# A priori solar radiation pressure model for QZSS Michibiki satellite

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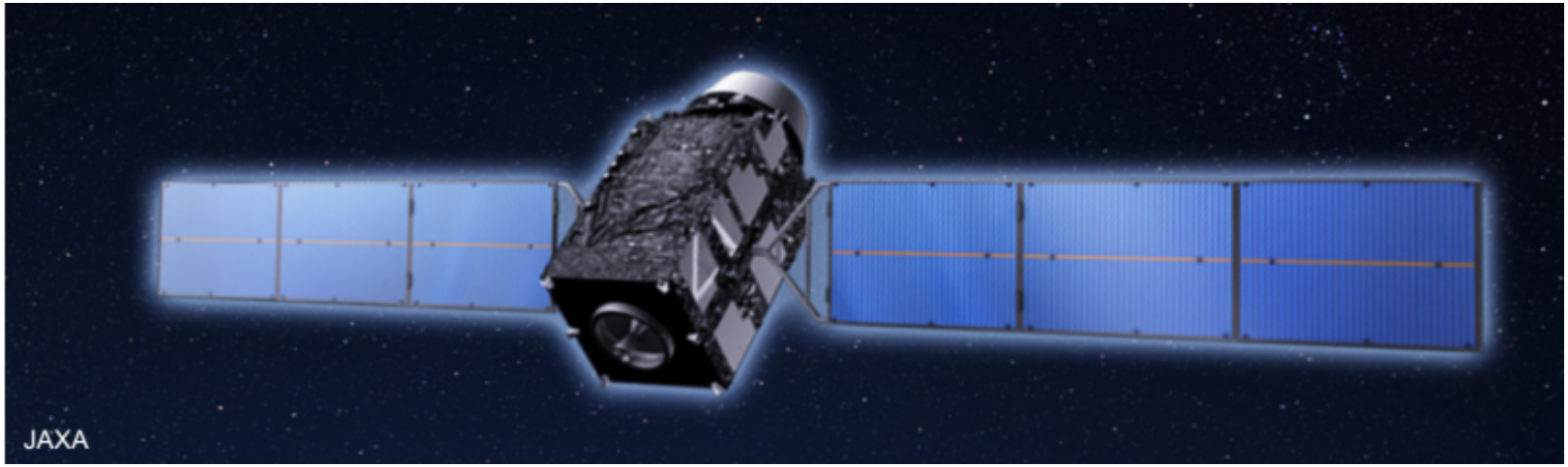
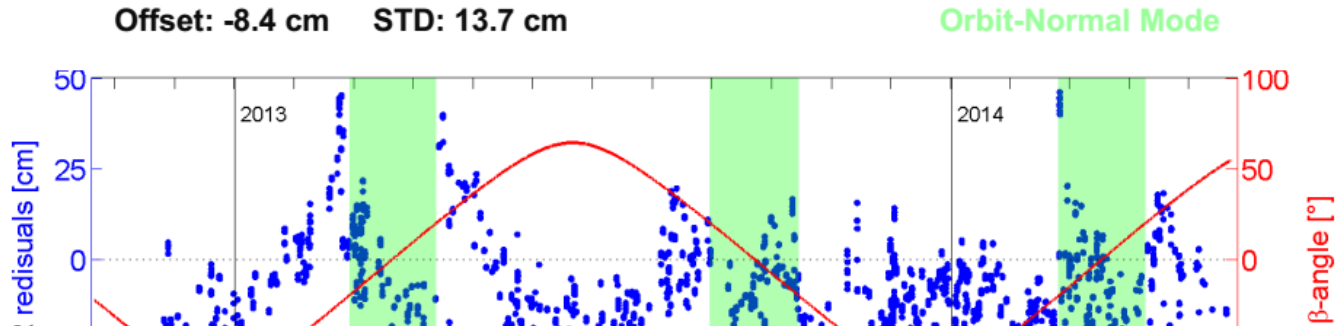
# Quasi-Zenith Satellite System (QZSS)

- Japanese regional augmentation system for GPS in the Asia-Pacific region
- Launch of QZS-1 in September 2010, PRN J01, Michibiki
- Inclined Geosynchronous Orbit (IGSO)
- Attitude control mode: Yaw-steering (YS) and Orbit-Normal (ON) mode



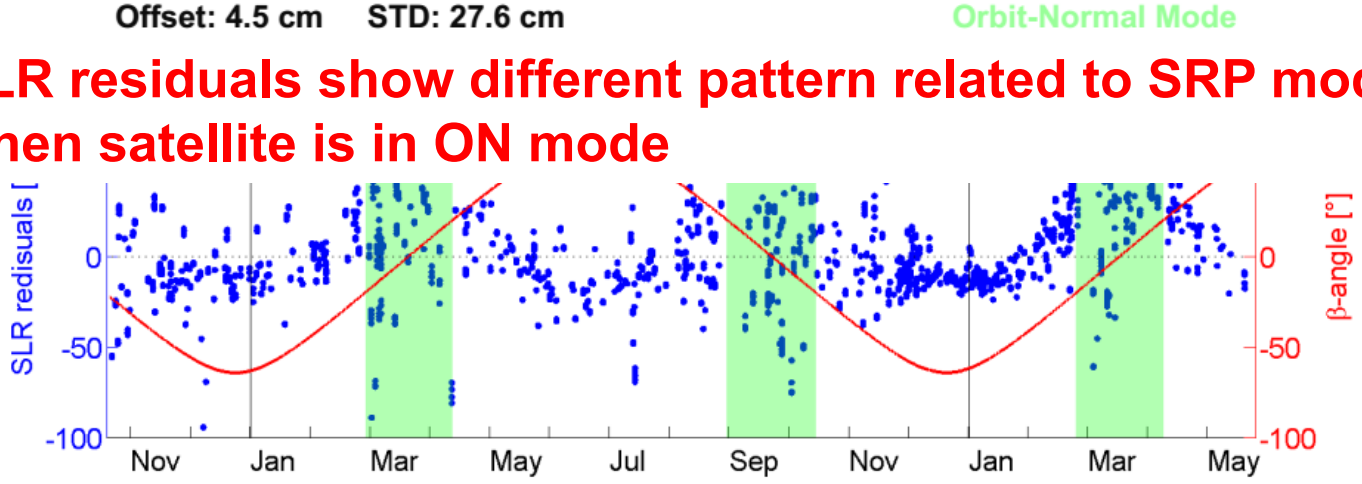
# QZSS orbit: SLR residuals

QZF




**SLR residuals show clear β-angle dependent systematic errors when satellite is in YS mode**

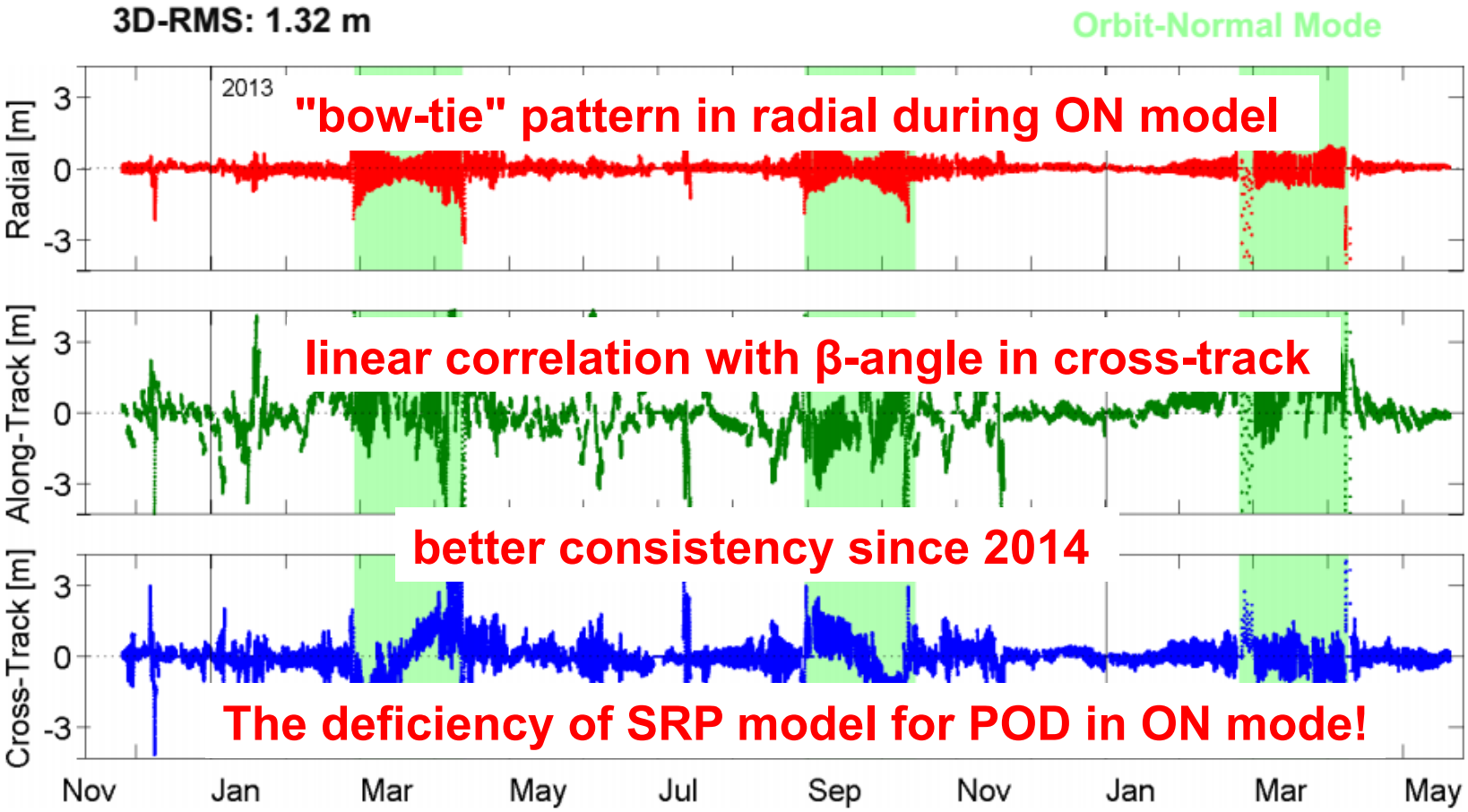
TUM




**SLR residuals show different pattern related to SRP model used when satellite is in ON mode**

Steigenberger P, Kogue S (2014) IGS-MGEX: QZSS orbit and clock determination

# QZSS orbit: comparison (QZF vs TUM)




Steigenberger P, Kogue S (2014) IGS-MGEX: QZSS orbit and clock determination

# CODE SRP models

$$a_{emp} = a_0 + D(u)e_D + Y(u)e_Y + B(u)e_B$$

$$D(u) = D_0 + \sum_{i=1}^{n_D} \left\{ D_{i,c} \cos C_i^D u + D_{i,s} \sin S_i^D u \right\}$$

$$Y(u) = Y_0 + \sum_{i=1}^{n_Y} \left\{ Y_{i,c} \cos C_i^Y u + Y_{i,s} \sin S_i^Y u \right\}$$

$$B(u) = B_0 + \sum_{i=1}^{n_B} \left\{ B_{i,c} \cos C_i^B u + B_{i,s} \sin S_i^B u \right\}$$

| Reference | $n_D$ | $n_Y$ | $n_B$ | Periodic terms |
|---|---|---|---|---|
| (Beutler et al 1994) | 1 | 1 | 1 | $C_i^D, S_i^D = i; C_i^Y, S_i^Y = i; C_i^B, S_i^B = i$ |
| (Springer et al 1999) | 0 | 0 | 1 | |
| (Arnold et al 2015) | 2*i* | 0 | 2*i*-1 | $C_i^D, S_i^D = 2i; C_i^B, S_i^B = 2i - 1$ |

# Adjustable box-wing model

- Modelded the satellite as "a box and two wings" (Rodríguez-Solano et al 2012)

$$\boldsymbol{a}_{srp,sp} = -\lambda \frac{S_0}{c} \frac{A}{M} \left( \frac{r_{AU}}{\|\boldsymbol{r}_s\|} \right)^2 \cdot \left( 1 + \frac{2}{3}\nu + \frac{1}{3}\nu\mu \right) \boldsymbol{e}_n$$

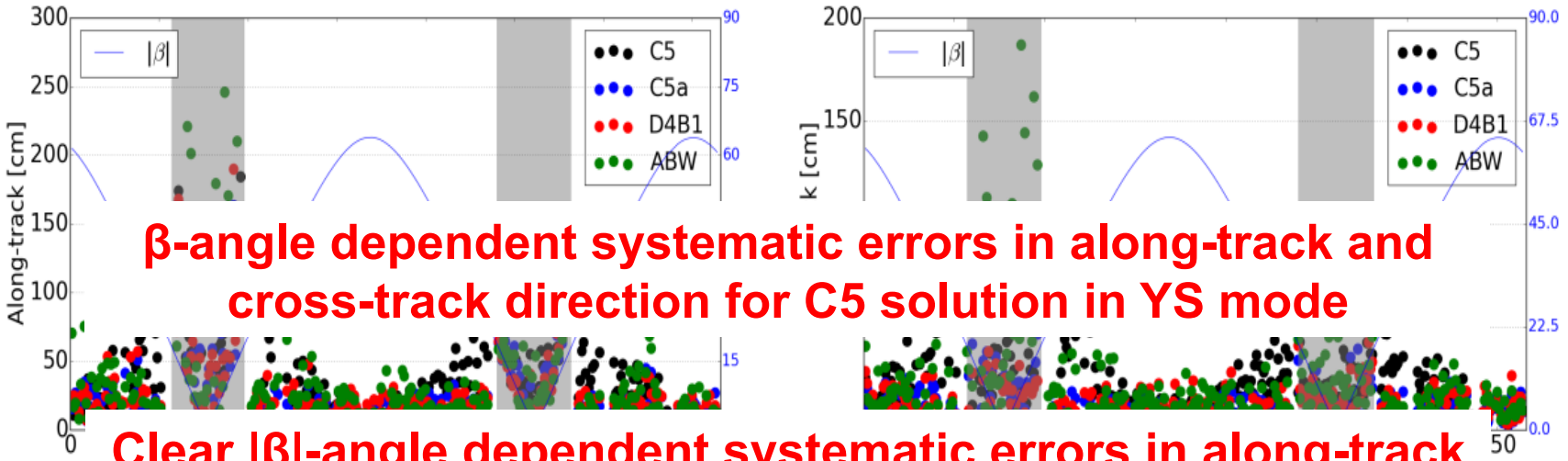
$$\boldsymbol{a}_{srp,body} = -\frac{S_0}{c} \frac{A}{M} \left( \frac{r_{AU}}{\|\boldsymbol{r}_s\|} \right)^2 \cdot \left\{ \left[ (1+\nu\mu)\cos^2\theta + \frac{2}{3}(1-\nu\mu)\cos\theta \right] \boldsymbol{e}_n + (1-\nu\mu)\cos\theta \sin\theta \boldsymbol{e}_s \right\}$$

- The similar POD accuracy has been achieved as CODE by adjustment of the optical coefficients of satellite bus and solar panels, and other parameters

# Comparison of different SRP models

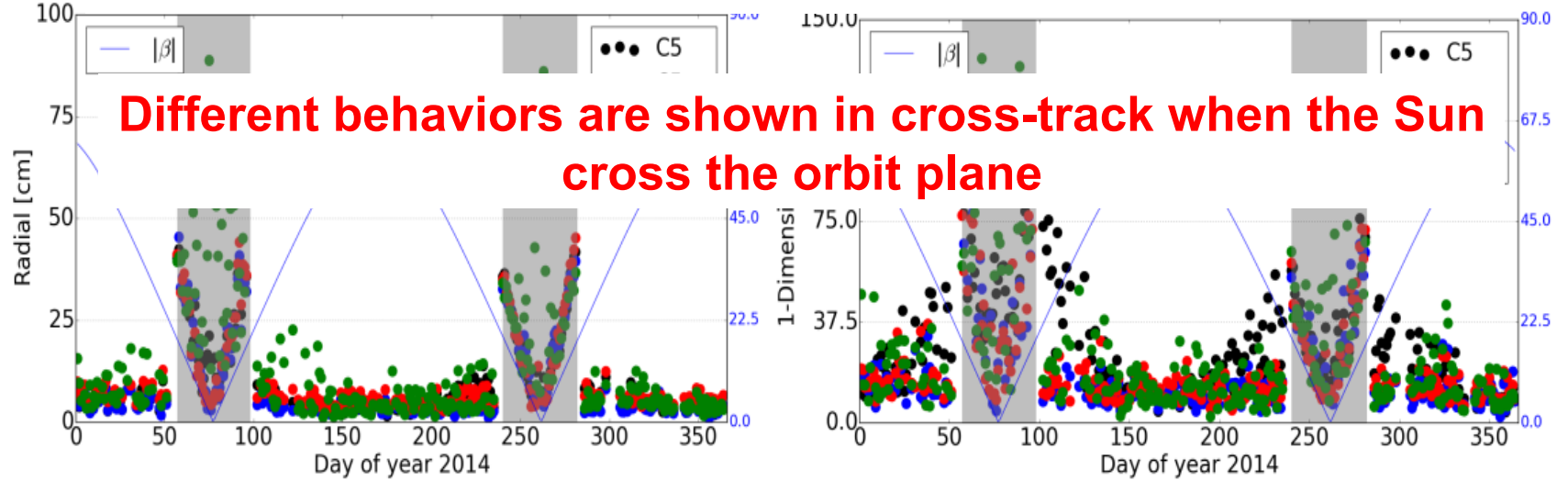
| Solutions | SRP model | Parameters to be estimated |
|---|---|---|
| C5 | CODE model without a priori model (Beulter et al 1994) | D0, Y0, B0, Bc1, Bs1 |
| C5a | CODE model without a priori model but with an empirical constant acceleration bias in along-track (Guo et al 2015) | D0, Y0, B0, Bc1, Bs1, A0 |
| D4B1 | Extended CODE model ( Arnold et al 2015) | D0, Y0, B0, Dc2, Ds2, Dc4, Ds4, Bc1, Bs1 |
| ABW | Adjustable Box-wing model (Rodríguez-Solano et al 2012) | SP, SB, Y0, +XAD, +ZAD, -ZAD, +XR, +ZR, -ZR |

# Overlapping orbit



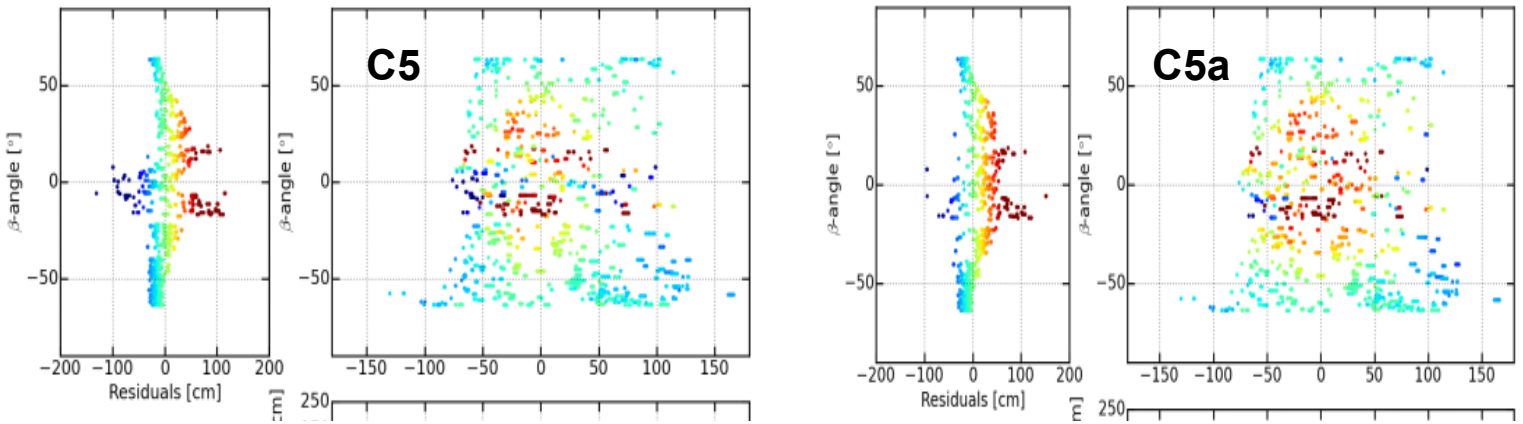

β-angle dependent systematic errors in along-track and cross-track direction for C5 solution in YS mode

Clear |β|-angle dependent systematic errors in along-track and radial direction for all solutions in ON mode



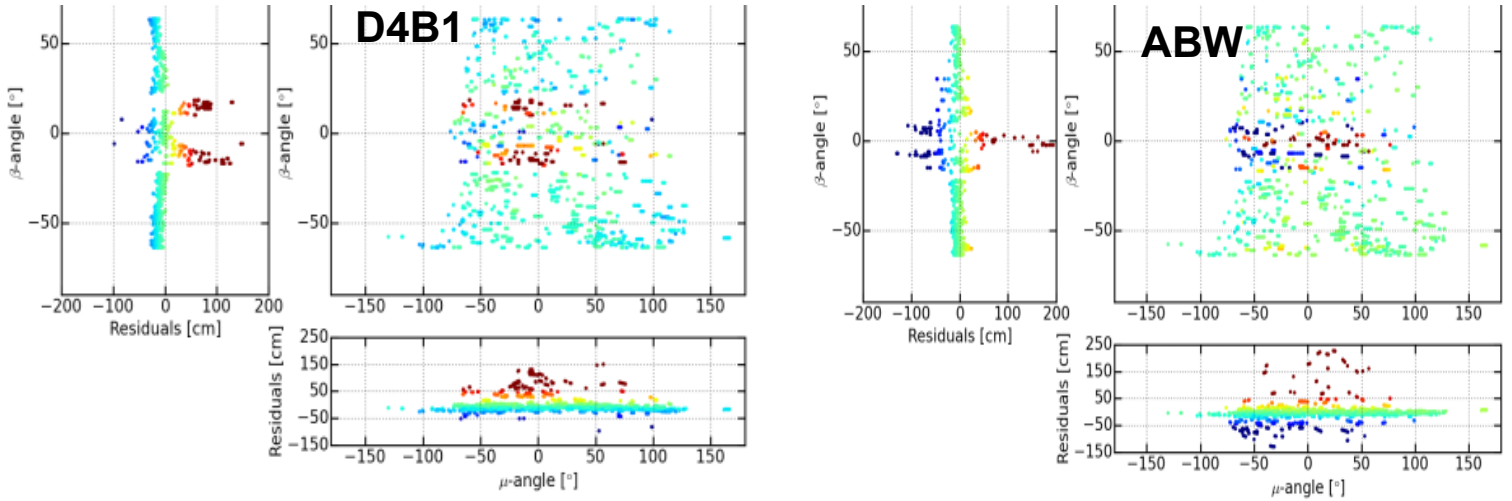

Different behaviors are shown in cross-track when the Sun cross the orbit plane

# SLR residuals

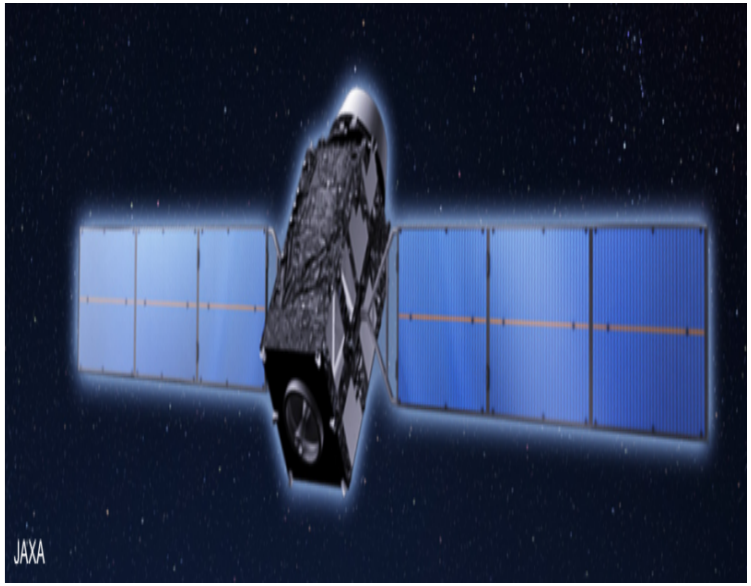


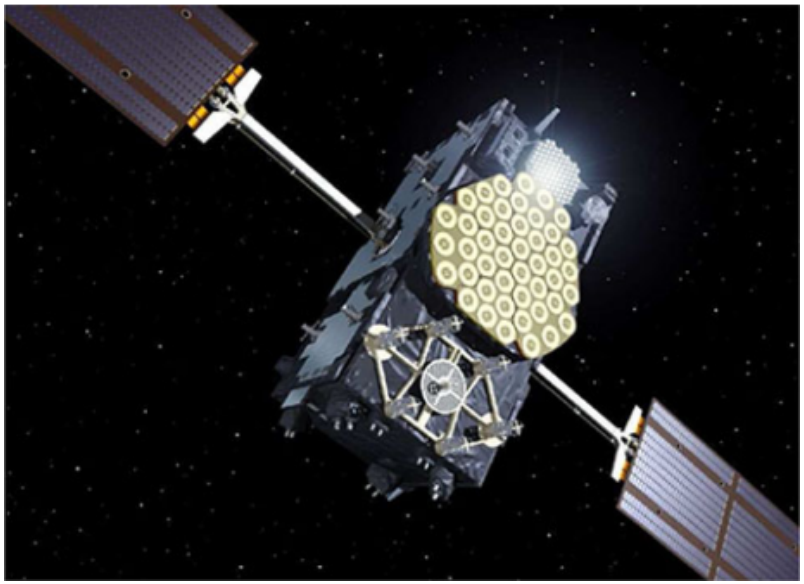

**No clear β-angle dependent systematic error in ABW solution, but this error is exist for others.**

**However, lower performance for ABW solution in ON mode**



# QZSS Michibiki vs Galileo



Cuboid, stretch in Z axis
L-band antenna

Cuboid, stretch in X axis

Cuboid box model ?

$$a = \frac{1\ \mathrm{AU}^2}{r_\odot^2}\left(a_{\mathrm{box}} + a_{\mathrm{emp}}\right)$$

Montenbruck et al (2015)

# Enhanced SRP model for Galileo

- Montenbruck et al (2015)
- Deduced for box-wing shape in yaw-steering regime
- Without considering the acceleration in solar panels

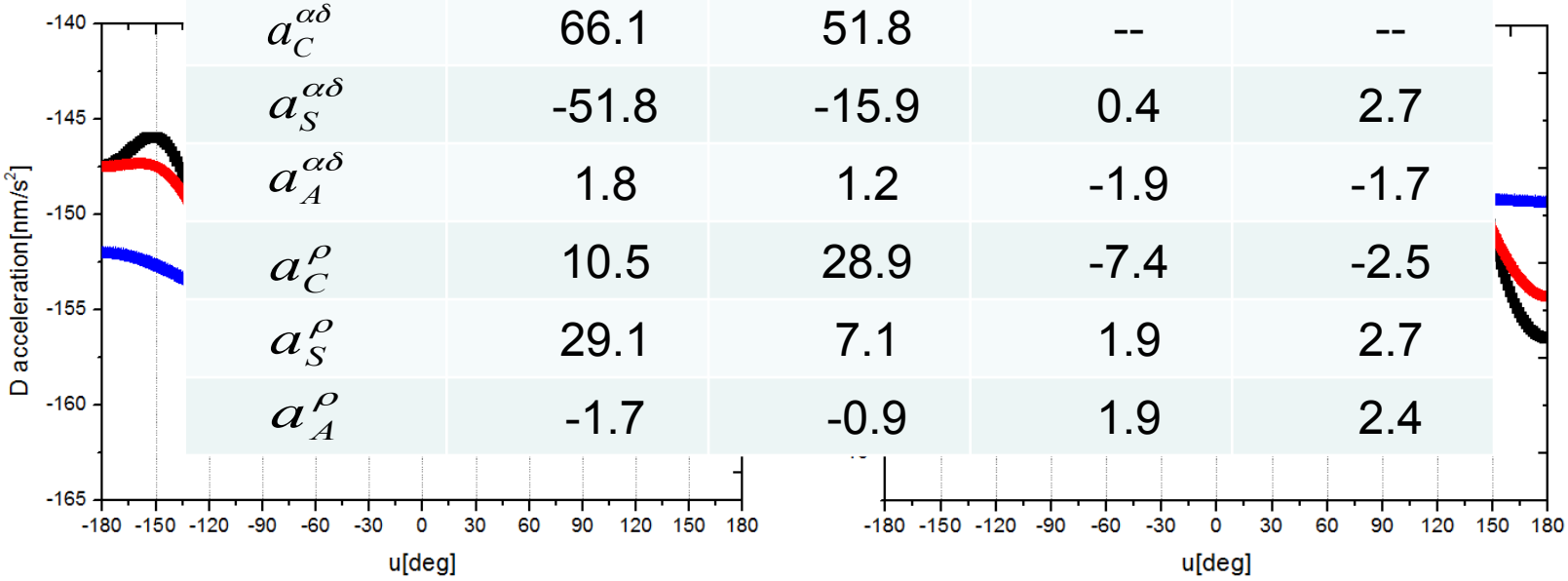
$$a_D = -a_C^{\alpha\delta}\left(\left|\cos\varepsilon\right| + \sin\varepsilon + \frac{2}{3}\right) - a_S^{\alpha\delta}\left(\left|\cos\varepsilon\right| - \sin\varepsilon - \frac{4}{3}\sin^2\varepsilon + \frac{2}{3}\right)$$
$$- a_A^{\alpha\delta}\left(\left|\cos\varepsilon\right| + \frac{2}{3}\left|\cos\varepsilon\right|\cos\varepsilon\right) - 2a_C^{\rho}\left(\left|\cos\varepsilon\right|\cos^2\varepsilon + \sin^3\varepsilon\right)$$
$$- 2a_S^{\rho}\left(\left|\cos\varepsilon\right|\cos^2\varepsilon - \sin^3\varepsilon\right) - 2a_A^{\rho}\cos^3\varepsilon$$

$$a_B = -\frac{4}{3}a_S^{\alpha\delta}\left(\cos\varepsilon\sin\varepsilon\right) - \frac{2}{3}a_A^{\alpha\delta}\left(\left|\cos\varepsilon\right|\sin\varepsilon\right)$$
$$- 2a_C^{\rho}\left(\left(\left|\cos\varepsilon\right| - \sin\varepsilon\right)\cos\varepsilon\sin\varepsilon\right) - 2a_S^{\rho}\left(\left(\left|\cos\varepsilon\right| + \sin\varepsilon\right)\cos\varepsilon\sin\varepsilon\right)$$
$$- 2a_A^{\rho}\cos^2\varepsilon\sin\varepsilon$$

# A prior SRP model for QZSS Michibiki

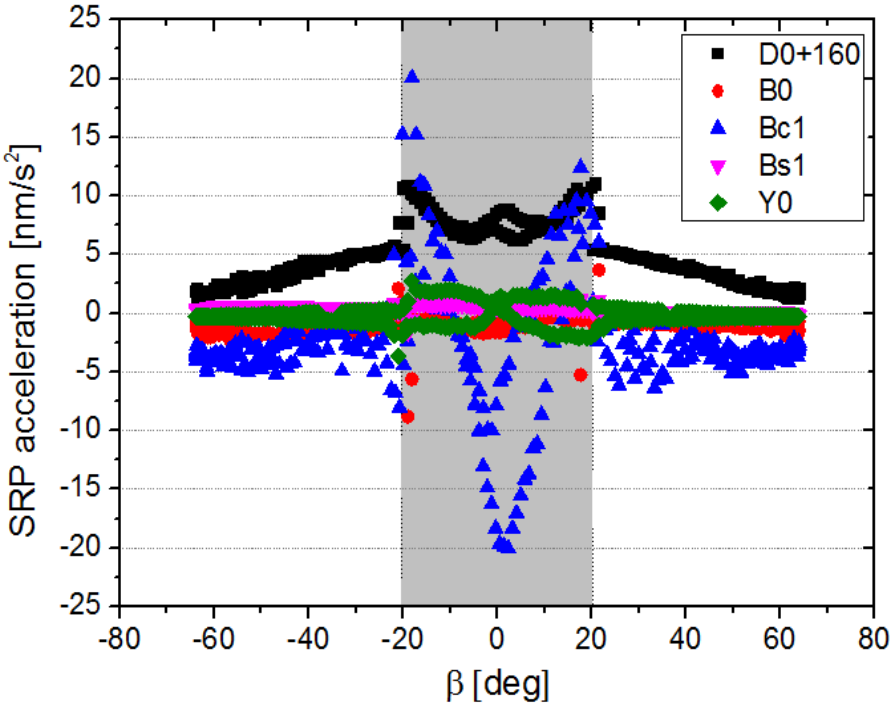
- The acceleration reconstruction in D and B axis
- Fittir
  acce

| Coefficients | D (nm/s²) | | B (nm/s²) | |
|---|---|---|---|---|
| | YS | ON | YS | ON |
| $a_C^{\alpha\delta}$ | 66.1 | 51.8 | -- | -- |
| $a_S^{\alpha\delta}$ | -51.8 | -15.9 | 0.4 | 2.7 |
| $a_A^{\alpha\delta}$ | 1.8 | 1.2 | -1.9 | -1.7 |
| $a_C^{\rho}$ | 10.5 | 28.9 | -7.4 | -2.5 |
| $a_S^{\rho}$ | 29.1 | 7.1 | 1.9 | 2.7 |
| $a_A^{\rho}$ | -1.7 | -0.9 | 1.9 | 2.4 |

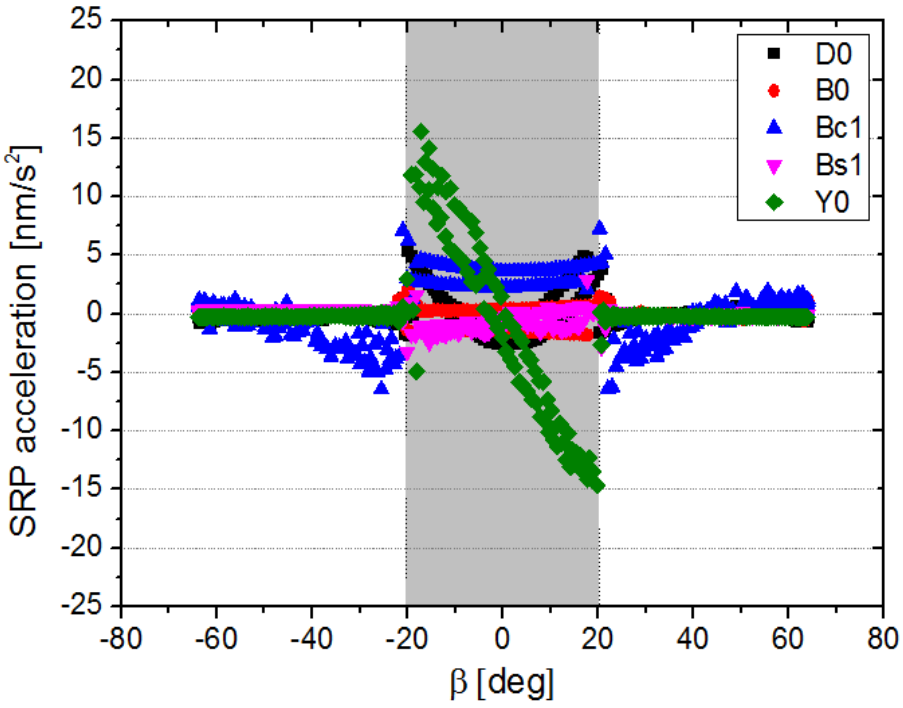



The accelerations in solar panels cause the different values for that same coefficients in D and B.

# Improvement of the prior model in ON mode




C5 without priori model




C5 with priori model

Periodic variations for Bc, Bs, and B0; Linear variations for Y0

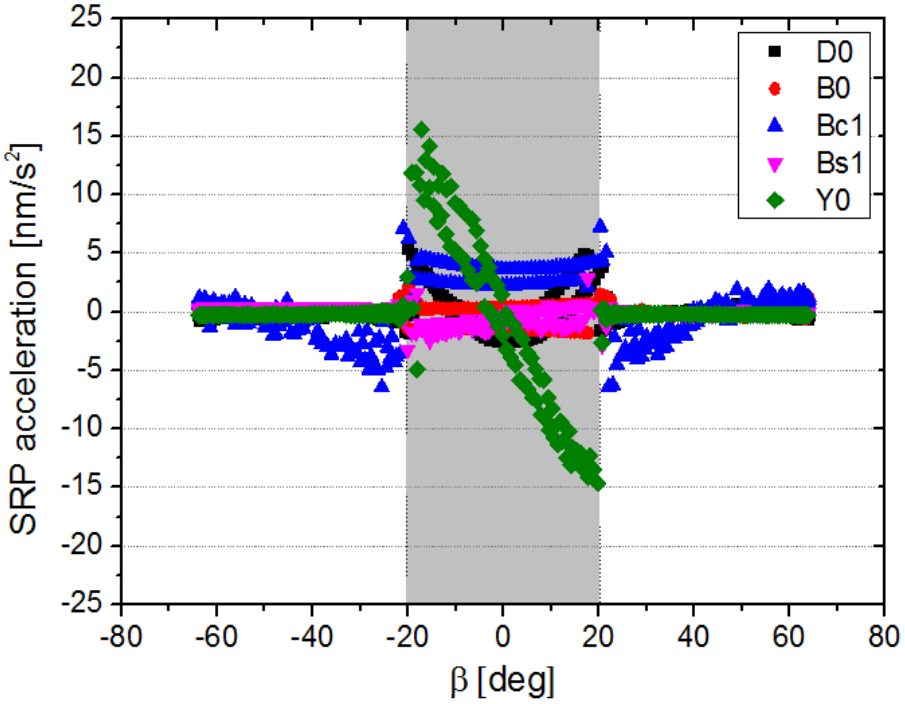
# Improvement of the prior model in ON mode

$$a_D^{ON} = a_D + a_{D0}^{fit}$$
$$a_B^{ON} = a_B + a_{B0}^{fit} + a_{Bc1}^{fit}\cos(u) + a_{Bs1}^{fit}\sin(u)$$
$$a_Y^{ON} = a_{Y0}^{fit}$$
$$a_*^{fit} = c_* + d_*\beta + e_*\beta^2$$

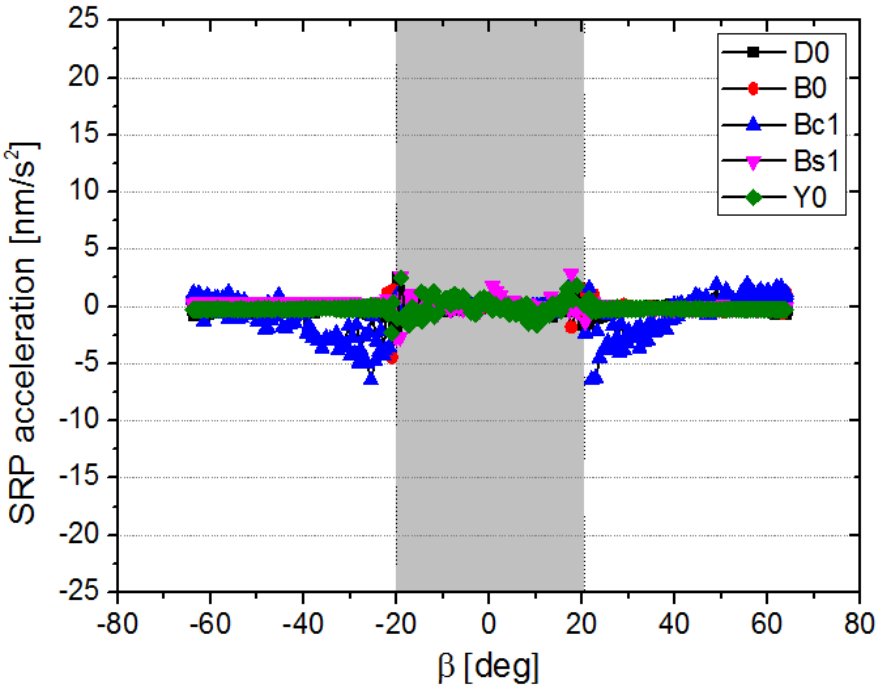
| | Coefficients | | |
|---|---|---|---|
| | $c_*$ | $d_*$ | $e_*$ |
| $a_{D0}^{fit}$ | -0.88 | 0.78 | 59.11 |
| $a_{B0}^{fit}$ | -1.42 | -0.33 | -2.36 |
| $a_{Bc1}^{fit}$ | 2.27 | 0.03 | 5.51 |
| $a_{Bs1}^{fit}$ | -1.16 | 1.11 | -2.24 |
| $a_{Y0}^{fit}$ | -1.75 | -43.17 | 5.38 |

Fitting the parameters during ON season with a two-order polynomial against beta angle

# The variations of CODE SRP parameters

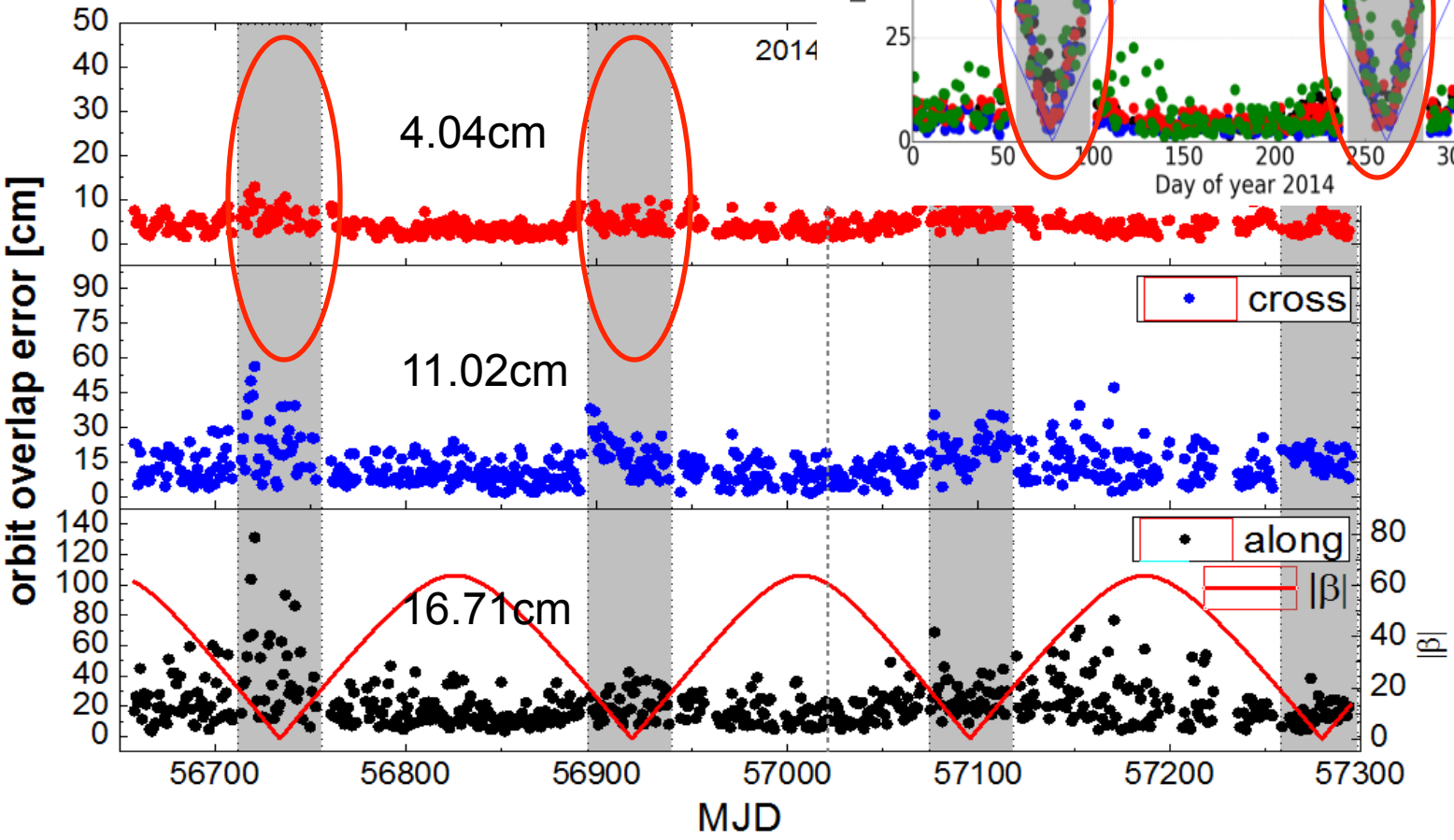


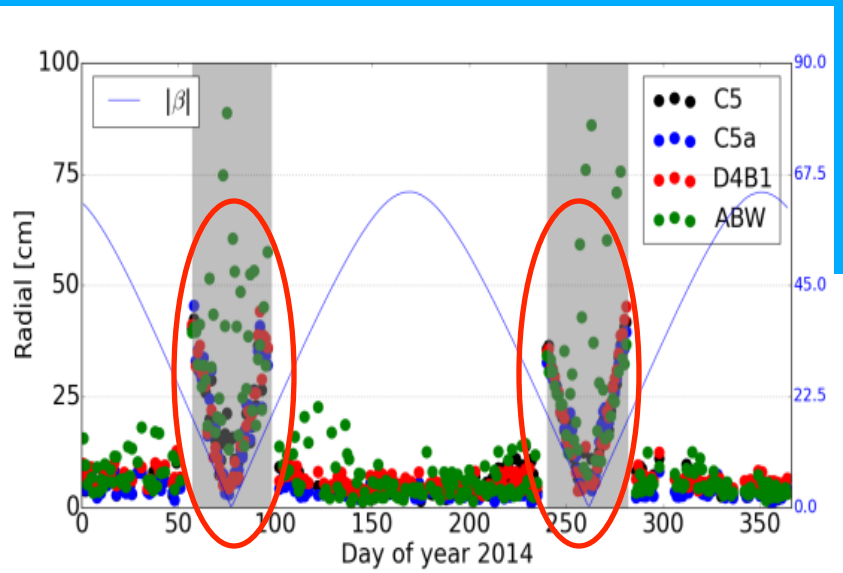

C5 with priori model




C5 with improved priori model

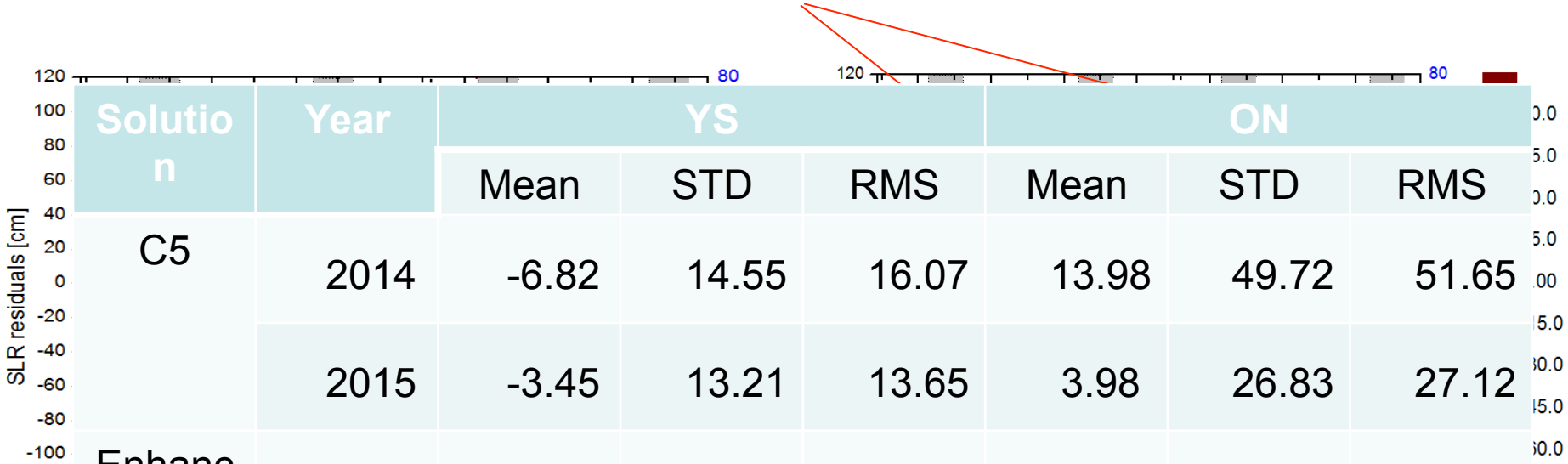
# Validation: overlapping orb



# Validation: SLR residuals

**SLR biases when the Sun is cross the orbit plane from the below to the above**




| Solution | Year | YS | | | ON | | |
|---|---|---|---|---|---|---|---|
| | | Mean | STD | RMS | Mean | STD | RMS |
| C5 | 2014 | -6.82 | 14.55 | 16.07 | 13.98 | 49.72 | 51.65 |
| | 2015 | -3.45 | 13.21 | 13.65 | 3.98 | 26.83 | 27.12 |
| Enhanced SRP for QZSS | 2014 | -5.59 | 5.98 | 8.19 | 7.22 | 14.62 | 16.31 |
| | 2015 | -4.46 | 7.04 | 8.33 | 1.88 | 14.11 | 14.23 |

# Conclusion

- For QZSS Michibiki, there is β-angle dependent systematic error, and the CODE and its extend SRP models are hard to eliminate the error
- Furthermore, CODE SRP and its modified version has deficiency for modelling the SRP acting on satellite in ON mode
- With reconstructed acceleration, a priori SRP model has been established for QZSS Michibiki satellite
- The model is consists two parts for YS and ON mode, respectively
- With this model, the systematic errors in SLR residuals has been removed, and orbit accuracy has been improved significantly when satellite in ON mode
- However, there is still relative larger SLR biases when the Sun is cross the orbit plane from the below to the above. It needs further investigation.

# Thanks for your attention!

**Any question or advice will be appreciated!**