Modeling and estimating the orbits of GNSS: Key principles in history and future

Gerhard Beutler
Astronomical Institute, University of Bern, Switzerland

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1200 E California Boulevard
Pasadena, California 91125
USA

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Modeling Satellite Orbits

What laws do Earth satellites obey in their motion:

- **In the most general case the orbit of a satellite is assumed to be a trajectory in the field of a stochastic differential equation system, implying that some of the perturbing accelerations are only known in a statistical sense, e.g., by their expectation values and their variances.**
- **In the simplest case the satellite orbit solves a system of non-linear ordinary differential equations (deqs). These methods are also referred to as "dynamic".**
- **Orbits piecewise solve deqs (short-arc methods, pseudo-stochastic pulses, piecewise constant accelerations).**

Modeling Satellite Orbits

The perturbing forces acting on an artificial Earth satellite may be classified as follows:

- **Class-1**: Forces, which are assumed to be **known** (e.g., planetary perturbations due to gravity),
- **Class-2**: Forces, which are assumed to be **known as mathematical functions**, but some **parameters are estimated** (solar radiation pressure (spr) models, gravity field models, …).
- **Class-3**: Forces, for which we **know** only **their stochastic properties**, e.g., their expectation values and variances (as a function of time).

It is **not clear a priori**, **which force belong to which class**!

It may, e.g., well be that **in future** some of the **low-degree & order terms of the geopotential** will no longer be considered as "fully known" (**class-1**) in some GNSS analyses, but as estimable quantities.

Modeling GNSS Orbits

- **Lageos (LAser GEodetic Satellite); spherical, diameter 60cm, mass 405kg**
- **GNSS satellite: Body 2 x 2 x 2 m^3, “wings” 20 x 2 m^2, mass 500-1000kg**

Modeling GNSS Orbits

Ferraris are built to minimize non-gravitational forces, trucks not really (only “to some extent”).

From the p.o.v. of orbitography the Lageos is a Ferrari, the GNSS satellite is a truck.

Modeling GNSS Orbits

Class-2 Forces: In the “GNSS world” these forces are in essence caused by solar radiation pressure (srp) either directly or indirectly.

We may/should distinguish:

- **Physical models derived from the satellites’ surfaces reflective & absorption & re-radiation properties, the attitude, etc. (e.g., Fliegel’s Rock-models). Almost a class-1 force.**
- **Purely empirical models (e.g., CODE model ECOM)**
- **Empirical models based on physical properties (e.g., Rodriguez-Solano et al.)**

Modeling GNSS Orbits

JOURNAL OF GEOPHYSICAL RESEARCH
Solid Earth
AN AGU JOURNAL

Papers on Geodesy and Gravity Tectonophysics

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Global Positioning System Radiation Force Model for geodetic applications

1. H. F. Fliegel
2. T. E. Gallini
3. E. R. Swift

Henry Fliegel is the undisputed pioneer of SRP modeling for GPS satellites.

Fliegel's approach is based on engineering facts and simple physical laws.

The hope is to construct a perfect a priori srp model (to make spr a class-1 force).

Modeling GNSS Orbits

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JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 41, No. 5, September–October 2004

Generalized Analytical Solar Radiation Pressure Model Algorithm for Spacecraft of Complex Shape

Marek Ziebart*
University College London, London, England WC1E 6BT, United Kingdom

The theoretical background of solar radiation pressure modeling is presented. The attitude behavior of specific classes of spacecraft and how these attitudes make it feasible to model radiation pressure effects in the spacecraft

IPN Progress Report 42-159 November 15, 2004

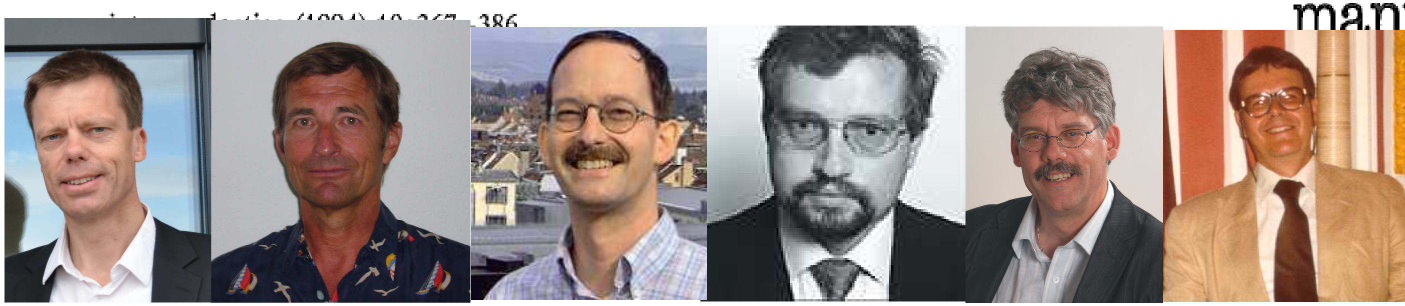
New Empirically Derived Solar Radiation Pressure Model for Global Positioning System Satellites

Y. Bar-Sever[1] and D. Kuang[1]

We describe the development and testing of a set of new and improved solar radiation pressure models for Global Positioning System (GPS) satellites that is based on four and one-half years of precise GPS orbital data. These empirical mod-

Marek Ziebart & Yoaz Bar-Sever further develop the Fliegel approach.

Modeling GNSS Orbits



Extended orbit modeling techniques at the CODE processing center of the international GPS service for geodynamics (IGS): theory and initial results

G. Beutler, E. Brockmann, W. Gurtner, U. Hugentobler, L. Mervart, M. Rothacher, and A. Verdun

Astronomical Institute, University of Berne, Sidlerstrasse 5, CH-3012 Bern, Switzerland

- Empirical resonance terms, Empirical CODE Orbit Model (ECOM), pseudo-stochastic parameters, albedo, gravity field parameters were studied here.
- The article was only written, because Leos Mervart wanted Beutler to learn LaTex. In this respect, the article was a success.
- E-Mail by Leos Mervart commenting this slide: Remember not the sins of my youth or my transgressions (Psalm 25:7)

Modeling GNSS Orbits

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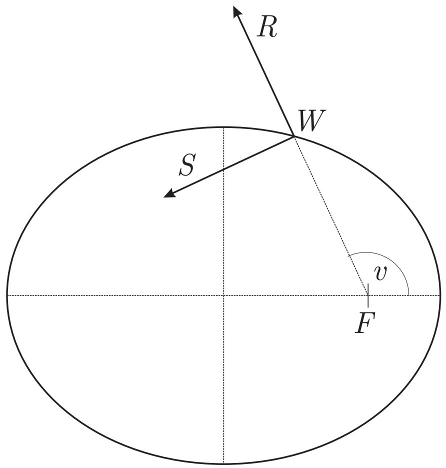
Colombo's proposed model

$$R = \sum_{i=0}^{\infty}\{R_{ci}\cos(iM) + R_{si}\sin(iM)\}$$

$$S = \sum_{i=0}^{\infty}\{S_{ci}\cos(iM) + S_{si}\sin(iM)\}$$

$$W = \sum_{i=0}^{\infty}\{W_{ci}\cos(iM) + W_{si}\sin(iM)\}$$

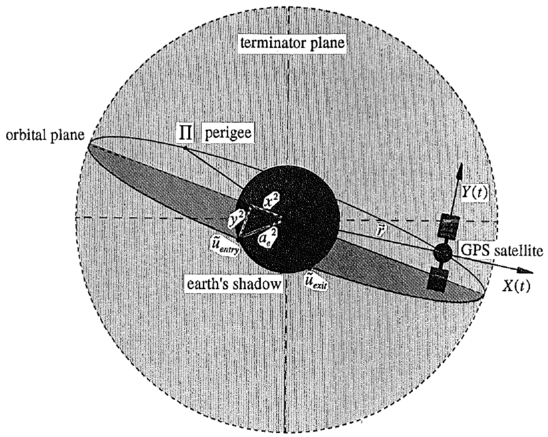


ECOM

$$D = \sum_{i=0}^{1}\{D_{ci}\cos(iu) + D_{si}\sin(iu)\}$$

$$Y = \sum_{i=0}^{1}\{Y_{ci}\cos(iu) + Y_{si}\sin(iu)\}$$

$$X = \sum_{i=0}^{1}\{X_{ci}\cos(iu) + X_{si}\sin(iu)\}$$



… by empirical models.

Oscar Colombo's original model (top)

ECOM (Empirical CODE Orbit model) (bottom).

M **is the mean anomaly,** $u \approx M + \omega$ **the argument of latitude.**

Improvement of ECOM (Empirical CODE Orbit Model):

(a) include higher order terms (i=2,3,…) in argument of latitude *u*.

(b) Replace *u* by $u - u_{sun}$

Modeling GNSS Orbits

$$\vec{a}_{drp} = \boxed{\vec{a}_{Rock}} + \boxed{X(t)\cdot\vec{e}_X + Y(t)\cdot\vec{e}_Y + Z(t)\cdot\vec{e}_Z} \quad (53)$$

where:

$\vec{e}_Z$ corresponds to $\vec{e}_0$ in eqn. (52),
$\vec{e}_Y$ corresponds to $\vec{e}_2$ in eqn. (52), and
$\vec{e}_X = \vec{e}_Y \times \vec{e}_Z$.

The coefficients $X(t)$, $Y(t)$, and $Z(t)$ are modeled with three parameters each:

$$\begin{aligned} X(t) &= X_0 + X_c \cdot \cos(u + \phi_X) \\ Y(t) &= Y_0 + Y_c \cdot \cos(u + \phi_Y) \\ Z(t) &= Z_0 + Z_c \cdot \cos(u + \phi_Z) \end{aligned} \quad (54)$$

where $u = M + \omega$ approximately is the argument of latitude. If the satellite is in the earth shadow, we have of course $X(t) = Y(t) = Z(t) = 0$. The Rock models

Excerpt from Beutler et al (1994).

The original intention was to absorb orbit model deficiencies rative to the best possible a priori model (red) with the at maximum nine parameters of the ECOM.

Formally, it is trivial to com-pletely ignore the a priori part of the model.

This was done later on, because

(a) the Rock models were far from perfect in the 1990 and

(b) There were (are) no Rock models for GLONASS.

Modeling GNSS Orbits

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Geodätisch-geophysikalische
Arbeiten in der Schweiz

(Fortsetzung der Publikationsreihe
«Astronomisch-geodätische Arbeiten in der Schweiz»)

herausgegeben von der

Schweizerischen Geodätischen Kommission
(Organ der Schweizerischen Akademie der Naturwissenschaften)

Sechzigster Band
Volume 60

Modeling and Validating Orbits and Clocks Using the Global Positioning System

Springer Link

Download PDF (5,334 KB)
GPS Solutions
January 1999, Volume 2, Issue 3, pp 50-62

A New Solar Radiation Pressure Model for GPS Satellites

Modeling GNSS Orbits

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J Geod (2014) 88:559–574
DOI 10.1007/s00190-014-0704-1

ORIGINAL ARTICLE

Reducing the draconitic errors in GNSS geodetic products

C. J. Rodriguez-Solano · U. Hugentobler ·
P. Steigenberger · M. Bloßfeld · M. Fritsche

The ECOM is replaced by the adjustable box-wing model, where (at maximum) 9 parameters are adjusted for each satellite.

Spurious effects on ERPs and other parameters are significantly reduced in combined GPS/GLONASS solutions.

→ This session, Dach, Hugentobler et al.

Modeling GNSS Orbits

The adjustable box-wing model (Rodriguez-Solano et al. 2012b) was created to compensate the effects of SRP impacting GPS satellites, using an intermediate approach between the physical/analytical models and the purely empirical models. The box-wing model is based on the physical interaction between solar radiation and satellite surfaces, simplifying the satellite to a box (satellite bus) and to a wing (solar panels). In addition, nine parameters can be adjusted (estimated) to fit best the GPS tracking data just as the CODE model does. The nine parameters are:

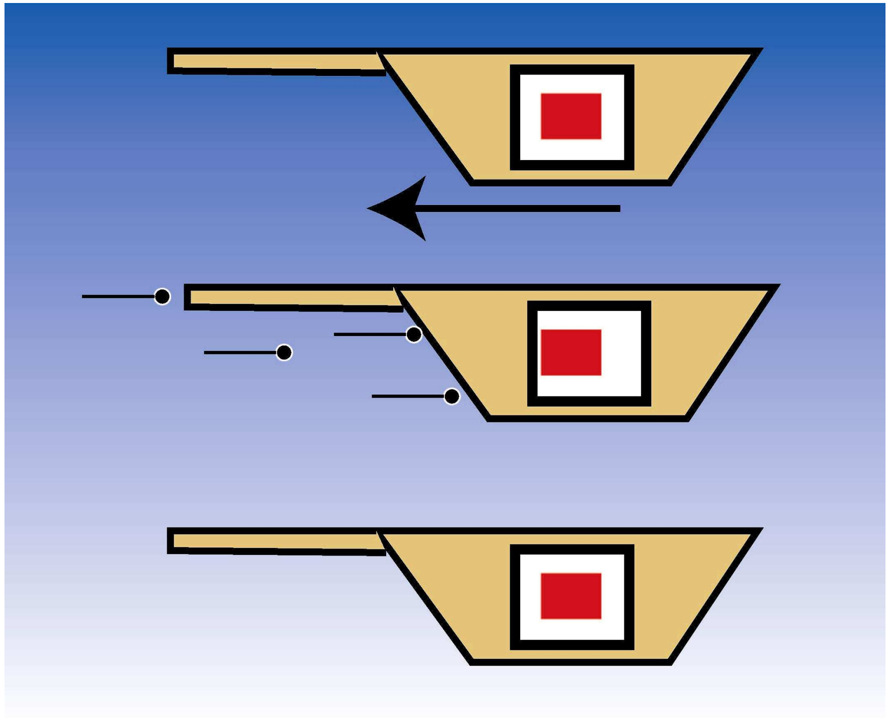
1. solar panel scaling factor $(1 + \rho + \frac{2}{3}\delta)$,
2. solar panel rotation lag,
3. Y-bias acceleration (Y_0 of CODE model),
4. absorption plus diffusion $(\alpha + \delta)$ of $+X$ bus,
5. absorption plus diffusion $(\alpha + \delta)$ of $+Z$ bus,
6. absorption plus diffusion $(\alpha + \delta)$ of $-Z$ bus,
7. reflection coefficient (ρ) of $+X$ bus,
8. reflection coefficient (ρ) of $+Z$ bus,
9. reflection coefficient (ρ) of $-Z$ bus.

Mathematically, the “Carlos-model” is an empirical model in the tradition of the Colombo- and ECOM-models.

Exactly like the former models it might be used on top of a truly physical model (based on the Fliegel tradition) or without a priori models.

Today, the ECOM *and* Carlos models are used without a priori models.

Modeling GNSS Orbits



Alternatively, one might measure the non-gravitational forces by accelerometers in the satellites. This would in essence remove the necessity to estimate the time variability of the non-gravitational forces.

The use of accelerometers in GNSS would enhance the usefulness of GNSS for precise applications.

This would put all modelers (empiric-al or physical) "out of action" (would it?).

By the way: accelerometers were on the last generation of Transit / Doppler satellites.

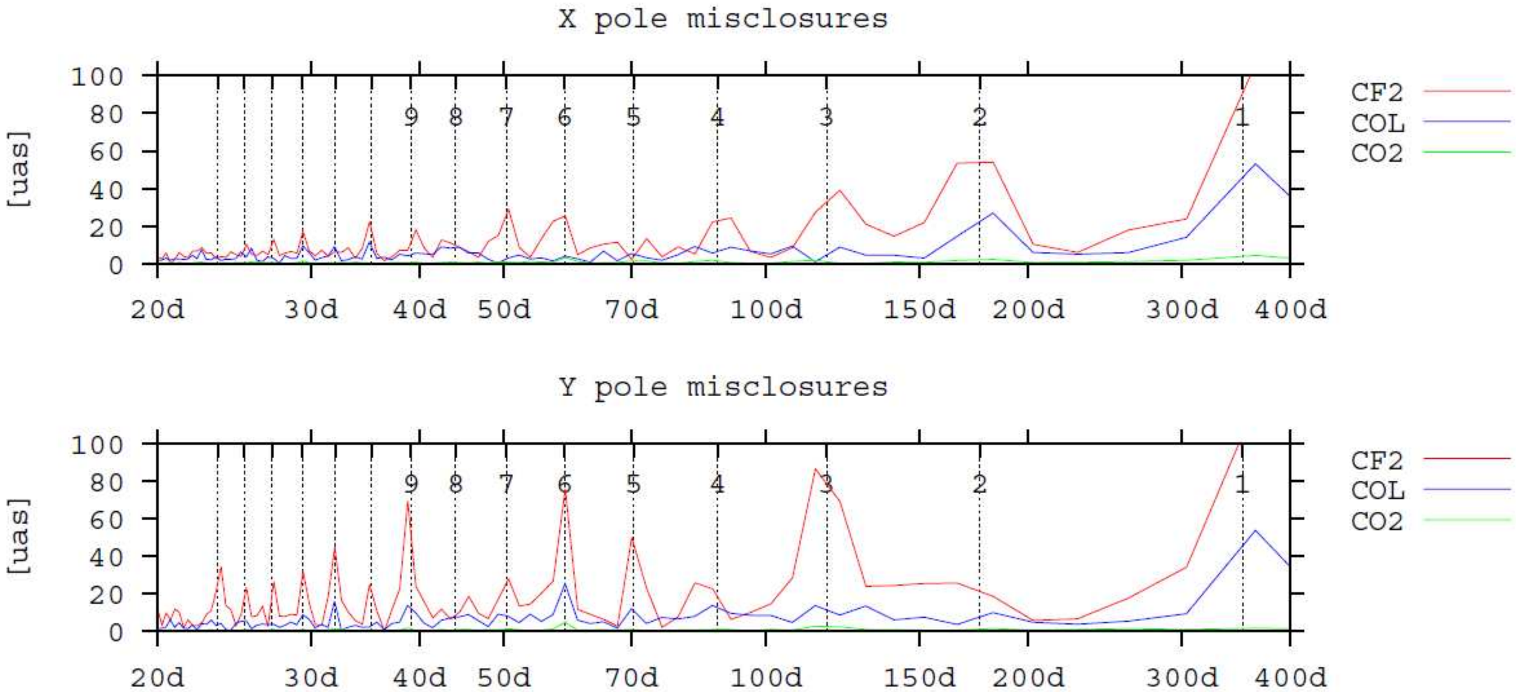
The Arc Length

Is there a natural arc length in satellite geodesy in general and in GNSS geodesy in particular?

- First (authoritative) answer: Certainly not! The revolution period *P* (or *2·P, 3·P*) would be good candidates – certainly not something strange like a solar day!
- Second answer (for orbit modelers): If parameters of the force field shall be estimated, the arc should be made as long as possible (many revolution periods) → learn from SLR analysis!
- Third answer (for "simple" geodesists): If force field parameters are of no interest and if the orbits should not bias other parameters of the adjustment, you may wish to "over-parameterize" the orbits to allow them to follow the observations as closely as possible (kinematic orbits?).
- Fourth answer (authoritative): There definitely is no natural arc length. The selection of the arc length is part of the "fine art" of Celestial Mechanics – and it depends on the application (e.g., IGS routine analysis, IGS reprocessing events, special exercises).
- Fifth answer (desparate/depressive CM): One (solar) day is the natural arc length in the IGS … and this probably will never change.

The arc length definitely has a significant impact on the propagation of orbit biases into other parameters (e.g., of geophysical interest):

Impact of arc length on other parameters



CODE Repro-2 Campaign (GPS & GLONASS). Spectrum of daily ERP misclosures (one-day, 3-d orbits/1-d ERPs, 3-d orbits & ERPs, old CODE classic)

→ **Length of the orbital arc has a significant impact on the amplitudes of the spurious spectral lines.**

From Simon Lutz et al, Session PY04

Multi-GNSS in the IGS

Protagonists and strong believers in GLONASS contributions to the IGS: Jim Slater (retired), Tim Springer (ESA), Rolf Dach (CODE), Gerhard Beutler (retired).

- **How was it done? GPS and GLONASS are analyzed "in one and the same program run".**
- **Why – philosophical answer: Both systems contribute to the determination of common parameters according to the number & quality of observations.**
- **Why – answer given by history: It could not be done in a different way when GLONASS was far from fully deployed (prior to about 2008).**

Drawback: Problems specific to a particular GNSS are brushed under the carpet.

Multi-GNSS in the IGS

Geodätisch-geophysikalische Arbeiten in der Schweiz

(Fortsetzung der Publikationsreihe «Astronomisch-geodätische Arbeiten in der Schweiz»)

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Schweizerischen Geodätischen Kommission
(Organ der Akademie der Naturwissenschaften Schweiz)

Dreiundachtzigster Band
Volume 83

Combined Analysis of Observations from Different Global Navigation Satellite Systems

J Geod
DOI 10.1007/s00190-014-0710-3

ORIGINAL ARTICLE

Homogeneous reprocessing of GPS, GLONASS and SLR observations

Mathias Fritsche · Krzysztof Sośnica · Carlos Javier Rodríguez-Solano · Peter Steigenberger · Kan Wang · Reinhard Dietrich · Rolf Dach · Urs Hugentobler · Markus Rothacher

Michael Meindl showed in 2011 that IGS-like GLONASS-only solutions are possible.

Fritsche et al. (2014) generated such a solution on the occasion of an IGS repro-exercise.

→ GNSS-specific solutions should be generated at least in the context of IGS Repro exercises!

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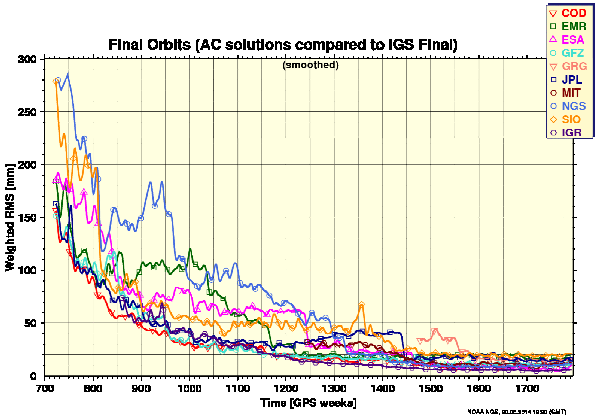
Summary & Conclusions



Orbit modeling in the IGS made great progress in the IGS since 1994.

Orbit validation and combination are of importance.

Consistency of solutions is important – in this context the IGS is in a much better shape today than 10 years ago thanks to the analysis coordination by NOAA/NGS – but …

- **Different orbit models (and different para-meterizations) should be allowed**
- **The arc length should be considered as an important attribute of the solutions!**
- **GNSS-specific solutions should be made regularly – at least in reprocessing exercises (and be it only for integrity monitoring);**
- **The three approaches (empirical, box-wing, physical modeling in the Fliegel tradition) must be further developed and validated.**
- **The parameterization of orbits (e.g., inclusion of low degree spherical harmonics → Sosnica et al., PY10) should be reconsidered – and not only for first degree terms (at least for repro-exercises).**