# An attempt to decorrelate geocenter motion from empirical accelerations

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## Introduction (A)

As a satellite-based technique, GNSS are theoretically sensitive to motions of the Earth's center of mass (CM). In particular, the net translations between the weekly solutions of the IGS Analysis Centers (ACs) and a secular frame such as ITRF2008 should approximate the non-linear motions of CM with respect to the Earth's center of figure. However, this sensitivity is limited by an insufficient knowledge of the non-gravitational forces acting on GNSS satellites. For precise orbit determination, ACs indeed need to estimate empirical accelerations, which correlate with the origin of the orbit frame.

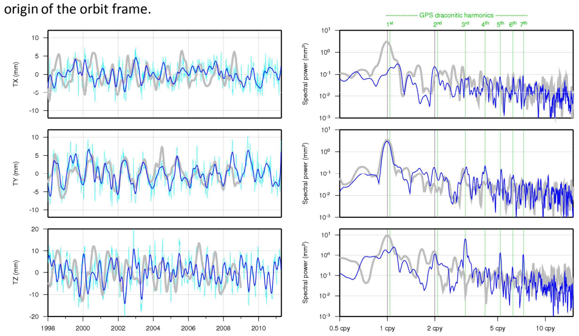

*Fig. 1: Comparison of SLR-derived and GNSS-derived geocenter motion time series*
- *Smoothed translations between weekly ILRS solutions and ITRF2008*
- *(Smoothed) translations between weekly ESA GNSS solutions and a stacked secular frame*

## Correlations between the ECOM parameters and the Z coordinate of CM (B)

If the Earth's CM was shifted by $\delta Z_{CM}$, the differential acceleration felt by a satellite in the Earth-centered inertial frame would be:

$$\delta\vec{a} = \frac{\partial\vec{a}}{\partial Z_{CM}}\delta Z_{CM} \approx \frac{GM}{r^3}\begin{bmatrix}-3xz/r^2\\ -3yz/r^2\\ 1-3z^2/r^2\end{bmatrix}\delta Z_{CM}$$

*Vector-valued function of time describing the sensitivity of the satellite acceleration to the Z coordinate of CM*

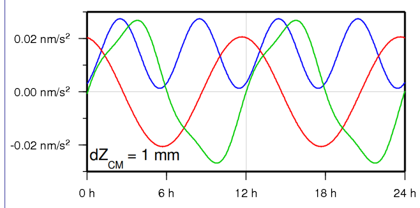

*Fig. 2: Example of accelerations that would be felt by a GPS satellite if CM was shifted by 1 mm in the Z direction. The accelerations are shown in the DYB frame of the satellite: Blue: D component; Red: Y component; Green: B component*

To model the non-gravitational forces acting on GNSS satellites, most IGS ACs freely estimate a subset of the Extended CODE Orbit Model (ECOM; Beutler et al., 1994) parameters:

- constant accelerations in the D, Y, B directions ($D_0$, $Y_0$, $B_0$)
- 'once-per-rev' accelerations in the B direction ($B_C$, $B_S$)

The highlighted parameters can potentially absorb the highlighted terms of the Fourier expansion of $\partial\vec{a}/\partial Z_{CM}$ in Table 1.

| Frame | Inertial frame | DYB frame | | |
|---|---|---|---|---|
| Axes | X, Y, Z | D | Y | B |
| Frequencies | constant, 2ω | constant, 2ω | ω, 3ω, 5ω… | ω, 3ω, 5ω… |

*Table 1: Frequencies found in the Fourier series expansion of $\frac{\partial\vec{a}}{\partial Z_{CM}}$ along different axes (ω = satellite revolution frequency)*

For various positions of the Sun above the orbital plane, we computed the power fraction of $\partial\vec{a}/\partial Z_{CM}$ absorbable by the ECOM parameters. This fraction is an indicator of the correlation between the Z coordinate of CM and the ECOM parameters of a single satellite.

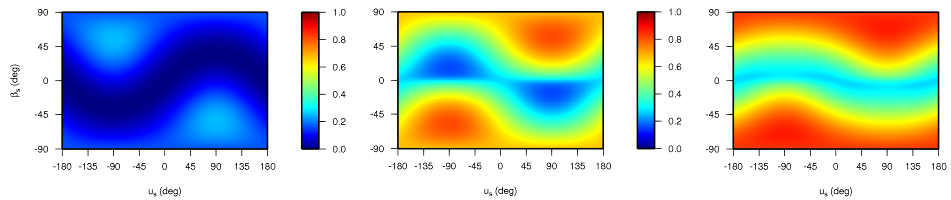

*Fig. 3: Power fractions of $\partial\vec{a}/\partial Z_{CM}$ absorbable by the ECOM parameters, plotted as functions of $u_S$ (argument of latitude of the Sun in the orbital plane) and $\beta_S$ (elevation of the Sun above the orbital plane)*

*Left: Power fraction of the constant term along D (i.e. absorbable by $D_0$)*
*Middle: Power fraction of the 'once-per-rev' term along B (i.e. absorbable by $B_C$, $B_S$)*
*Right: Sum of both (i.e. total power fraction absorbable by the ECOM parameters)*

## Decorrelation constraint (C)

In a (daily) least-squares adjustment of GNSS observations, it should be possible to decorrelate the Z coordinate of CM from the empirical satellite accelerations by introducing the following constraint equation:

$$\sum_{i=1}^{n_{sat}}\int_{0h}^{24h}\frac{\partial\vec{a}^i(t)}{\partial Z_{CM}}\cdot\vec{a}^i_{emp}(t)\,dt = K \Leftrightarrow A\cdot\begin{bmatrix}D_0^1\\ \vdots\\ B_S^{n_{sat}}\end{bmatrix} = K$$

where $\vec{a}^i_{emp}(t)$ denotes the total empirical acceleration estimated for satellite i, i.e.:

$$\vec{a}^i_{emp}(t) = D_0^i\,\vec{e}^i_D(t) + Y_0^i\,\vec{e}^i_Y(t) + \left(B_0^i + B_C^i\cos u^i(t) + B_S^i\sin u^i(t)\right)\vec{e}^i_Y(t) + \ldots$$

We applied this constraint to a daily normal equation provided by ESA for various values of K. The impact of the constraint was then assessed by:

- estimating translations between the obtained terrestrial frames and a reference solution obtained without constraint (Fig. 4),
- comparing the obtained terrestrial frames with the IGS combined frame of the week (Fig. 5),
- comparing the obtained orbits with the IGS combined orbits of the day (Fig. 6).

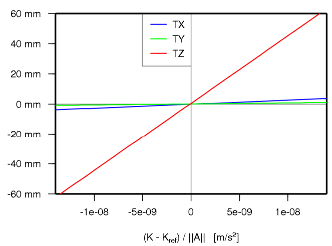

*Fig. 4: Terrestrial frame translations resulting from the application of the decorrelation constraint*

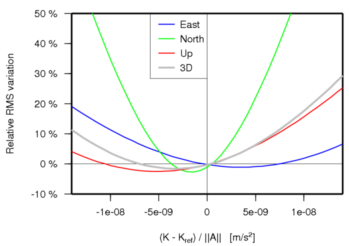

*Fig. 5: Comparison of the obtained terrestrial frames with igb08P1487.snx – Relative residual RMS variations due to the application of the decorrelation constraint*

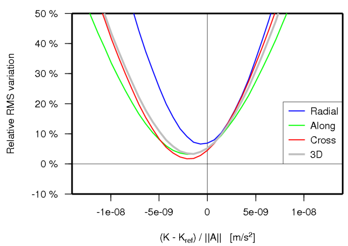

*Fig. 6: Comparison of the obtained orbits with igs14871.sp3 – Relative residual RMS variations due to the application of the decorrelation constraint*

- This example shows that the proposed decorrelation constraint mainly acts on the Z component of the frame origin, as expected.
- The Z component of the frame origin can vary in a range of 1-2 cm with a relatively small impact on the geometry of both the orbits and the terrestrial frame.
- A fundamental question remains however open:

**How should be chosen the 2nd member K of the constraint equation?**

## Experiment (D)

Unfortunately, the 2nd member of the decorrelation constraint equation cannot be taken zero: some a priori knowledge of the ECOM parameters is necessary. We thus tried the following experiment:

1) Invert daily normal equations provided by ESA over 1995-2008 without decorrelation constraints.
2) For each GPS satellite, build an a priori model for the parameter $D_0$ by fitting a $\beta_S$-dependent function to the daily $D_0$ estimates (cf. Springer et al., 1999).
3) Re-invert the daily normal equations with decorrelation constraints applied with respect to the a priori $D_0$ models from 2).

Fig 7. shows the result of the experiment: applying the decorrelation constraints with respect to our a priori $D_0$ models has almost no impact on TZ! One probable explanation is that errors in the unconstrained $D_0$ estimates were transmitted to our a priori $D_0$ models.

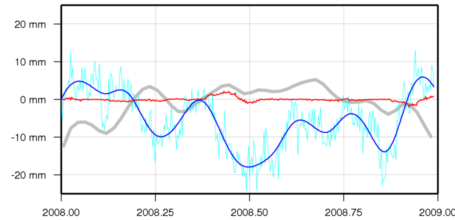

*Fig. 7: Impact of the decorrelation constraints on TZ, with K computed from empirical $\beta_S$-dependent models of the satellite $D_0$ parameters*
- *Smoothed Z-translations between weekly ILRS solutions and ITRF2008*
- *(Smoothed) Z-translations between the daily solutions from step 1) and IGS08*
- *Z-translations between the daily solutions from step 3) and those from step 1) (i.e., impact of the decorrelation constraints)*

## Conclusions (E)

- In global GNSS solutions where solar radiation pressure is estimated in the form of ECOM parameters, there is a high correlation between the Z component of the frame origin and a **known** linear combination of the ECOM parameters.
- A sufficient a priori knowledge of the ECOM parameters would allow to break this correlation, by means of the proposed decorrelation constraint.
- To obtain a precision of 1 mm on TZ, and assuming white noise errors and 30 satellites, the precision of the a priori model should be ≈ $4.10^{-10}$ m/s².
- Moreover, empirical a priori models such as the one tried here do not seem appropriate, as they can absorb geocenter-related errors.
- Improvements in purely GNSS-derived Z geocenter motion estimates thus only seems achievable through:
  - either precise external (i.e. analytical) solar radiation pressure models,
  - or other, less correlated parameterizations of the satellite empirical accelerations.

## References

Beutler G, Brockmann E, Gurtner W, Hugentobler U, Mervart L, Rothacher M (1994) Extended orbit modeling techniques at the CODE processing center of the International GPS Service for Geodynamics (IGS): Theory and initial results, Manuscripta Geodetica, 19(6):367-386

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