1}{f_1^2} \\ \frac{1}{f_2^2} \\ \frac{1}{f_5^2} \end{bmatrix} q + \begin{bmatrix} \tilde{b}_1 \\ \tilde{b}_2 \\ \tilde{b}_5 \end{bmatrix} + \begin{bmatrix} \varepsilon_{p1} \\ \varepsilon_{p2} \\ \varepsilon_{p5} \end{bmatrix}, \quad \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \rho - \begin{bmatrix} \frac{1}{f_1^2} \\ \frac{1}{f_2^2} \\ \frac{1}{f_5^2} \end{bmatrix} q - \begin{bmatrix} \lambda_1 \tilde{N}_1 \\ \lambda_2 \tilde{N}_2 \\ \lambda_5 \tilde{N}_5 \end{bmatrix} + \begin{bmatrix} \varepsilon_{\varphi 1} \\ \varepsilon_{\varphi 2} \\ \varepsilon_{\varphi 5} \end{bmatrix}$$

$$\rho = \left| X_r - X^s \right| + CLK_r - CLK^s + T_r^s$$

$$\boldsymbol{\varepsilon}_P = \begin{bmatrix} \varepsilon_{P_1} & \varepsilon_{P_2} & \varepsilon_{P_5} \end{bmatrix}^T, \quad \boldsymbol{\varepsilon}_\phi = \begin{bmatrix} \varepsilon_{\phi_1} & \varepsilon_{\phi_2} & \varepsilon_{\phi_5} \end{bmatrix}^T$$

Statistical models:

$$E(\boldsymbol{\varepsilon}_p) = \mathbf{0}, Cov(\boldsymbol{\varepsilon}_p) = \boldsymbol{\Sigma}_p, E(\boldsymbol{\varepsilon}_\phi) = \mathbf{0}, Cov(\boldsymbol{\varepsilon}_\phi) = \boldsymbol{\Sigma}_\phi$$

We aim to estimate the covariance matrices $\boldsymbol{\Sigma}_\mathbf{P}, \boldsymbol{\Sigma}_\phi$ **from time to time**

Geometry-free/Ionosphere-free (GFIF) - functional models

- Four linearly independent and the effects of code and phase noises are separable (Estey & Meertens, 1999, for GFIF1 to GFIF3)

GFIF$_1$: $$P_1 - \frac{\beta_2+1}{\beta_2-1}\phi_1 + \frac{2}{\beta_2-1}\phi_2 = \left(\tilde{b}_1 + \frac{\beta_2+1}{\beta_2-1}\lambda_1\tilde{N}_1 - \frac{2}{\beta_2-1}\lambda_2\tilde{N}_2\right) + \varepsilon_{mp1}$$

GFIF$_2$: $$P_2 - \frac{2\beta_2}{\beta_2-1}\phi_1 + \frac{\beta_2+1}{\beta_2-1}\phi_2 = \left(\tilde{b}_2 + \frac{2\beta_2}{\beta_2-1}\lambda_1\tilde{N}_1 - \frac{\beta_2+1}{\beta_2-1}\lambda_2\tilde{N}_2\right) + \varepsilon_{mp2}$$

GFIF$_3$: $$P_5 - \frac{2\beta_3}{\beta_3-1}\phi_1 + \frac{\beta_3+1}{\beta_3-1}\phi_5 = \left(\tilde{b}_5 + \frac{2\beta_3}{\beta_3-1}\lambda_1\tilde{N}_1 - \frac{\beta_3+1}{\beta_3-1}\lambda_5\tilde{N}_5\right) + \varepsilon_{mp5}$$

GFIF$_{WL}$: $$\alpha\frac{f_1\phi_1 - f_2\phi_2}{f_1-f_2} + (1-\alpha)\frac{f_2\phi_2 - f_5\phi_5}{f_2-f_5} - \phi_1 = \lambda_1\tilde{N}_1 - \alpha\frac{f_1\lambda_1\tilde{N}_1 - f_2\lambda_2\tilde{N}_2}{f_1-f_2} - (1-\alpha)\frac{f_2\lambda_2\tilde{N}_2 - f_5\lambda_5\tilde{N}_5}{f_2-f_5} + \varepsilon_{mpWL}$$

Lumped signal bias (LSB)

where,

$$\tilde{b}_i \equiv b_{i,r} - b_i^s \qquad \tilde{N}_i \equiv N_i - B_{i,r} + B_i^s \qquad \alpha = \frac{f_1^2 + f_2 f_5}{f_1^2 - f_1 f_5}, \beta_2 = \frac{f_1^2}{f_2^2}, \beta_3 = \frac{f_1^2}{f_5^2}$$

GFIF stochastic models

$$\boldsymbol{\varepsilon}_P = \begin{bmatrix} \varepsilon_{P_1} & \varepsilon_{P_2} & \varepsilon_{P_5} \end{bmatrix}^T$$

$$\boldsymbol{\varepsilon}_\phi = \begin{bmatrix} \varepsilon_{\phi_1} & \varepsilon_{\phi_2} & \varepsilon_{\phi_5} \end{bmatrix}^T$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{mp1} & \varepsilon_{mp2} & \varepsilon_{mp5} & \varepsilon_{mpWL} \end{bmatrix}^T$$

$$\boldsymbol{\varepsilon} = \mathbf{B}_1 \boldsymbol{\varepsilon}_P + \mathbf{B}_2 \boldsymbol{\varepsilon}_\phi$$

$$E(\boldsymbol{\varepsilon}) = \mathbf{0}, \qquad Cov(\boldsymbol{\varepsilon}_{mp}) = \boldsymbol{\Sigma}$$

$$\boldsymbol{\Sigma} = \mathbf{B}_1 \boldsymbol{\Sigma}_P \mathbf{B}_1^T + \mathbf{B}_2 \boldsymbol{\Sigma}_\phi \mathbf{B}_2^T$$

$$\mathbf{B}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_2 = \begin{bmatrix} -(\frac{2}{\beta_2 - 1} + 1) & \frac{2}{\beta_2 - 1} & 0 \\ \frac{-2\beta_2}{\beta_2 - 1} & \frac{2\beta_2}{\beta_2 - 1} - 1 & 0 \\ \frac{-2\beta_3}{\beta_3 - 1} & 0 & \frac{2\beta_3}{\beta_3 - 1} - 1 \\ \frac{f_1(f_2 + f_5)}{(f_1 - f_2)(f_1 - f_5)} & \frac{-f_2^2(f_1 + f_5)}{f_1(f_1 - f_2)(f_2 - f_5)} & \frac{f_5^2(f_1 + f_2)}{f_1(f_2 - f_5)(f_1 - f_5)} \end{bmatrix}$$

Contribution of code and phase noise terms

	GFIF	Coefficients of linear GFIF equations 1,2,5,WL						Contribution of pseudorange and phase noise terms (unit: cm).	
		P_1	P_2	P_5	ϕ_1	ϕ_2	ϕ_5	$\sigma_P=$ 30cm	$\sigma_\phi=$ 2 mm
GPS signals	$GFIF_1$	1	0	0	-4.0915	3.0915	0	**30**	**1.03**
	$GFIF_2$	0	1	0	-5.0915	-4.0915		**30**	**1.16**
	$GFIF_3$	0	0	1	-3.5212		2.5212	**30**	**0.87**
	$GFIF_{WL}$	0	0	0	27.293	-147.961	120.668	**0**	**38.57**
Beidou signals	$GFIF_1$	1	0	0	-4.8874	3.88740	0	**30**	**1.25**
	$GFIF_2$	0	1	0	-4.9743	0	3.9743	**30**	**1.27**
	$GFIF_3$	0	0	1	-5.8874	4.8874	0	**30**	**1.53**
	$GFIF_{WL}$	0	0	0	37.3188	-158.8910	121.5722	**0**	**40.70**

Multivariate Multiple Regression (MMR)

- Seemingly Unrelated Regression (SUR) models (Zellner,1962):
  ① Use p-degree polynomial function to describe a GFIF time series:

$$y_{j,i} = \mu_{i0} + (t_j - t_0)\mu_{i1} + \ldots + (t_j - t_0)^{p_i} \mu_{ip_i} + \varepsilon_{j,i}$$

  ② Group all the GFIF time series over n epochs together:

$$\begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \\ \mathbf{Y}_3 \\ \mathbf{Y}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{H} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{H} \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \\ \boldsymbol{\mu}_3 \\ \boldsymbol{\mu}_4 \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_{Y1} \\ \boldsymbol{\varepsilon}_{Y2} \\ \boldsymbol{\varepsilon}_{Y3} \\ \boldsymbol{\varepsilon}_{Y4} \end{bmatrix}$$

where:

$$\mathbf{Y}_i = \begin{bmatrix} y_{1,i} \\ y_{2,i} \\ \vdots \\ y_{n,i} \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} 1 & (t_1 - t_0) & \cdots & (t_1 - t_0)^{p_i} \\ 1 & (t_2 - t_0) & \cdots & (t_2 - t_0)^{p_i} \\ \vdots & \vdots & & \vdots \\ 1 & (t_n - t_0) & \cdots & (t_n - t_0)^{p_i} \end{bmatrix}$$

$$\boldsymbol{\mu}_i = \begin{bmatrix} \mu_{i0} \\ \mu_{i1} \\ \vdots \\ \mu_{ip_i} \end{bmatrix}, \quad \boldsymbol{\varepsilon}_{Yi} = \begin{bmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \vdots \\ \varepsilon_{ni} \end{bmatrix}$$

MMR-Least square solution

③ Write compactly:

$$\mathbf{Y} = \mathbf{H}_Y \boldsymbol{\mu} + \boldsymbol{\varepsilon}_Y \quad E(\varepsilon_Y) = 0, \boldsymbol{\Sigma}_Y = \boldsymbol{\Sigma} \otimes \mathbf{I}_n$$

③ Least square estimation (Zellner, 1962):

$$\hat{\boldsymbol{\mu}} = (\mathbf{H}_Y^T \boldsymbol{\Sigma}_Y^{-1} \mathbf{H}_Y)^{-1} \mathbf{H}_Y^T \boldsymbol{\Sigma}_Y^{-1} \mathbf{Y}$$

④ Residual vector of each GFIF time series (Zellner, 1962):

$$\mathbf{v_i} = (\mathbf{I_n} - \mathbf{H}(\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T)\mathbf{Y_i}$$

⑤ The variance matrix $\sum$ is estimated by (4-by-4)

$$\hat{\boldsymbol{\Sigma}} = \frac{\mathbf{v}^T \mathbf{v}}{n - p - 1} \longleftarrow \quad \mathbf{v} = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3 \quad \mathbf{v}_4]$$

⑥ Or the variance matrix over a sliding /piece-wise data window:

$$\hat{\boldsymbol{\Sigma}}(j) = \frac{\mathbf{v}^T(j)\mathbf{v}(j)}{m - 1}$$

Epoch-by-epoch updated 4-by-4 covariance matrix

'

MMR-Variance Component Estimation

⑦ Estimation of components of Σ_P and Σ_ϕ

- The apiori variance matrix:

Σ_p^0 : the upper-left 3-by-3 submatrix of $\hat{\Sigma} = \dfrac{\mathbf{v}^T\mathbf{v}}{n-p-1}$

Σ_ϕ^0 : $\Sigma_\phi^0 = \Sigma_p^0 \times 10^{-4}$

- Variance Component Estimation for θ_1, θ_2

$$\boldsymbol{\Sigma}_Y = (\theta_1 B_1 \boldsymbol{\Sigma}_p^0 B_1^T + \theta_2 B_2 \boldsymbol{\Sigma}_\phi^0 B_2^T) \otimes \mathbf{I}_n$$

VCE results (eg. MINQUE, LS-VCE)

(1) $\hat{\Sigma}_P = \hat{\theta}_1 \Sigma_p^0$, $\hat{\Sigma}_\phi = \hat{\theta}_2 \Sigma_\phi^0$

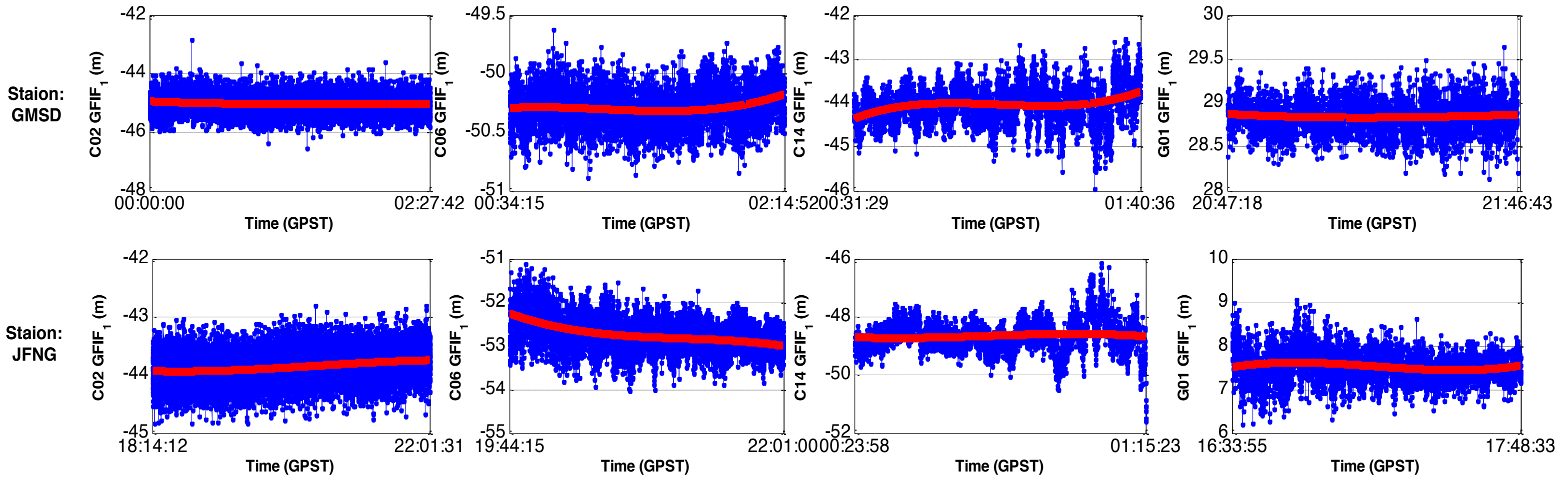
(2) $\hat{\Sigma}_P(j) = \hat{\theta}_1(j) \Sigma_p^0(j)$, $\hat{\Sigma}_\phi(j) = \hat{\theta}_2(j) \Sigma_\phi^0(j)$ → **Epoch-by-epoch updating**

For j=m,m+1,…,n

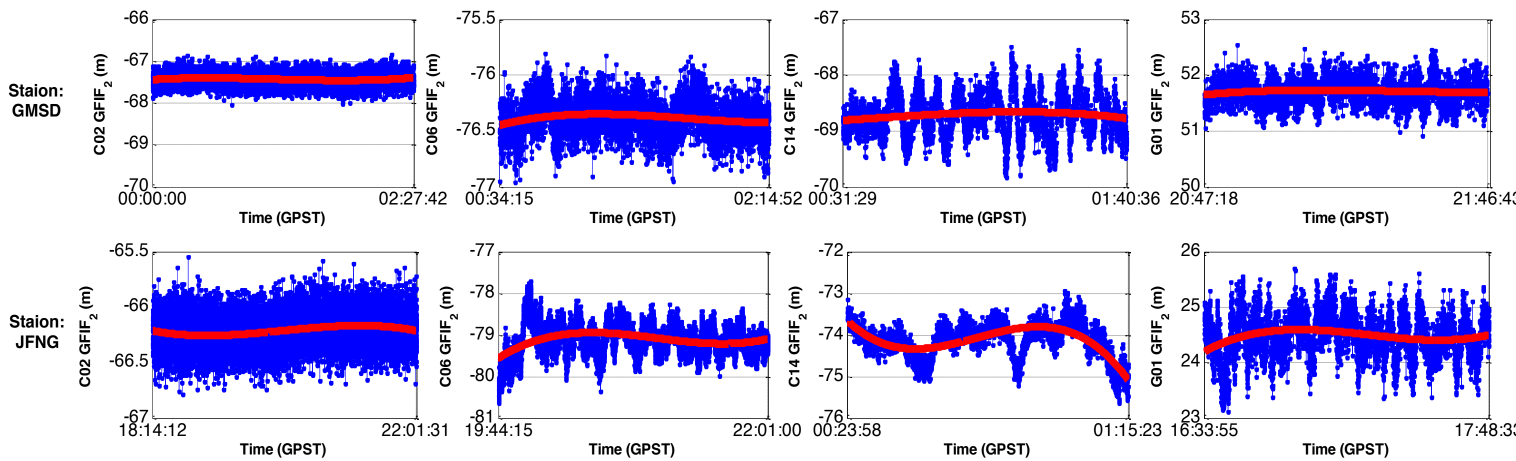
Estimation of overall variance scale parameters

GFIF vs. LSB fitting (GMSD, JFNG)

$GFIF_1$

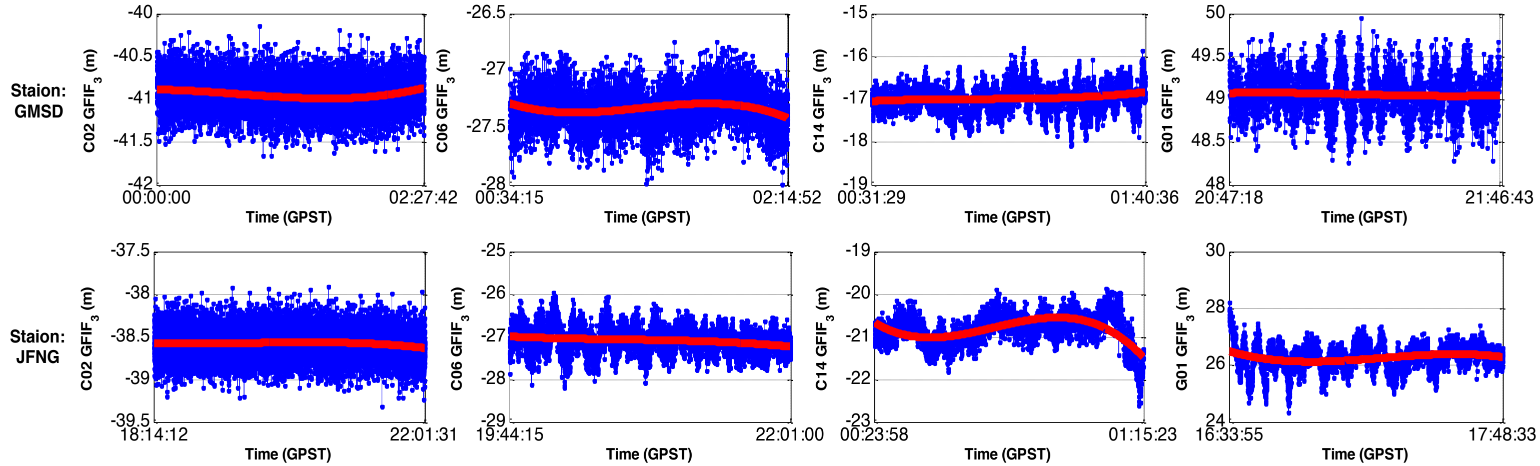


$GFIF_2$

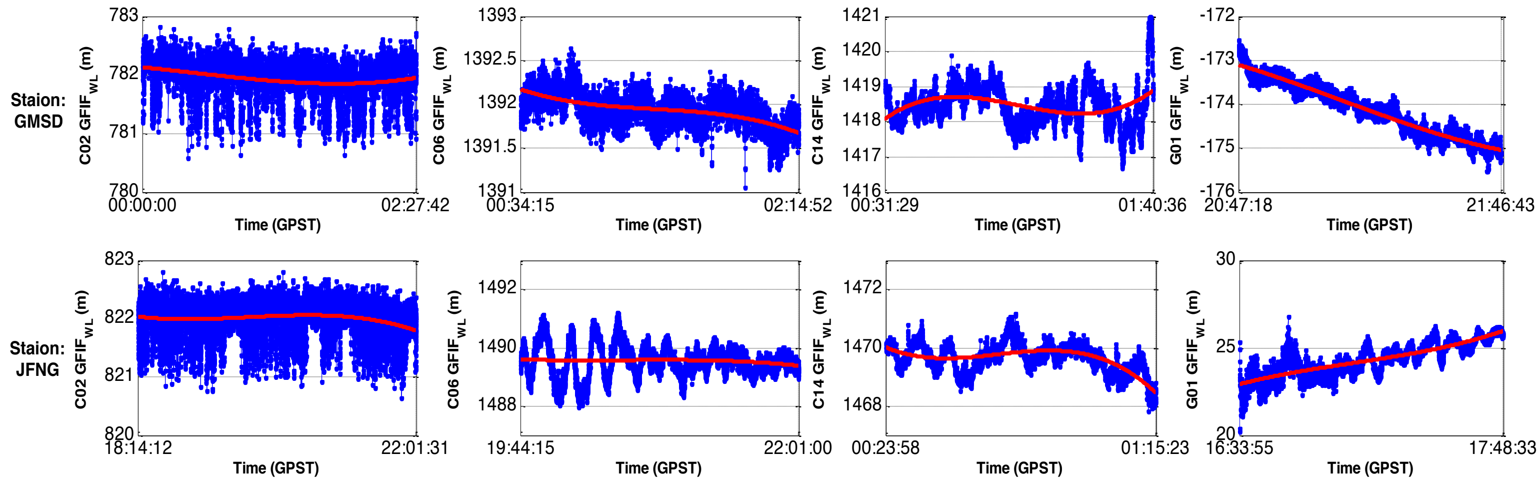


GFIF vs. LSB fitting (GMSD, JFNG)

$GFIF_3$



$GFIF_{WL}$



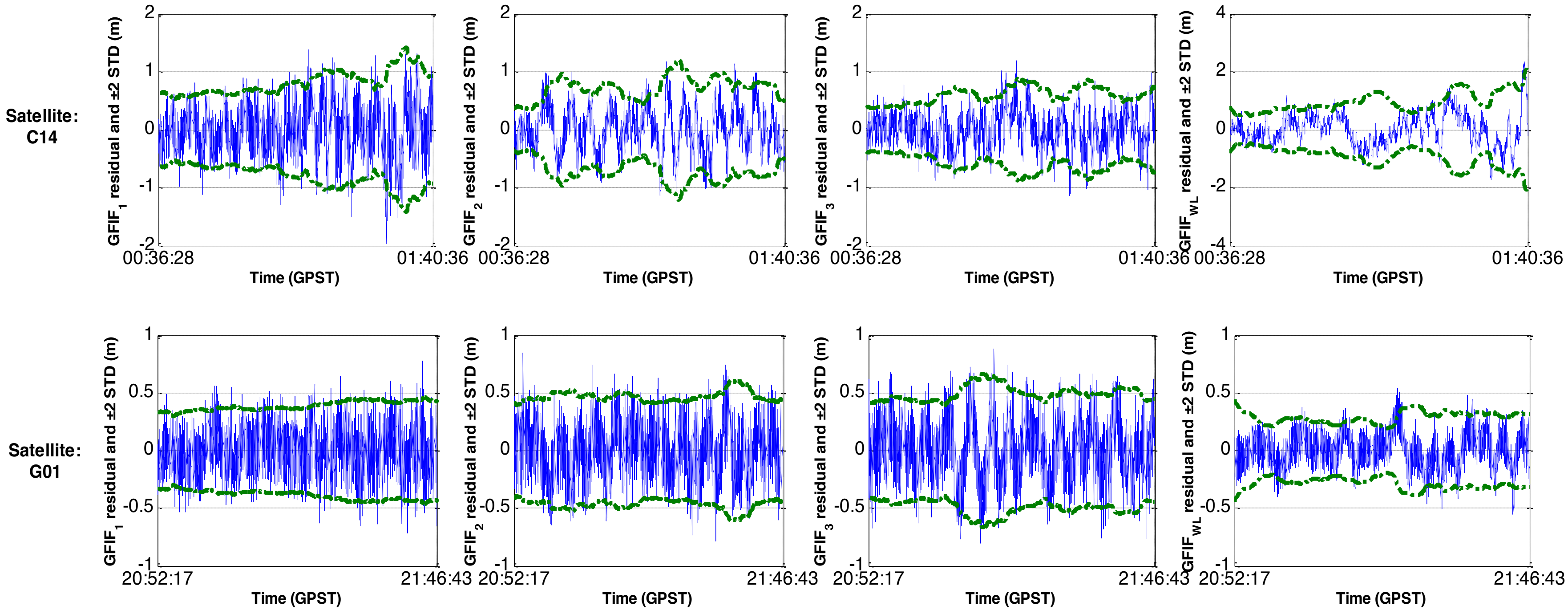
GFIF LSB fitting results:

- The lumped signal biases in all the GFIF time series are slowly time varying and are well-fitted by a p-degree polynomial (here p=3) over hours.

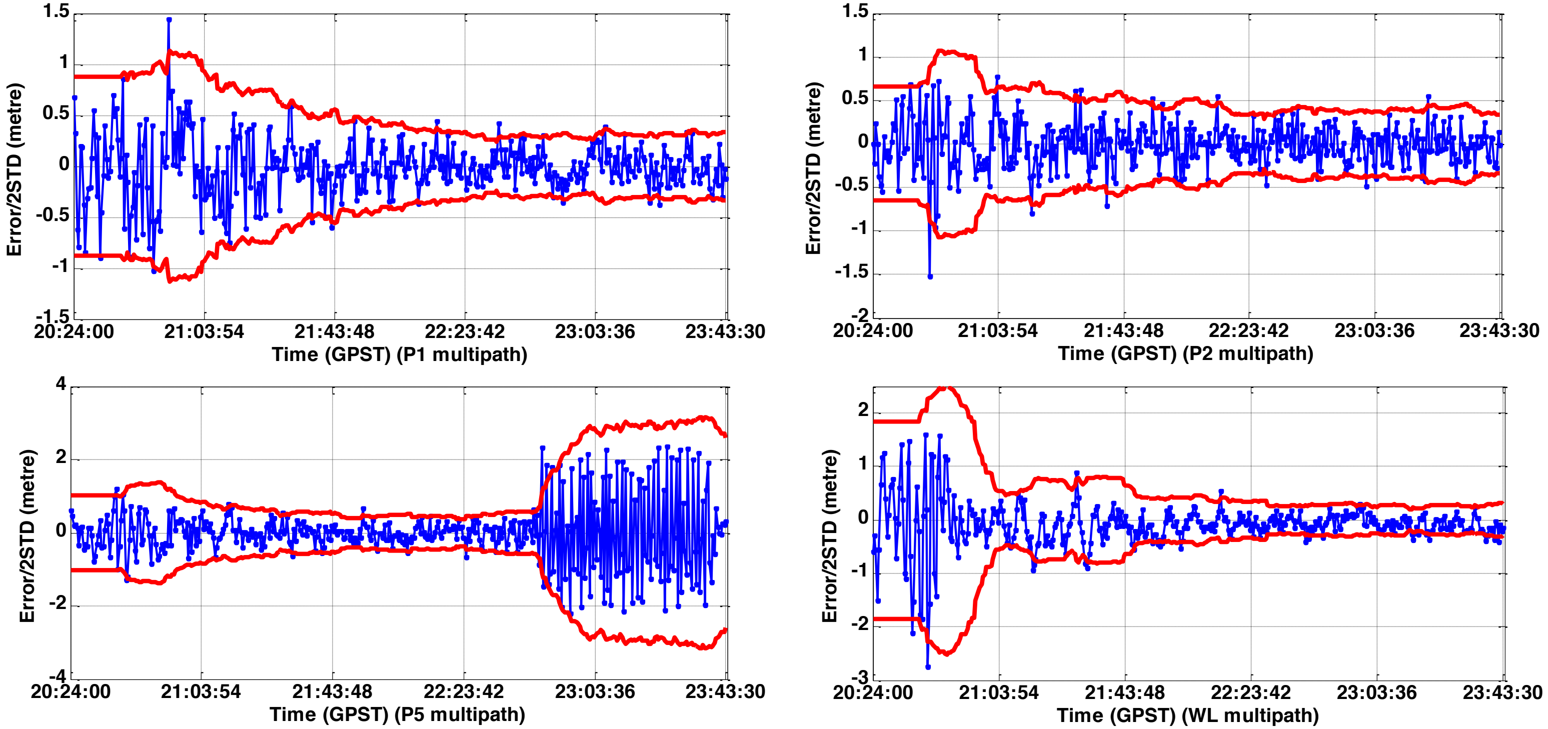
- All the $GFIF_{1\sim3}$ plots show a small range of variation within one meter. The $GFIF_{WL}$ bias curves for G01 show the variation of up to a few meters.

- The impact of fitting period and degrees of polynomials on the covariance analysis should be small, but is yet to be further investigated

GFIF residuals vs. ±2σ curves



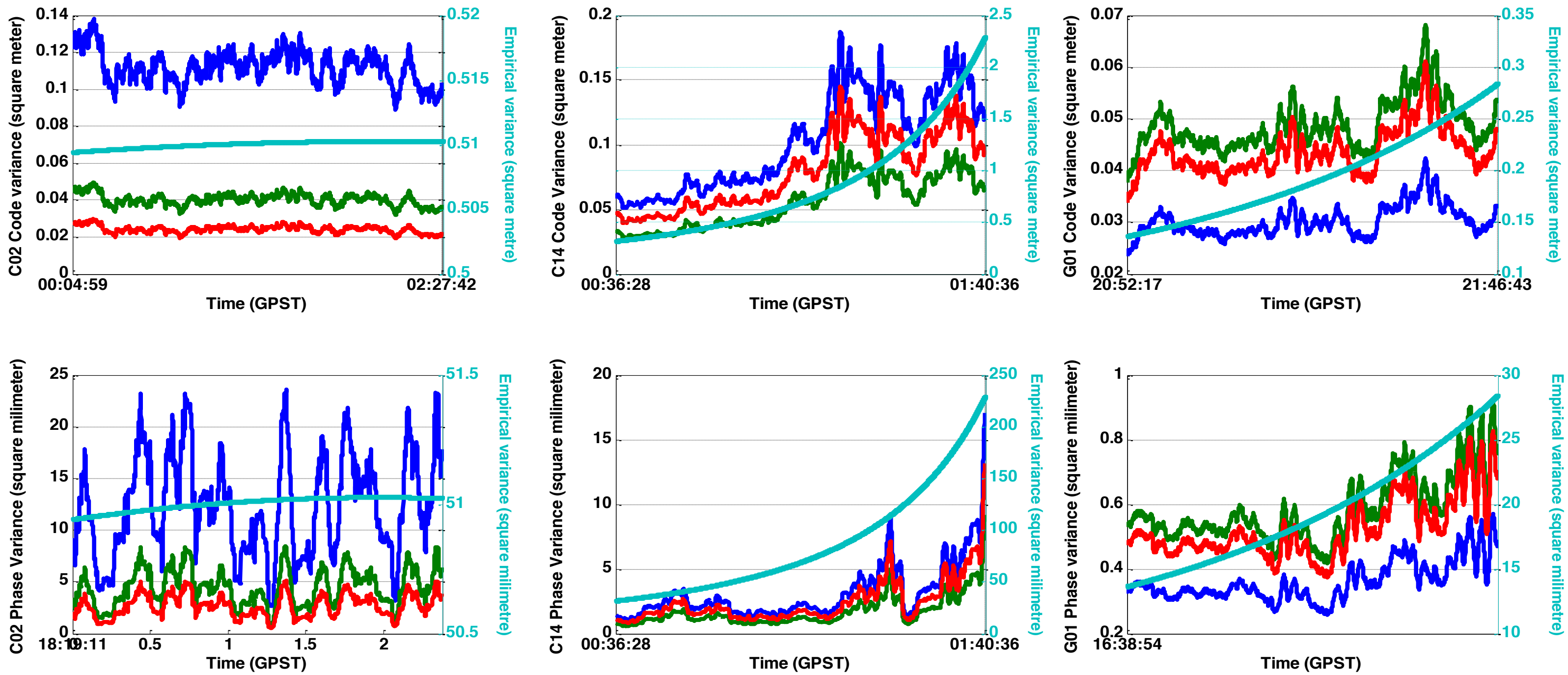
The slowly varying ±2 σ curves bound the peak-to-peak amplitude of each error time series tightly (within 95%), showing a good agreement between the estimated σ values and the actual GFIF errors.



The residuals of four GFIF models for the GMSD-G01 direction, against their ±2 σ curves (23 November 2013) over a moving data window of 15 min.

Variances (σ^2) of codes and phases

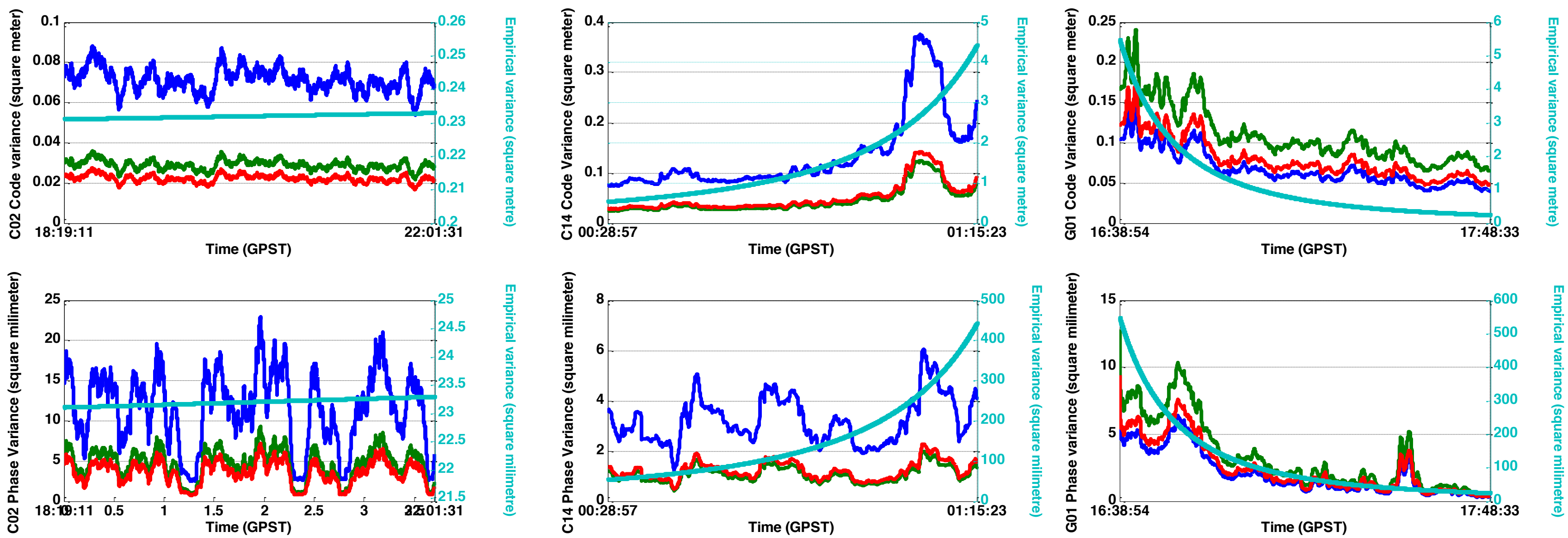
- Station GMSD



An empirical model:

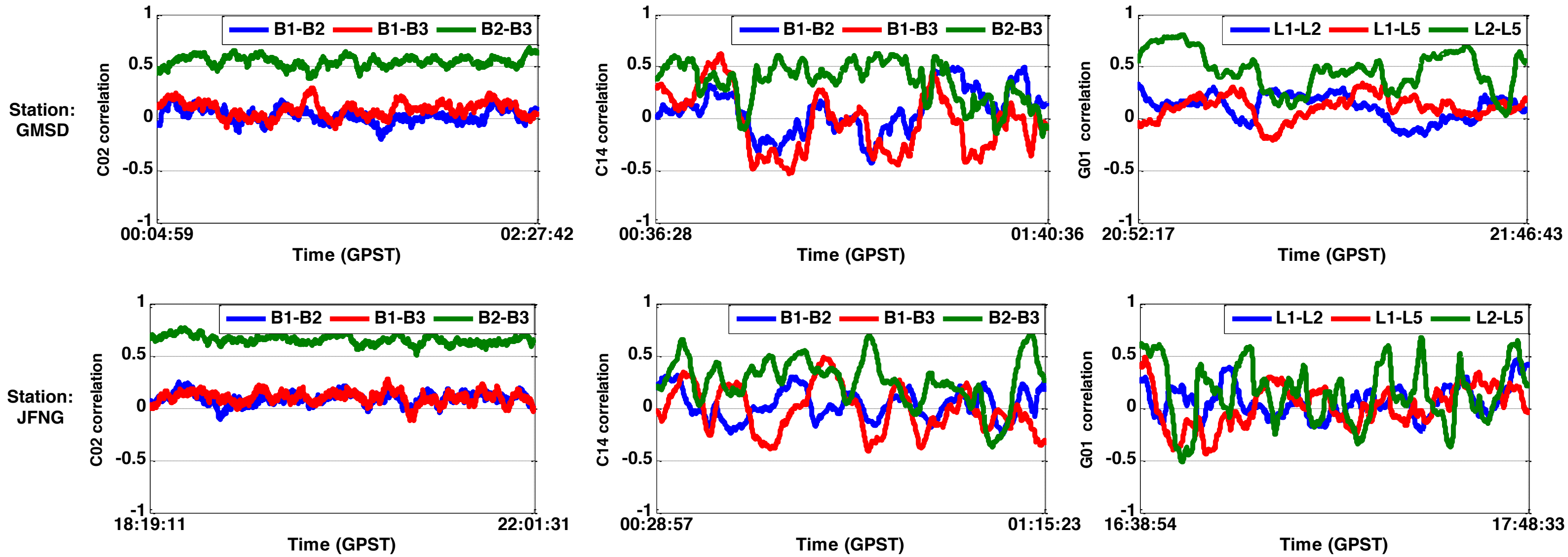
$$\sigma^2_{p(ele)} = \frac{\sigma_0^2}{\sin^2(ele)}, \sigma^2_{\phi(ele)} = \frac{10^{-4}\sigma_0^2}{\sin^2(ele)}$$

Variances σ^2 of codes and phases



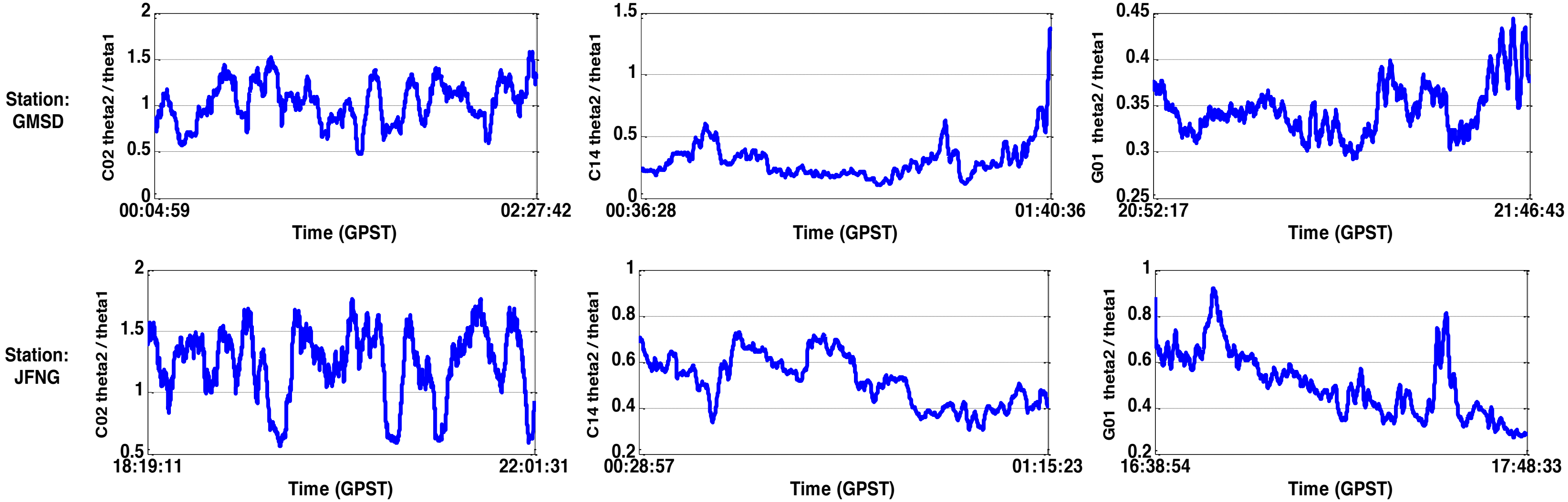
- ➢ **Match the trend of the elevation-dependence in general**
- ➢ **Reflect the spontaneous amplitude changes of the errors**
- ➢ **Noise levels are different and are not just related to elevation only.**
    - ➢ **phase σ : 1 to 4 mm, code σ : 15 to 30 cm, sometimes, 60cm**

Cross-correlation (codes)



- **B2 and B3 (L2 and L5) shows higher cross-correlation.**

Ratio of phase σ to code σ values



- **Assumed value: 1/100**
- **Estimated value: 1/500 ~ 1/50**

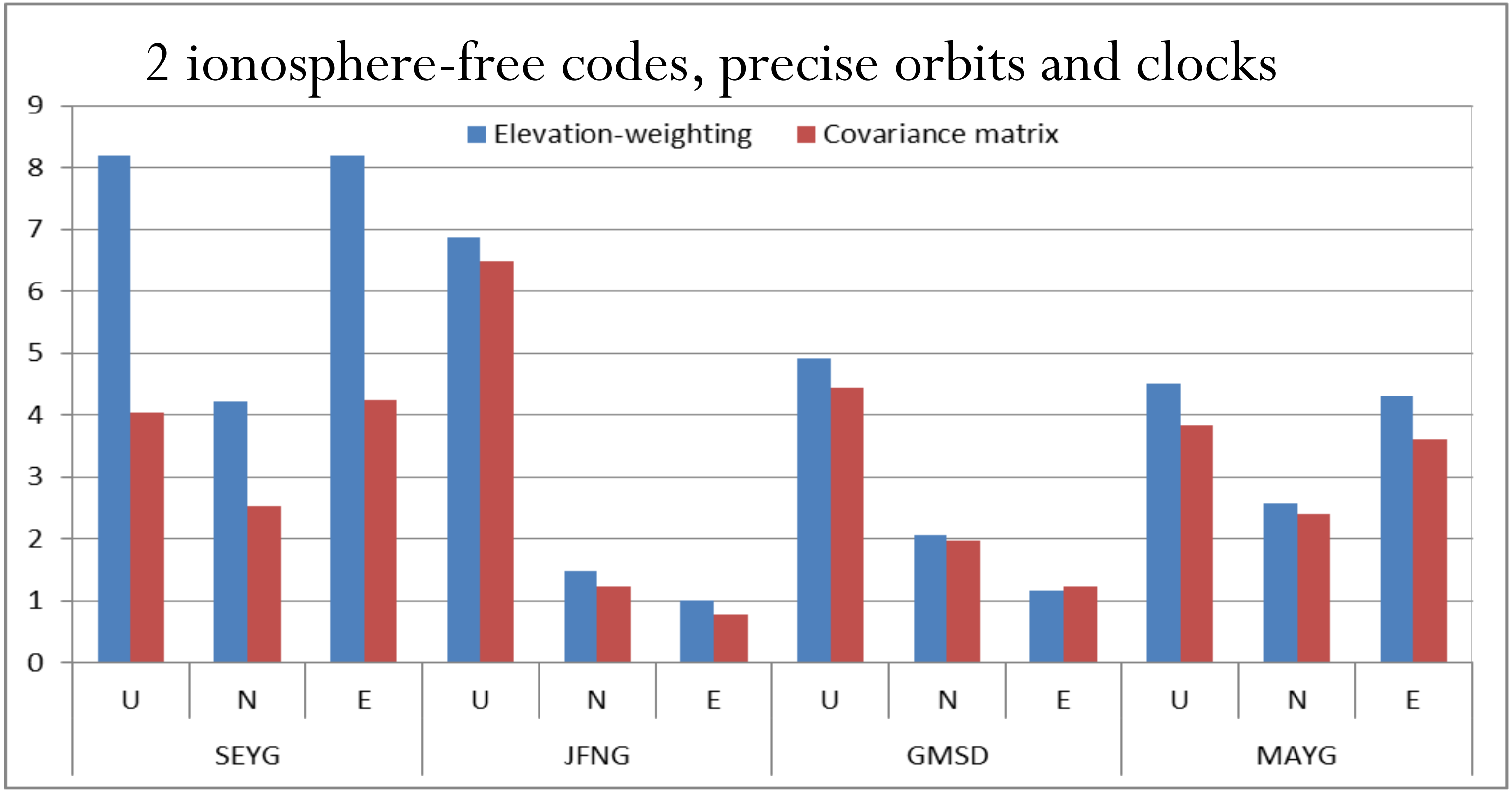
Receiver covariance matrix exchange - A rinex-like format

```
> 2015 01  02 13 34 10.0000000 10
C01  2.7136 0.2866 0.2959 2.6616 1.7037 2.0595 1.6225 0.1714 0.1770 1.5914 1.0187 1.2314
C02  9.7385 1.2565 0.7537 4.4240 1.7188 2.6069 9.2857 2.1982 1.3186 7.7396 3.0070 4.5608
C03  4.5923 0.6752 0.4655 2.9355 1.7417 2.2010 3.6825 0.5414 0.3733 2.3540 1.3967 1.7650
C04  4.6486 0.5576 0.4011 3.4688 1.8767 2.3892 1.0932 0.1311 0.0943 0.8158 0.4413 0.5619
C05  9.6412 0.4831 0.0367 4.2388 1.3722 2.3946 3.4281 0.1310 0.0099 1.1495 0.3721 0.6494
C06  9.2404 0.1113 0.2019 6.1956 0.3975 8.5236 0.7277 0.0088 0.0159 0.4879 0.0313 0.6712
C07  3.1136 0.2851 0.0027 2.2133 1.1418 2.3682 1.0761 0.0985 0.0009 0.7649 0.3946 0.8185
C08  5.2063 0.3947 0.3435 3.0696 1.0100 5.2927 0.8850 0.0671 0.0584 0.5218 0.1717 0.8996
C09  5.8737 0.5899 1.9516 4.4030 0.5675 6.8229 0.8876 0.0891 0.2949 0.6654 0.0858 1.0311
C10  3.6962 0.3206 0.1520 3.0677 0.9479 3.0721 0.6698 0.0581 0.0275 0.5559 0.1718 0.5567
> 2015 01  02 13 34 11.0000000 10
C01  2.7059 0.2858 0.2951 2.6541 1.6990 2.0537 1.6274 0.1719 0.1775 1.5963 1.0218 1.2352
C02  9.7522 1.2578 0.7545 4.4287 1.7206 2.6097 9.8528 2.2541 1.3521 7.9366 3.0835 4.6768
C03  4.5729 0.6723 0.4635 2.9231 1.7344 2.1917 3.7091 0.5453 0.3760 2.3709 1.4068 1.7777
C04  4.6068 0.5525 0.3975 3.4376 1.8598 2.3677 1.0920 0.1310 0.0942 0.8148 0.4408 0.5612
C05  9.6307 0.4827 0.0367 4.2353 1.3711 2.3926 3.4325 0.1312 0.0100 1.1510 0.3726 0.6502
C06  9.2453 0.1114 0.2021 6.1989 0.3977 8.5282 0.7316 0.0088 0.0160 0.4905 0.0315 0.6749
C07  3.1142 0.2852 0.0027 2.2137 1.1420 2.3686 1.0785 0.0988 0.0009 0.7667 0.3955 0.8203
C08  5.2152 0.3953 0.3441 3.0748 1.0117 5.3017 0.8908 0.0675 0.0588 0.5252 0.1728 0.9056
C09  5.8946 0.5920 1.9585 4.4186 0.5695 6.8471 0.8876 0.0891 0.2949 0.6653 0.0857 1.0310
C10  3.7187 0.3225 0.1529 3.0863 0.9537 3.0908 0.6640 0.0576 0.0273 0.5511 0.1703 0.5519
```

Units: square decimetre (1-6) & square millimetre (7-12)

Single point positioning RMS results(m)-BDS

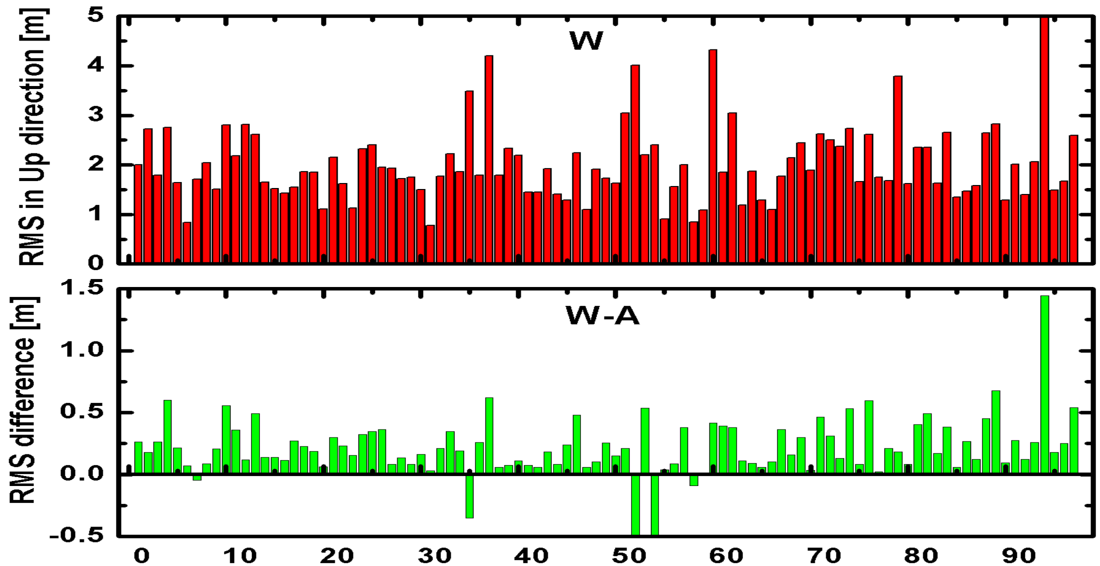
BDS SPP at 4 MGEX stations SEYG, MAYG, JFNG,GMSD



Unit: metre	Station: SEYG			Station: JFNG			Station: GMSD			Station: MAYG		
	U	**N**	**E**	**U**	**N**	**E**	**U**	**N**	**E**	**U**	**N**	**E**
Elevation-based Variance	8.20	4.23	8.19	6.87	1.47	1.01	4.91	2.06	1.16	4.51	2.59	4.31
Estimated variance	4.05	2.53	4.24	6.49	1.23	0.78	4.43	1.97	1.23	3.85	2.40	3.61
Improvement %	**50.1**	40.3	48.2	5.5	15.8	23.1	9.7	4.3	-5.9	**16.6**	**7.4**	**16.2**

Single point positioning RMS results(m)-GPS

- RMS statistics from 95 MGEX stations (DOY 182 2014)
  (1 ionosphere-free code, IGS precise orbits and clocks)



	U	N	E	3D
Scheme W: elevation	2.03	0.91	0.80	2.36
Scheme A: covariance	1.81	0.81	0.70	2.10
W-A	0.22	0.10	0.10	0.26

11%

Conclusion

- The time variation of GNSS code covariance matrices in each line-of-sight can be monitored using GFIF combinations through the proposed approach.
- The ratio of phase to code noise σ values varies from time to time, thus the variation of phase covariance matrices can also be obtained
- The derived σ reflects the varying observational environment better than the elevation-based model.
- SPP results derived with the estimated covariance matrices show the evident improvement as compared with the SPP results from an empirical model, based on extensive data analysis from 99 stations

Thank you!
Questions?