Thermal Re-Radiation Acceleration in the GNSS Orbit Modelling Based on Galileo Clock Parameters

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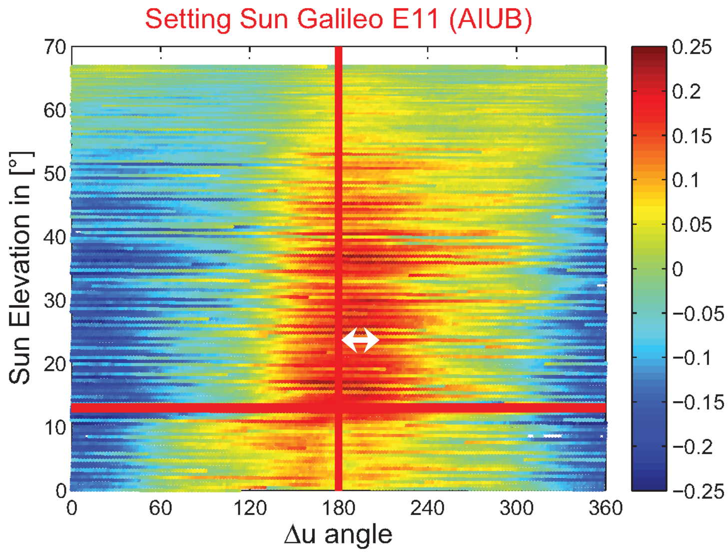
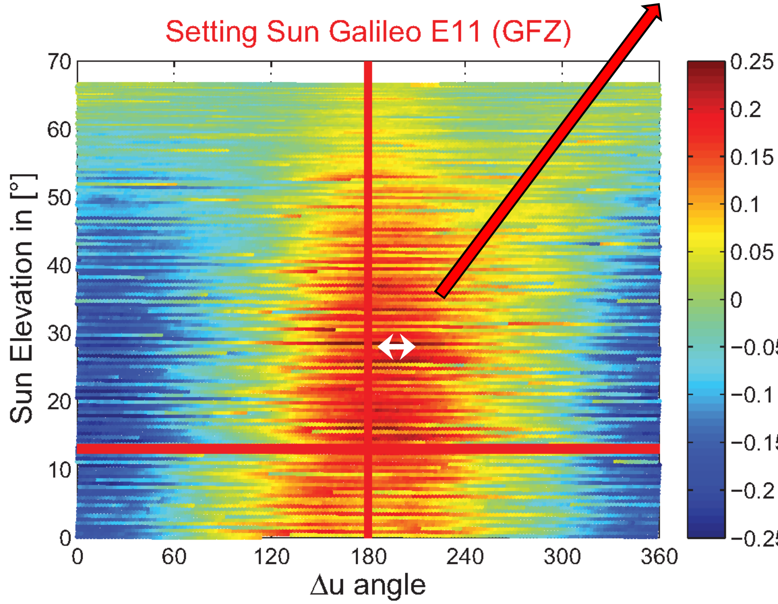
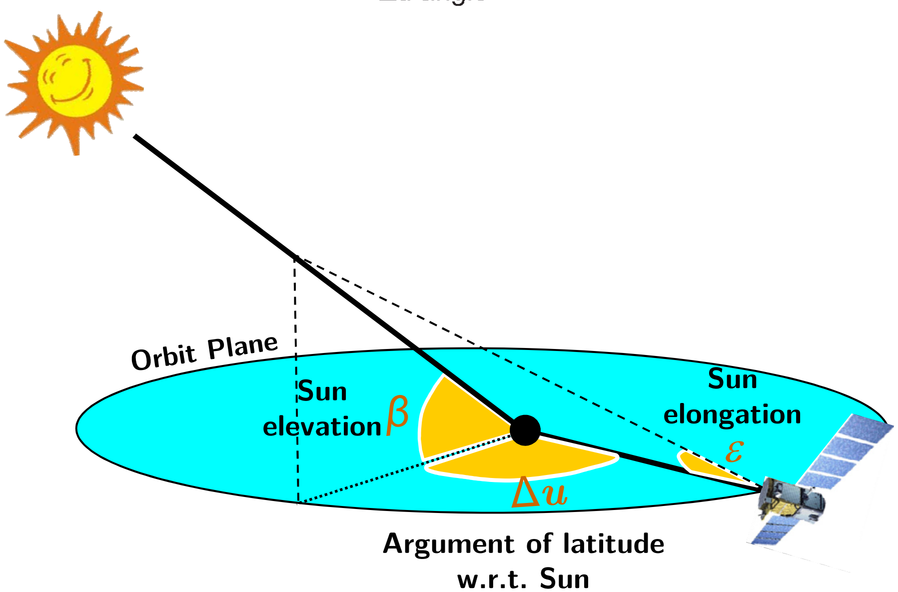
Galileo Clock Parameters in the Sun-fixed Frame (daily time drift/bias removed)

Thermal inertia



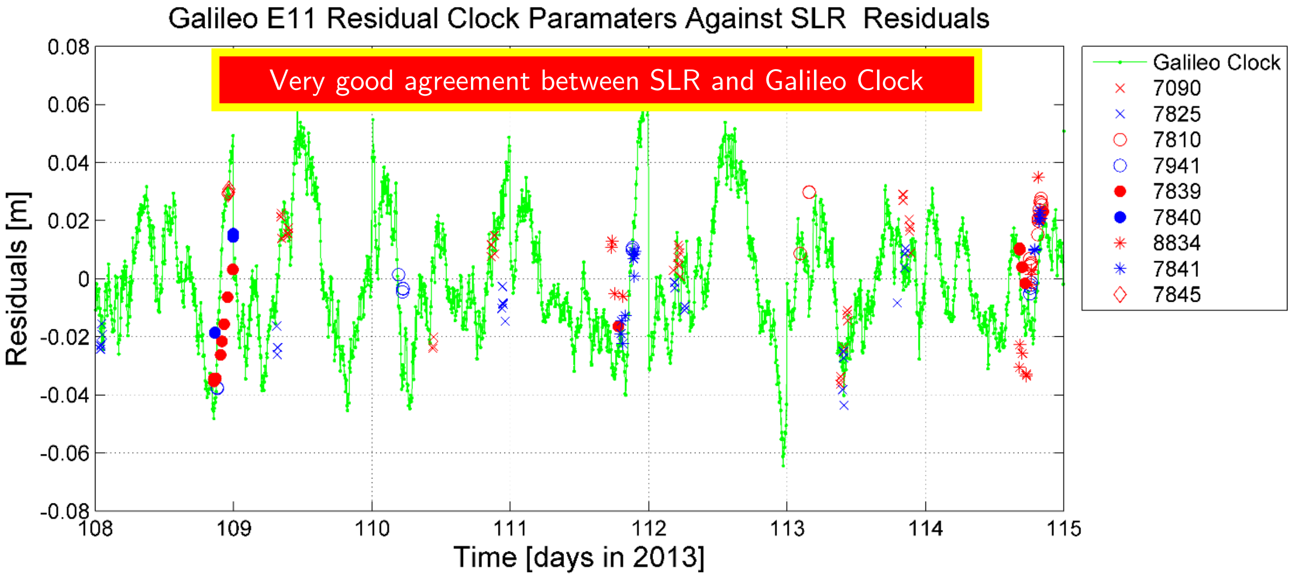




$$\Delta r = A \cdot \cos\beta \cos\Delta u$$

$$\delta_{clk} = \Delta r = -A \cdot \cos\varepsilon$$

Comparison of SLR and Galileo Clock

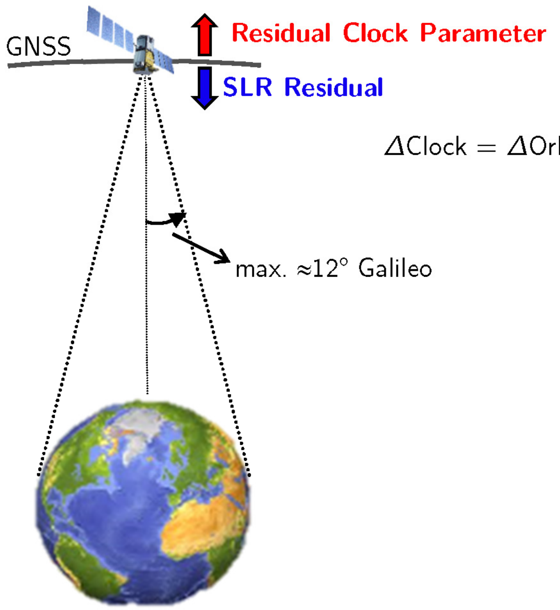


STD(MGEX CLOCK)=20.7 mm

STD(Simulated CLOCK)=15.5 mm

STD(Orbit)=14 mm

STD(SLR)=25.3 mm

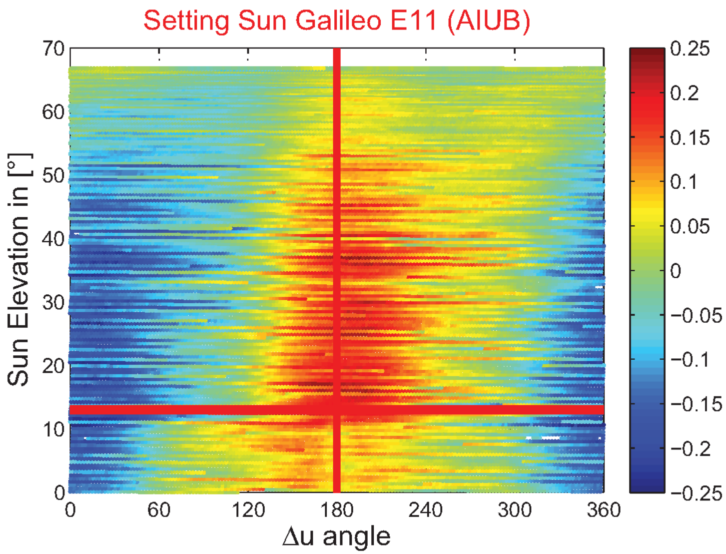


ΔClock = ΔOrbit = – ΔSLR

Galileo clock is showing smaller standard deviation compared to SLR!
(MGEX and ground test)

Relation Between Clock, SLR Bias and Orbit Translation (geocenter)

Galileo Clock



$$\Delta r = A \cdot \cos\beta \cos\Delta u$$
$$\delta_{clk} = \Delta r = -A \cdot \cos\varepsilon$$

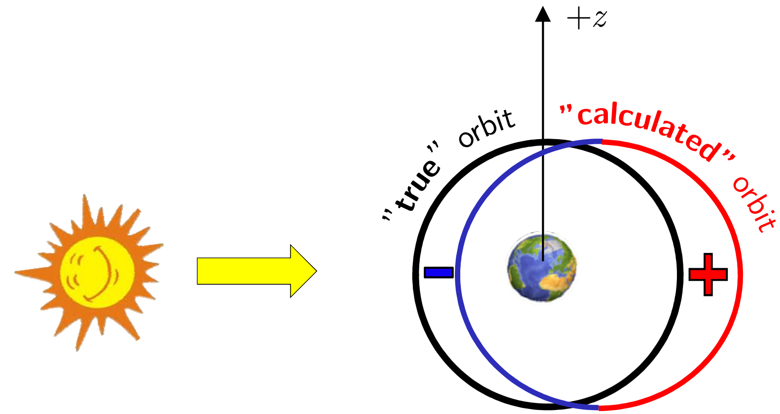
SLR Bias:

$$\bar{\delta}_{SLR} = \bar{\delta}_{const} + \frac{\sum n_i \cdot A \cos\varepsilon_i}{\sum n_i} = -2.4\ cm - 4.1\ cm = \boxed{-6.5\ cm}$$

Observed SLR Bias

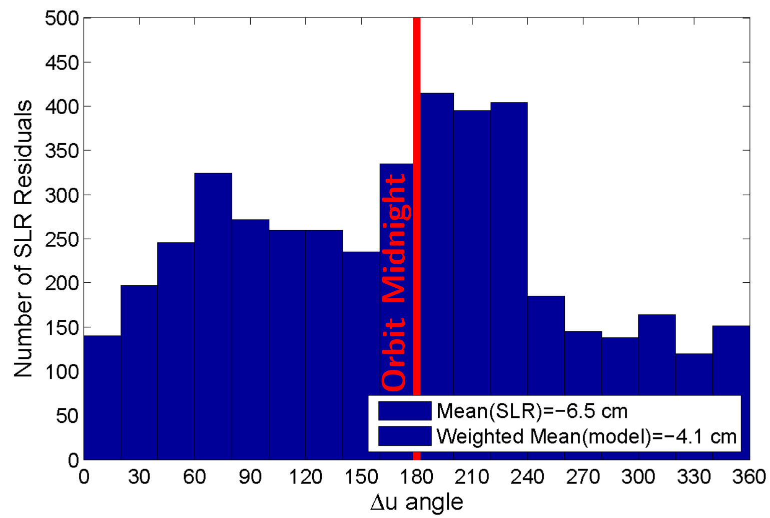
albedo? (wrong sign! increases the SLR bias)
antenna thrust? (wrong sign! increases the SLR bias)

Orbit Translation



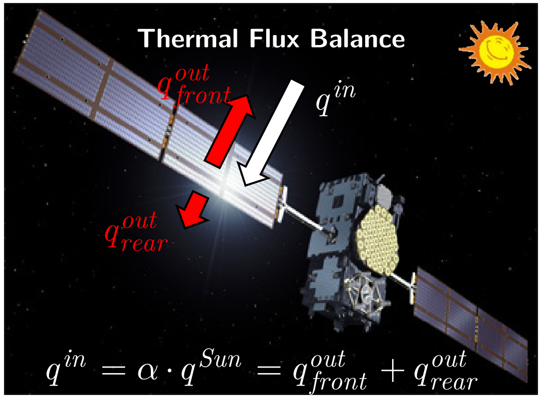
$$\Delta Z_{geocenter} \sim \sin\beta$$

SLR Residuals ="observed" - "calculated"

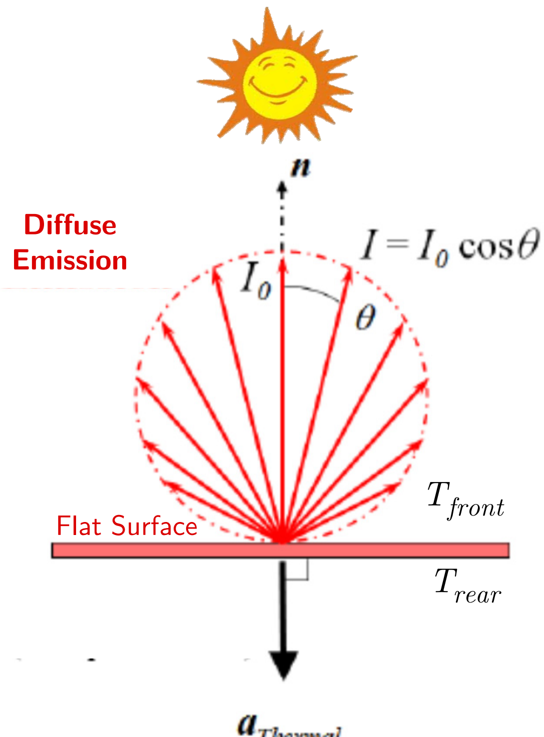


Thermal Re-Radiation Acceleration

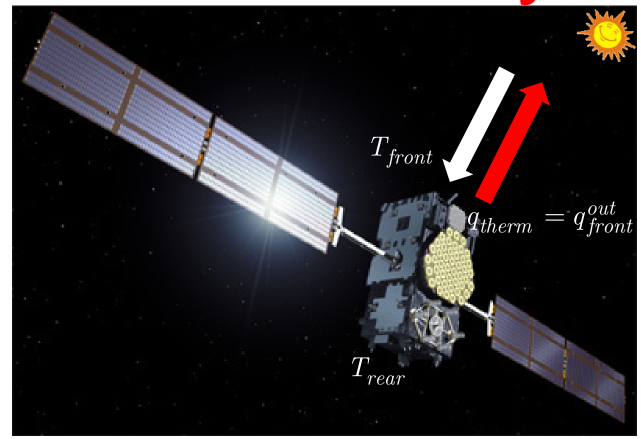
Solar Panels



absorptivity



Satellite Body



Stefan-Bolzman Law:

Thermal Flux

$$q_{therm} = q^{out}_{front} - q^{out}_{rear} = 5.67 \times 10^{-8} \left(\varepsilon_{front} T^4_{front} - \varepsilon_{rear} T^4_{rear} \right) \qquad \left[W/m^2 \right]$$

Stefan-Boltzman constant

Acceleration induced by the thermal radiation emission

Surface temperature (front, rear)

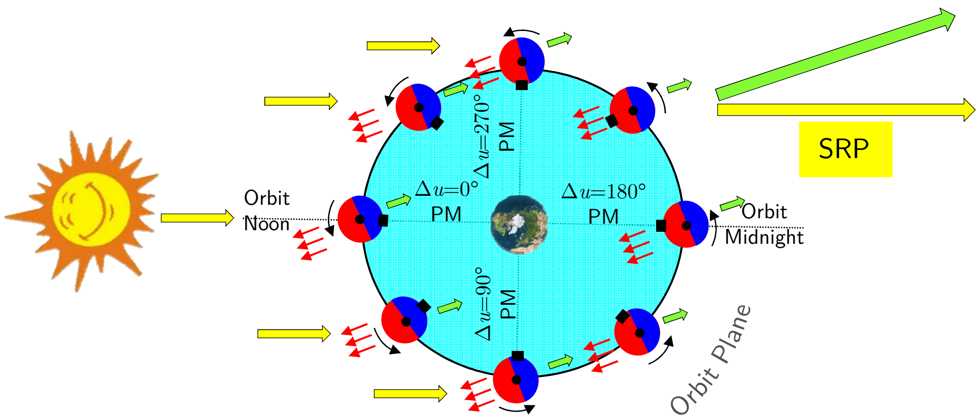
$$\boxed{\boldsymbol{a}_{therm} = -\frac{2}{3} C_{therm} \frac{A}{m} \left(\varepsilon_{front} T^4_{front} - \varepsilon_{rear} T^4_{rear} \right) \boldsymbol{n}}$$

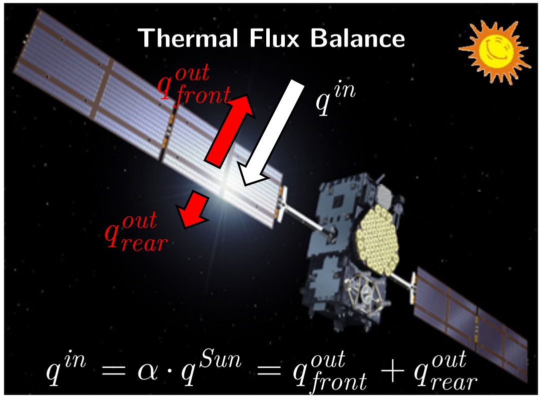
$$C_{therm} = \frac{5.67 \times 10^{-8}}{c}$$

Thermal coefficient

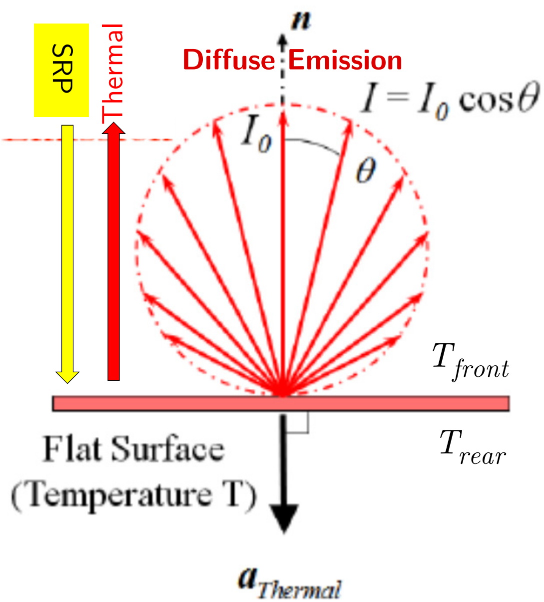
Thermal emissivities

Thermal Re-Radiation Effect and Thermal Inertia (Yarkovsky Effect)

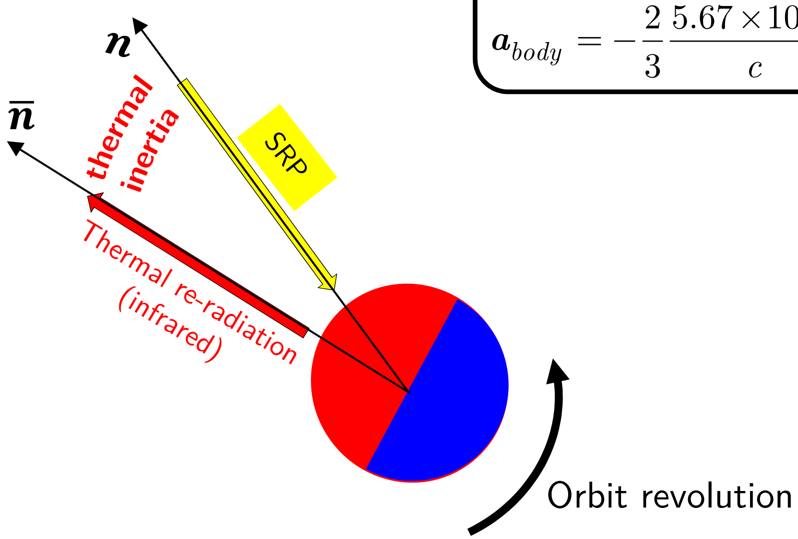




Solar Panel:



Satellite Body:



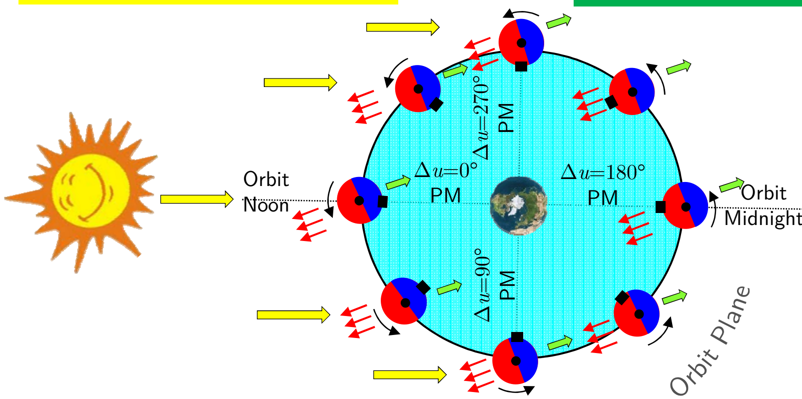
$$\boldsymbol{a}_{panel} = -\frac{2}{3}\frac{5.67\times 10^{-8}}{c}\frac{A}{m}\left(\varepsilon_{front}T^4_{front} - \varepsilon_{rear}T^4_{rear}\right)\boldsymbol{n}$$

$$\boldsymbol{a}_{body} = -\frac{2}{3}\frac{5.67\times 10^{-8}}{c}\frac{A}{m}\varepsilon_{body}T^4_{body}\bar{\boldsymbol{n}}$$

Thermal Re-Radiation Effect and Thermal Inertia (Yarkovsky Effect)

Solar radiation (SRP)

Thermal (infrared) re-radiation acceleration



Hill Equations:

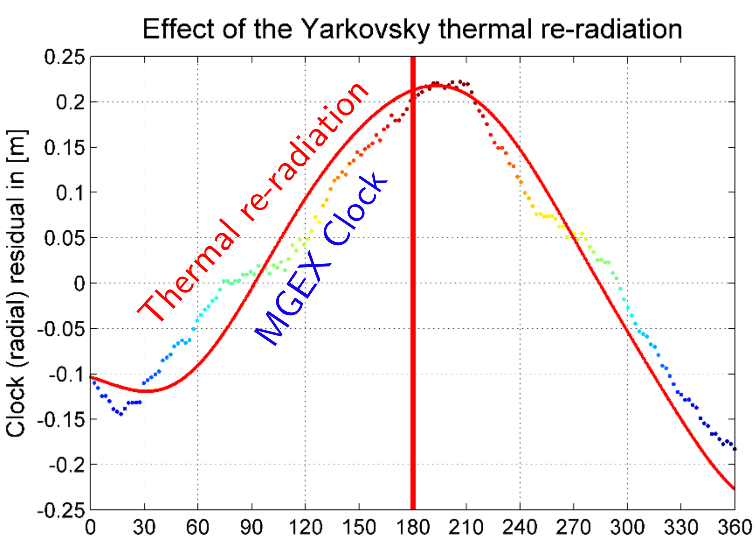
exact solution for the periodic (orbit) acceleration

$$\Delta a_i = A_i \cos nt + B_i \sin nt + C_i \qquad i = \begin{Bmatrix} \text{radial} \\ \text{along} \\ \text{across} \end{Bmatrix}$$

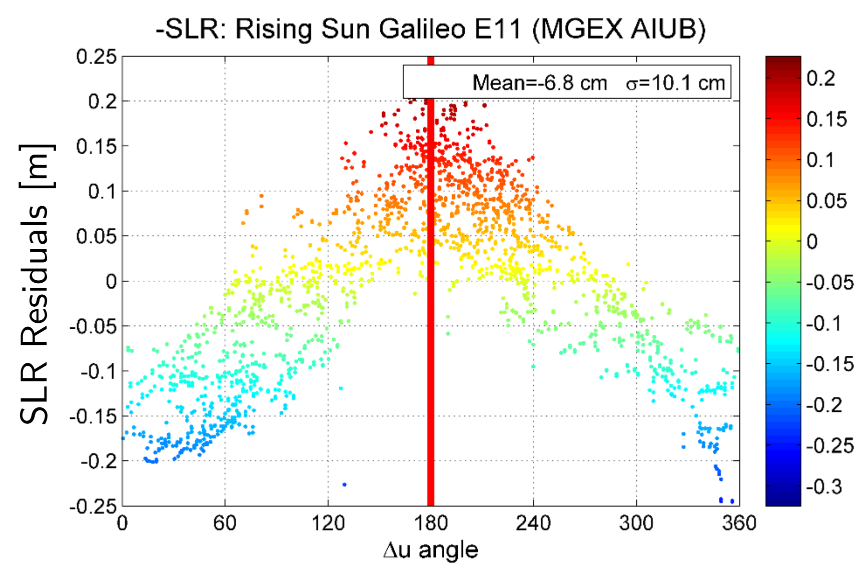
$$\Delta x_i = f(A_i, B_i C_i, n) \qquad i = \begin{Bmatrix} \text{radial} \\ \text{along} \\ \text{across} \end{Bmatrix}$$

(Colombo, 1989)

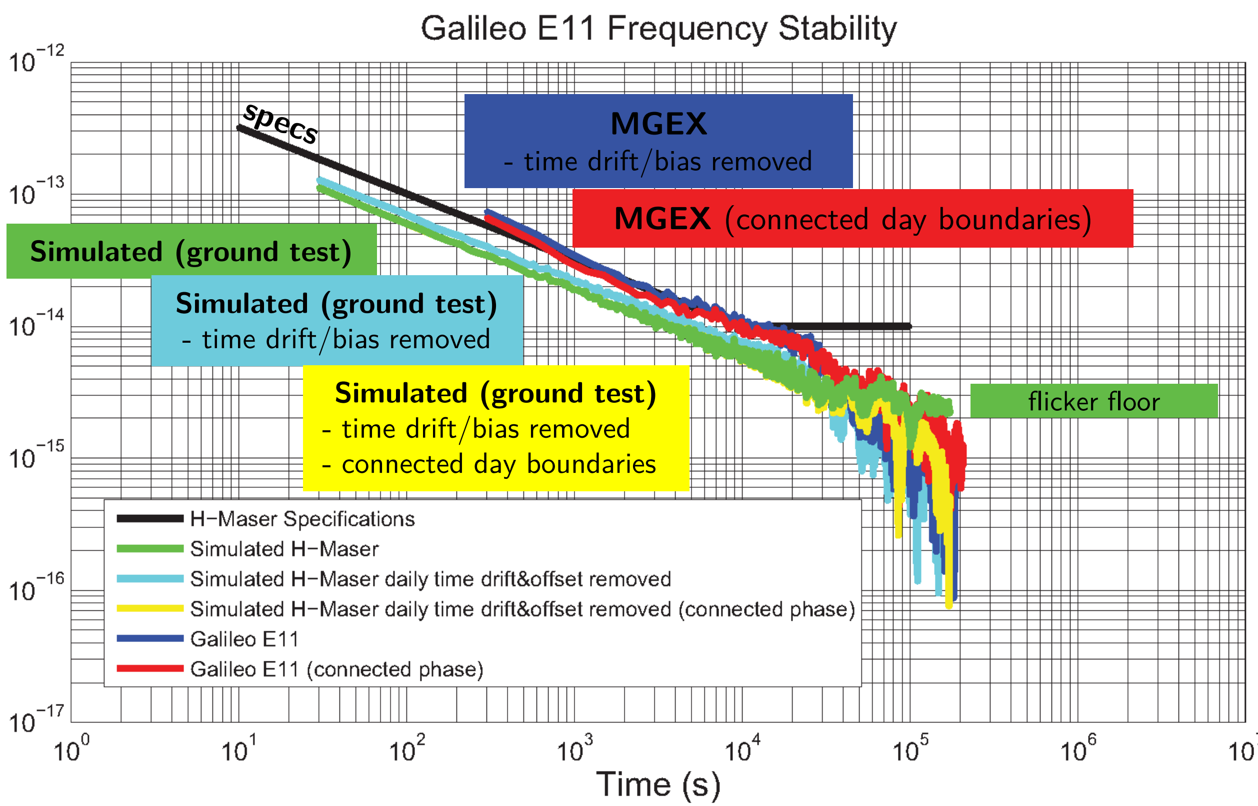
Estimated thermal inertia = 4.7 min



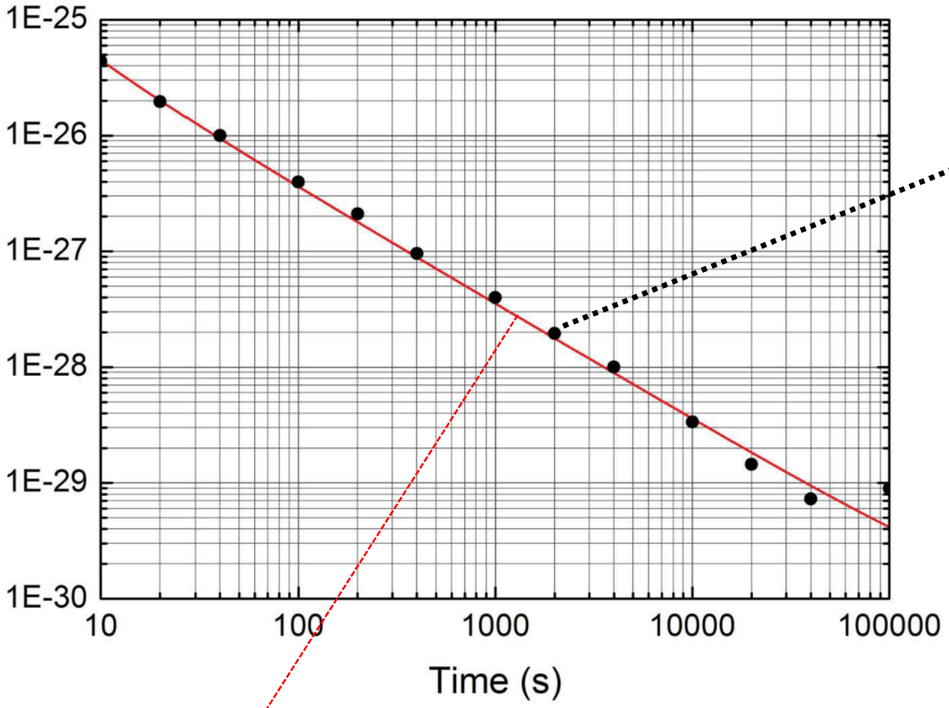
SLR Residuals (1/2 draconic year)



Allan Deviation



Clock Noise Model: Overlapping Allan Variance from the Ground Test



Ground test of the Galileo H-Maser

$$f(\tau) = \frac{A^2}{\tau^2} + \frac{B^2}{\tau} + C^2 + D^2\tau$$

Noise model function

White phase noise: **9.8×10^{-13}** short-term $\tau<10$ s

White frequency noise: **5.9×10^{-13}** short-to-medium term

Flicker frequency noise: **7.9×10^{-16}** long-term $\tau>6$ h

Frequency drift: **1.2×10^{-20}/s** (from flight model test)

Clock Model: MGEX vs. Simulated

Simulated Galileo H-maser (only clock noise)

Simulated Galileo H-Maser, (σ in mm)									
N	0.2 h	0.25 h	0.5 h	1.0 h	1.5 h	6 h	12 h	14 h	24 h
1	1.2	1.4	2.0	2.7	3.4	6.8	9.3	11.2	15.5
2	1.0	1.1	1.5	2.2	2.7	5.7	7.7	8.8	10.3
3	0.8	0.9	1.3	1.9	2.3	4.7	6.5	7.8	9.8
4	0.7	0.8	1.2	1.7	2.1	4.3	5.8	6.6	8.7
5	0.8	0.9	1.1	1.5	1.9	3.8	5.2	5.6	7.8

polynomial

Polynomial removed:

- N=1 **Linear model**
- N=2 **Quadratic model**
- N=3
- N=4
- N=5

MGEX (AIUB)

MGEX (AIUB) Clock Parameters, (σ in mm)									
N	0.2 h	0.25 h	0.5 h	1.0 h	1.5 h	6 h	12 h	14 h	24 h
1	-	1.8	2.9	4.2	5.3	10.9	16.2	18.3	20.2
2	-	-	2.3	3.7	4.2	8.3	12.5	14.1	17.8
3	-	-	1.8	3.1	3.8	7.1	10.4	12.4	16.9
4	-	-	1.3	2.8	3.4	6.4	9.3	10.4	12.9
5	-	-	-	2.8	3.2	5.6	8.5	9.6	11.9

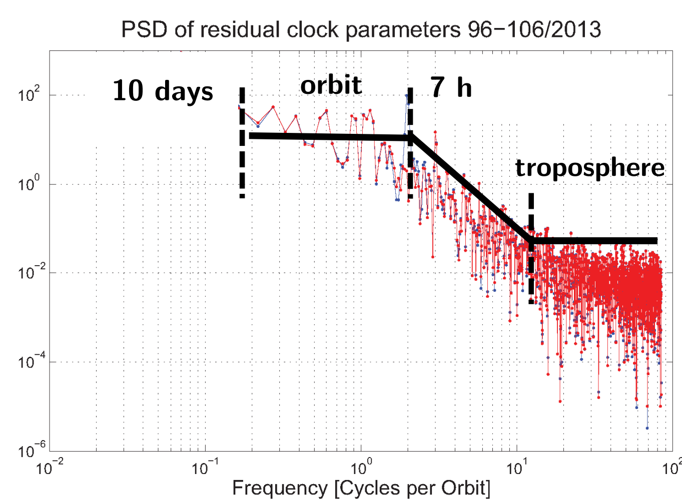
polynomial

$$\textbf{DIFFERENCE} = \sqrt{\textbf{MGEX}^2 - \textbf{SimulatedHmaser}^2}$$

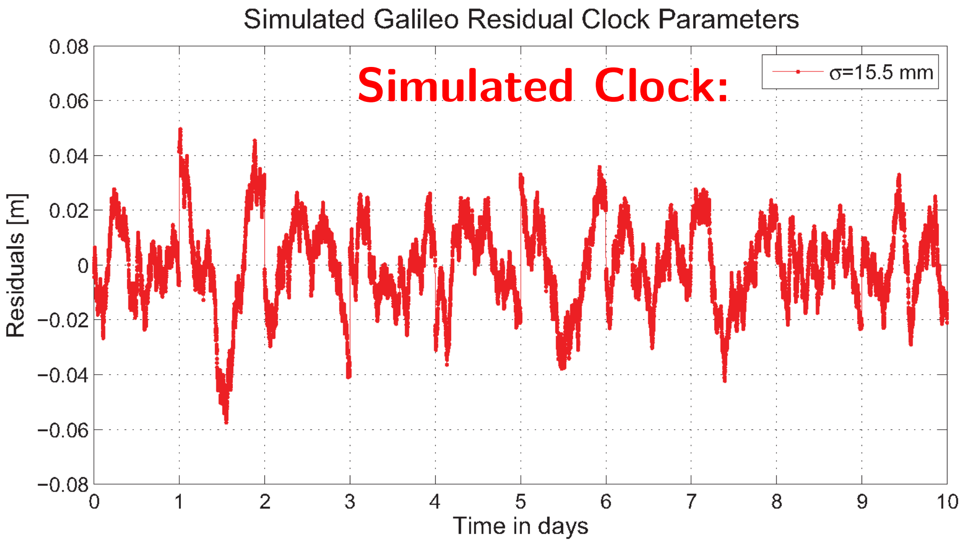
Difference MGEX-Simulated, (σ in mm)									
N	0.2 h	0.25 h	0.5 h	1.0 h	1.5 h	6 h	12 h	14 h	24 h
1	-	1.1	2.1	3.2	4.0	8.5	13.3	14.5	13.1
2	-	-	1.7	2.9	3.2	6.1	9.8	11.0	14.4
3	-	-	1.2	2.4	3.0	5.3	8.1	9.7	13.8
4	-	-	0.5	2.2	2.7	4.7	7.3	8.1	9.6
5	-	-	-	2.3	2.6	4.2	6.7	7.9	9.0

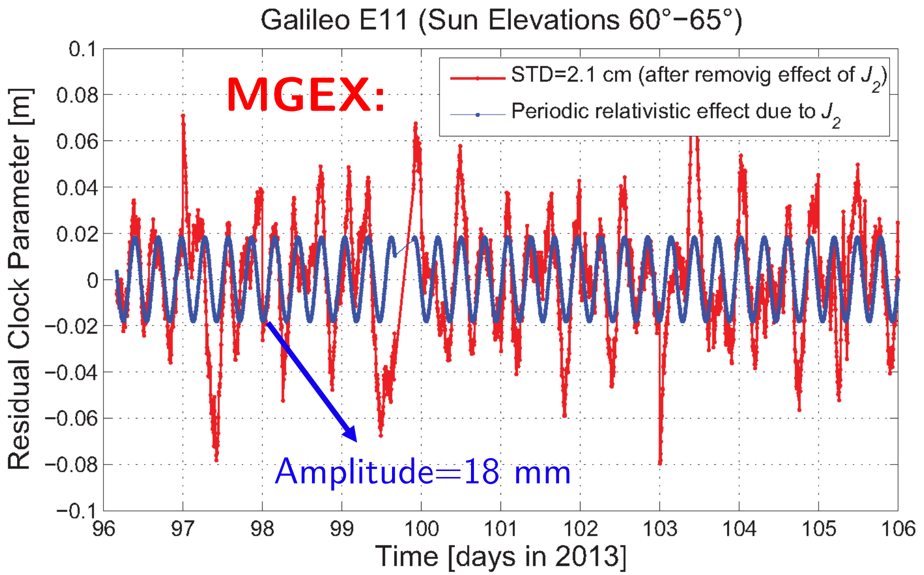
polynomial

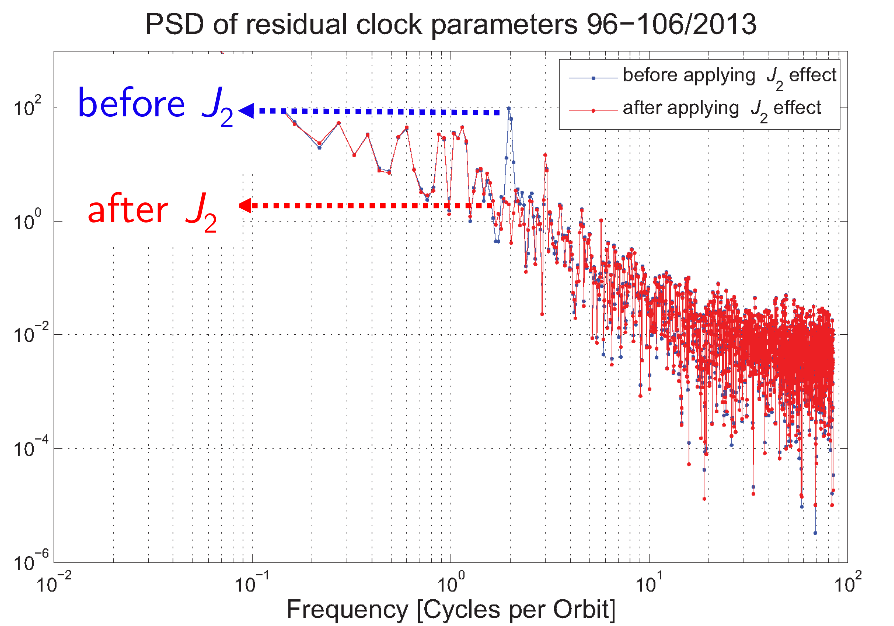
Ground network noise:



J_2 Periodic Relativistic Effect







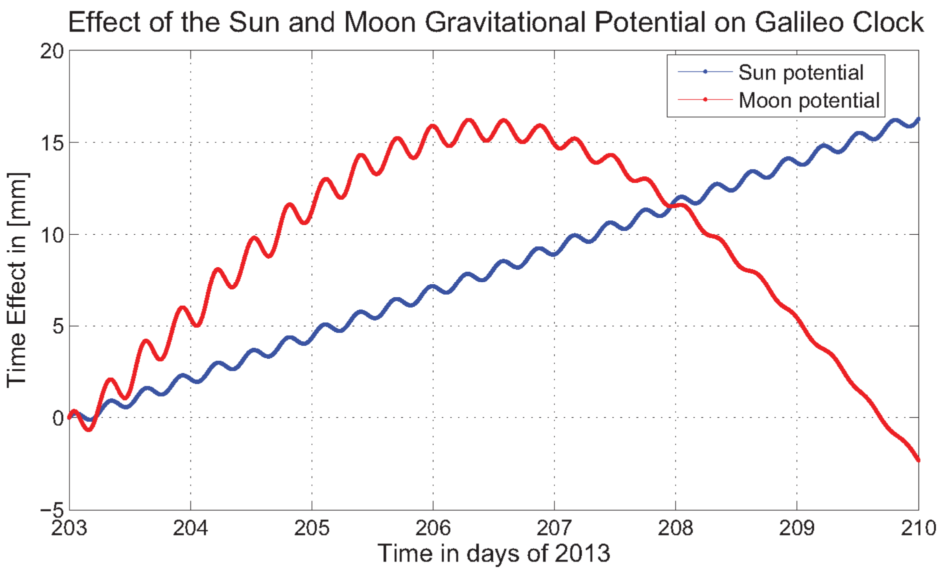
J_2 Periodic Relativistic Correction

$$\Delta t(J_2)_{per} = -\frac{3}{2}\frac{a_e^2}{a^2c^2}J_2\sqrt{GMa}\cdot\sin^2 i\sin 2u$$

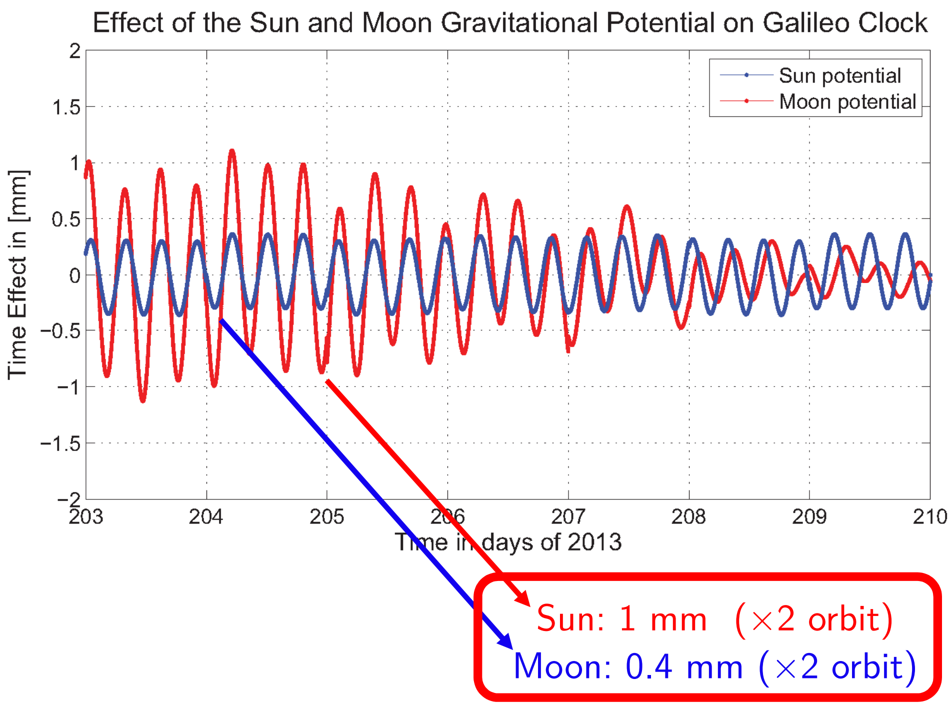
(Kouba, 2004)

Gravitational Potential of Sun and Moon

Accumulated time over 7 days



After removing daily time drift/bias



Proper time of the clock:

$$\frac{d\tau}{dt} = 1 - \frac{1}{c^2}\left[\frac{\mathbf{v}^2}{2} + U_E(\mathbf{x}) + \underbrace{V(\mathbf{X}_A) - V(\mathbf{X}_E) - x_A^i \partial_i V(\mathbf{X}_E)}\right]$$ (Petit and Luzum, 2010)

contribution from Sun and Moon:

$$V(\mathbf{X}_A) - V(\mathbf{X}_E) - x_A^i \partial_i V(\mathbf{X}_E) = \sum_{A \neq E} GM_A \left[\frac{1}{r_{AP}} - \frac{1}{r_{AE}} + \frac{\mathbf{x}_{AE}\mathbf{x}_{EP}}{r_{AE}^3}\right]$$ (Wolf and Petit, 1995)

Effect of the Earth's Magnetic Field

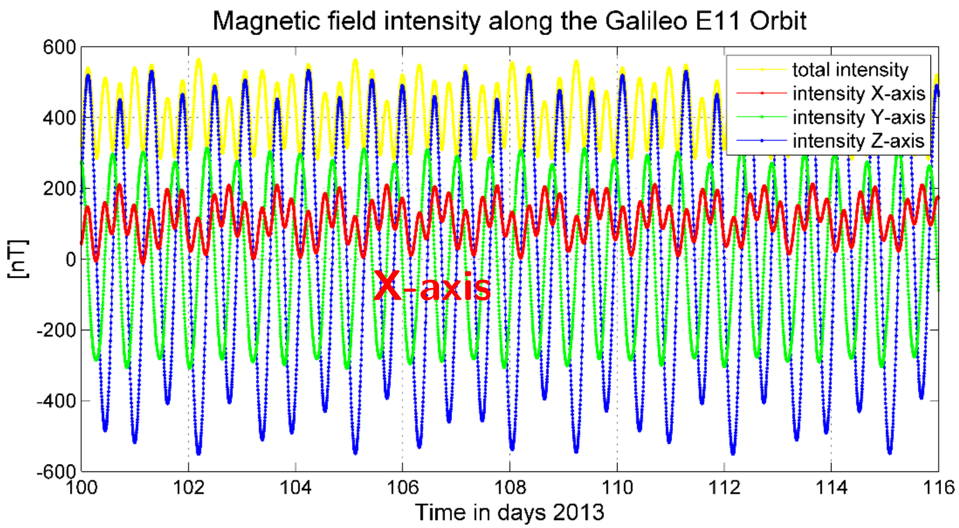
Magnetic Sensitivity:

(as measured during ground tests)

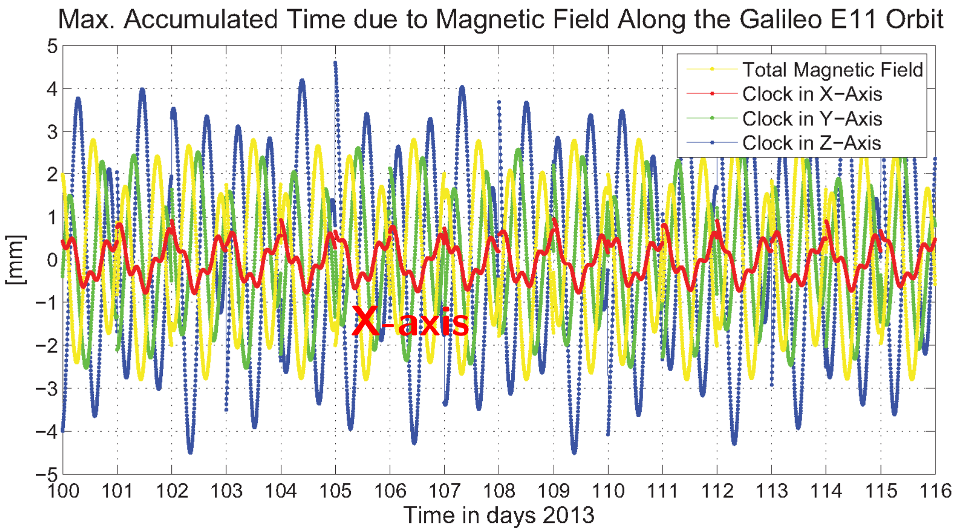
$<3\times10^{-13}$/Gauss (Boving et al., 2009)

(1 Gauss $=10^{-4}$T)

IGRF model:



Linear model (time drift/bias) removed



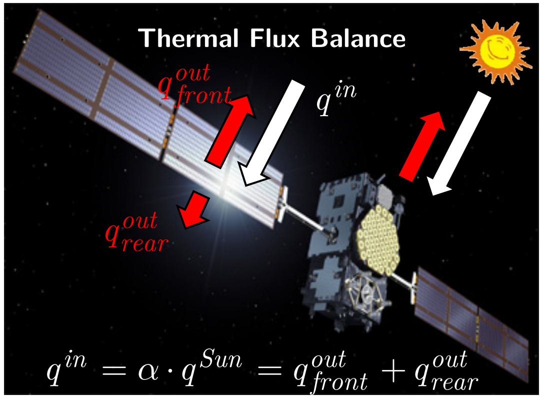
IGRF model:

total magnetic field variation 300-550 nT → **max. effect < 0.8 mm**

assuming the orientation of the Galileo maser cavity along the satellite X-axis (that never faces Sun)

Thermal Sensitivity

Satellite Surface Temperature Along the Orbit:



thermal balance:
$$\alpha \cdot J_{Solar} = \varepsilon \cdot J_{re-radiated}$$

$$\alpha \cdot \boxed{\frac{3.856 \times 10^{26}}{4\pi d^2}} = \varepsilon \cdot \boxed{5.67 \times 10^{-8} T^4}$$

Solar radiation intensity

Thermal re-radiation intensity

emittance

absorptance

$$\left.\begin{aligned} \alpha \,/\, \varepsilon \text{ (black paint)} &= 1.08 \\ \alpha \,/\, \varepsilon \text{ (kapton)} &= 0.63 \end{aligned}\right\} \rightarrow$$

Max. temperature variation orbit noon-midnight:
$$\Delta T = 0.07°C - 0.08°C$$

Clock Thermal Sensitivity:

(as measured during ground tests)

$\boxed{\lesssim 2 \times 10^{-14} /°C}$ (Boving et al., 2009)

(Mattioni et al., 2002)
ground platform temperature variations of 5°C

temperature stabilized within **3 m°C** (by cavity thermal control)

$$\boxed{\Delta f / f << 10^{-15}}$$

In addition:
orientation of the Galileo maser cavity is
along the satellite +X-axis
(that never faces the Sun)

Conclusions

- **Thermal re-radiation (and thermal inertia) can explain the distinct clock/orbit pattern over a draconic year!**

- SLR bias in Galileo (and GPS) orbits can be explained by orbit shift opposite to the Sun direction due to the thermal re-radiation of the satellite body (SRP is too small for satellite body).

- **Geometrical Mapping of Orbit Perturbations** using onboard GNSS clock is a new technique to monitor orbit errors and was successfully applied to the modelling of thermal re–radiation acceleration (thermal inertia)

- **Galileo clock (MGEX and ground test) is showing smaller standard deviation compared to SLR**

- Simulated Galileo residual clock parameters show a standard deviation of σ=15.5 mm, when time bias and time drift (linear model) is removed over 24 h intervals from the simulated epoch-wise Galileo clock parameters over 10 days, whereas this standard deviation is reduced to σ =11.2 mm when the linear model is removed every 14 h (orbit period), down to σ =2.7 mm after time bias and time drift removal on the 1 h.

- The main perturbation affecting the Galileo clock parameters for high Sun elevation (>60°) is the the periodic relativistic effect due to J_2 gravity field coefficient (amplitude of about **18 mm**)

- Accumulated time along the Galileo orbit due to the gravitational potential of Sun and Moon after removing daily time bias and time drift shows distinct 2x per orbit effect below **0.4 mm for the Sun** and **1 mm for the Moon potential**.

- Environmental effects, such as variations in temperature and magnetic field were integrated along the orbit, but did not give a significant impact on the Galileo residual clock parameters. The max. effect of magnetic field is below **0.8 mm** whereas temperature perturbations are well below 1×10^{-15}.

Absolute Code Biases: DCBs Without TEC Maps

Differential Code Bias:

$$DCB_{P_1,P_2} := AB_1 - AB_2$$

Absolute Code Bias on P_1

Absolute Code Bias on P_2

Graphic Linear Combination:

$$LP_1 = \frac{1}{2}(P_1 + L_1) = \rho + \frac{1}{2}\lambda_1 N_1 + c\delta t - c\delta t^s + \boxed{\frac{1}{2}AB_1} + \varepsilon(LP_1)$$

Iono-Free Linear Combination:

$$L_{iono\text{-}free} = \frac{f_1^2}{f_1^2 - f_2^2}L_1 - \frac{f_2^2}{f_1^2 - f_2^2}L_2 = \rho + \lambda_N N_1 + \frac{1}{2}(\lambda_W - \lambda_N)N_W + c\delta t - c\delta t^s$$

Ambiguity-free condition:

$$\kappa_1^{af}\lambda_N + \kappa_2^{af}\frac{\lambda_1}{2} := 0$$

Ambiguity-Free Linear Combination:

$$AF_1 = \rho + \frac{f_1}{f_1 - f_2}AB_1 - \underbrace{\frac{c \cdot f_2}{(f_1 - f_2)^2}}_{\lambda = 0.67\ \mathrm{m}}N_W + c\delta t - c\delta t^s$$

Geometry-Free:

$$AF_1 - P_{iono\text{-}free} = \kappa_1^{af}L_{iono\text{-}free} + \kappa_2^{af}LP_1 - P_{iono\text{-}free} = \frac{\kappa_2^{af}}{2}AB_1$$

IGS Clock Convention ("Iono-Free Clocks" based on P_1 and P_2)

Absolute Code Biases with Third Frequency

$$L^{1,2}_{iono\text{-}free} = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = \rho + \lambda_N N_1 + \frac{1}{2}\left(\lambda_W - \lambda_N\right) N_W$$

$$L^{2,5}_{iono\text{-}free} = \frac{f_1^2}{f_1^2 - f_2^2} L_1 - \frac{f_2^2}{f_1^2 - f_2^2} L_2 = \rho + \lambda_{N(2,5)} N_1 - \lambda_{N(2,5)} N_W + \frac{1}{2}\left(\lambda_{W(2,5)} - \lambda_{N(2,5)}\right) N_{W(2,5)}$$

Iono-Free Linear Combinations:

Ambiguity-free condition:

$$\kappa_1^{af*} \lambda_N + \kappa_2^{af*} \lambda_{N(2,5)} := 0$$

$$L^{af*}_{iono\text{-}free} := \kappa_1^{af*} L^{1,2}_{iono\text{-}free} + \kappa_2^{af*} L^{2,5}_{iono\text{-}free}$$

$$:= \rho + \left[\frac{\kappa_1^{af*}}{2}\left(\lambda_W - \lambda_N\right) - \kappa_2^{af*} \lambda_{N(2,5)}\right] N_W + \frac{\kappa_2^{af*}}{2}\left(\lambda_{W(2,5)} - \lambda_{N(2,5)}\right) N_{W(2,5)}$$

$$AF_1 = \rho - \frac{c \cdot f_2}{(f_1 - f_2)^2} N_W + \boxed{\frac{f_1}{f_1 - f_2} AB_1} + c\delta t - c\delta t^s$$

Ambiguity-free condition:

$$-\frac{c \cdot f_2}{(f_1 - f_2)^2} \kappa_1^{af**} + \kappa_2^{af**} \lambda_W^{af*} := 0$$

Two-frequency Ambiguity-Free LC (previous slide)

Ambiguity-Free Linear Combination:

Geometry-Free:

$$AB_1 = \frac{f_1 - f_2}{\kappa_1^{af**} f_1}\left[\left(\kappa_1^{af**} AF_1 + \kappa_2^{af**} L_3^{af*} - P_{iono\text{-}free}\right) - \underbrace{\kappa_2^{af**} \lambda^{af*}_{W(2,5)}}_{\lambda = 3.41\ \text{m}} N_{W(2,5)}\right]$$

IGS Clock Convention ("Iono-Free Clocks")

CODE DCBs vs. DCBs Based on the Absolute Code Biases