

# Achieving sub-centimetre RTK positioning accuracy by kinematic estimation of relative zenith troposphere-delays

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## Abstract

Precise Point Positioning (PPP) and Real Time Kinematic (RTK) GNSS techniques have been extensively developed to offer centimeter level positioning capabilities, after the solutions are converged or phase ambiguities are correctly fixed to their integers. It is a common perception that the achievable PPP/RTK accuracy is limited to the order of a few centimeters and the uncertainty of vertical component is around two times as high as those of horizontal components. Very little research has been made to further reduce this positioning uncertainty.

We examine the modeling and estimation challenges to achieve higher kinematic positioning accuracy. A relative/residual ZTD parameter is introduced along with the user's coordinate parameters to compensate the effects of residual tropospheric delays and multipath in the kinematic position estimation. Experimental results from 11 baselines of Shanghai CORS network with the lengths of up to 90 kilometers show that through additional modeling and estimation, the RTK solution accuracy can be improved from several centimeters to the sub-centimetre in all coordinate components. More evidently, the accuracy of the height component is even higher, for instance, better than 5 mm.

## Motivation

Regardless of PPP-RTK and RTK algorithms, there are common limitations that hinder their capabilities for more precise applications

- the achievable kinematic accuracy remains in the level of a few centimeters and the uncertainty of the vertical component is around two times as high as those of horizontal components after the phase ambiguities are correctly fixed to their integer values;
- the errors of the current PPP-RTK or RTK solutions show both systematic and random characteristics, which not only are sensitive to, or grow in proportion to, the inter-station separation, but also depend on the locations of the reference stations and user locations

Ambiguity resolution is just an intermediate step in obtaining the precise position solutions. While most users seem to passively accept centimeter-level positioning accuracy, little research attention has been paid to further improvement of the RTK precision after the ambiguity-fixed observations are obtained. In fact, this issue is also very challenging for two reasons:

- After correct ambiguity resolution, the remaining systematic errors still affect the position solutions.
- Filtered RTK solutions can take advantages of measurements of multiple epochs to improve the accuracy, but still in centimetre level
- It is preferred that position estimation is done using a single epoch observation.

## Procedures

The proposed sub-centimetre RTK positioning procedure adopt a troposphere-decorrelation procedure after ambiguity resolution, the whole procedure follows four steps:

**Step 1:** The the ambiguity-floated RTK solutions with a single or multiple constellations can be calculated with standard least-squares or filter. Single epoch or multiple epochs observation may be required to obtain the float solutions and covariance matrix suitable for reliable ambiguity resolution.

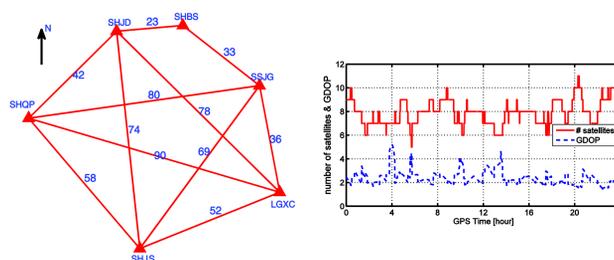
**Step 2:** Perform ambiguity resolution for L1 and L2 carriers or their combinations of DD phase measurements (in single-base or network-base RTK case). This process usually involved an integer search and validation based on the float ambiguity estimates and their covariance matrix from Step 1.

**Step 3:** With correctly fixed ambiguity parameters, the linear RTK model can be given as:  
 $y = Hx + \epsilon, cov(y) = \sigma_0^2 Q$   
The rovers coordinates can be estimated using the least squares (LS) algorithms without introducing the ZTD parameter. the LS solution is given as:  
 $\hat{x}_{LS} = (H^T Q^{-1} H)^{-1} H^T Q^{-1} y, cov(\hat{x}_{LS}) = \sigma_0^2 (H^T Q^{-1} H)^{-1}$   
The user location solutions with centimeter level accuracy is known as the LS-RTK solutions.

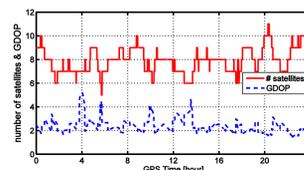
**Step 4:** With the assistance of the ambiguity-fixed LS-RTK solutions, the troposphere-decorrelation solution can be given as  
 $\hat{x}_{TD} = N^{-1} H^T Q^{-1} y, cov(\hat{x}_{TD}) = \sigma_0^2 N^{-1} H^T Q^{-1} H N^{-1}$   
with  $N = H^T Q^{-1} H + \alpha S$ . The solution is evaluated by mean squared error (MSE), which is given as  
 $M_{TD} = E[(\hat{x}_{TD} - x)(\hat{x}_{TD} - x)^T] = \sigma_0^2 N^{-1} H^T Q^{-1} H N^{-1} + \alpha^2 N^{-1} S x x^T S N^{-1}$   
where  $x$  is the true parameter value. The troposphere decorrelation parameter can be computed with  $\alpha = \arg \min_{\alpha > 0} trace(M_{TD})$ ,  
with  $S = \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$  where  $I_3$  is a  $3 \times 3$  identity matrix. This RTK solution is known as TD-RTK solutions.

## Numerical Results

Six 24-h dual-frequency GPS data sets on day 199 in 2010 were collected at the 30s rate from the CORS network in Shanghai China and analyzed. The elevation mask was taken  $10^\circ$  in the computations. 11 baselines were formed with 6 stations and the baseline lengths range from 23 to 90 km. The coordinates of these 6 stations are known to a high precision and thus being used as the benchmark for the RTK assessment.

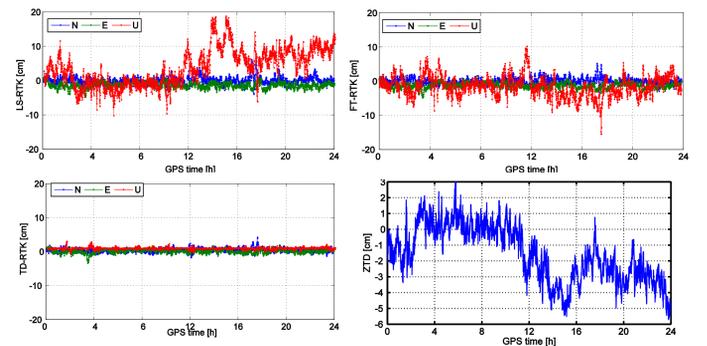


**Fig.1** The configuration of 6 stations and their formed 11 baselines with labeled baseline length in kilometers

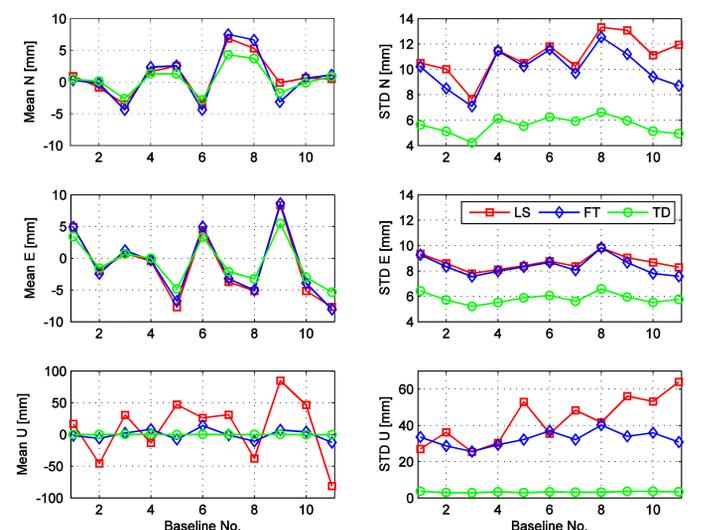


**Fig.2** The number of tracked satellites and GDOP values for 24 hour observation for SHJS-LGXC baseline.

Three models are compared in this study, the least-squares based RTK (LS-RTK), the filter based RTK (FT-RTK) and the troposphere decorrelated RTK (TD-RTK). The troposphere is ignored in LS-RTK estimates. The results are shown in Fig 3, Fig 4 and Table 1.



**Fig.3** The errors of LS-RTK (top-left), FT-RTK (top-right) and TD-RTK (bottom-left) solutions and the ZTD variation for the SHJS-LGXC baseline (52 km).



**Fig.4** Comparison of LS-RTK, FT-RTK and TD-RTK solutions in mean and STD statistics for three coordinate components over all 11 baselines

**Table 1.** Comparison of LS-RTK, FT-RTK and TD-RTK solutions in RMS statistics for three coordinate components over 11 baselines [mm]. TD-RTK clearly reaches sub-cm solutions.

Baseline km.	LS-RTK			FT-RTK			TD-RTK		
	N	E	U	N	E	U	N	E	U
23	10.5	10.6	31.7	10.2	10.5	33.6	5.6	7.3	3.7
33	10.0	8.8	58.1	8.5	8.7	29.3	5.1	5.9	3.0
36	8.4	7.8	39.8	8.3	7.6	25.7	5.0	5.2	2.8
42	11.6	8.1	33.1	11.7	8.0	30.4	6.3	5.5	3.4
52	10.8	11.4	70.9	10.6	10.7	33.1	5.7	7.6	3.0
58	12.3	9.8	44.1	12.4	10.0	39.4	6.9	6.9	3.4
69	12.3	9.1	57.4	12.3	8.7	32.1	7.3	6.0	3.1
74	14.3	11.1	56.4	14.2	11.1	41.6	7.6	7.3	3.2
78	13.1	12.3	101.6	11.6	12.2	34.6	6.2	8.1	3.6
80	11.1	10.1	70.6	9.4	8.7	36.2	5.1	6.3	3.6
90	11.9	11.3	103.4	8.8	11.1	33.3	5.0	7.9	3.4
mean	11.5	10.0	60.6	10.7	9.8	33.6	6.0	6.7	3.3

## Summary

Achieving the sub-centimeter RTK positioning accuracy is a challenging research direction. In our troposphere-decorrelation (TD) approach, ZTD is estimated kinematically along with the coordinates without filtering over epochs. The proposed TD-RTK method enables efficient separation of residual tropospheric delays from the coordinates, essentially attributing to epoch-by-epoch estimation of ZTDs. This is in contrast to the traditional treatment for ZTD parameter in RTK algorithms and ZTD parameter in PPP algorithms, where ZTD or ZTD are estimated from a filtering procedure or as an average ZTD over multiple epochs by using a random walk model. The traditional modeling cannot describe the instantaneous variation of the real ZTD. This should be the reason why the TD-RTK solutions have achieved much more accurate height solutions. The TD-RTK method is a promising technique for the next generation RTK systems.