Performance of the Selected Geopotential Models with the Empirical Accelerations in the Aspect of GOCE Satellite Orbit Computation

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Introduction: Beginning in the second half of 2009, the gravity field measurements - Satellite Gravity Gradiometry measurements (SGG) and the Satellite to Satellite Tracking data (SST) are transmitted by the Gravity Field andSteady – State Ocean Circulation Explorer (GOCE) to the ground segment of the mission. An important role in the processing of these kinds of data plays the knowledge of the satellite orbit, which estimation is based on the SST observations. The presented work is a part of an orbital research of the GOCE satellite. The aim of this research is to use the SST and SGG measurements in the process of the satellite orbit improvement. This work includes a comparison of selected gravity field models in an aspect of GOCE orbit computation. Such the comparison can be helpful in selecting the proper geopotential model for the mentioned process of orbit improvement.

Research and results: In order to perform of the comparison of selected gepotential models, the various variants of the GOCE orbit were computed and then compared with a reference orbit in J2000 inertial reference frame. These 1-day variants differ from each other with the satellite motion model. The reference orbit is the 1-day GOCE satellite orbit delivered by European Space Agency (ESA) and it is known as the reduced-dynamic Precise Science Orbit (RD PSO). This orbit with a sampling interval of 10 sec., has the centimeter-level accuracy and originally is expressed with respect to the ITRF2005 reference frame. Thus, this reference orbit was transformed into J2000 reference frame using given quaternions. The basic tool used in this work is the Toruń Orbit Processor system (TOP), which enables determination of a satellite orbit in the field of gravitational and non-gravitational perturbing forces. The TOP system is based on the Cowell numerical integration method of 8th order. All GOCE orbits were computed using a fixed initial state vector at the epoch 55136.99983 MJD (1 Oct 2009 23h 59m 45.0s UTC). This vector can be described in terms of the following keplerian elements: semi-major axis: 6643.270587 km, eccentricity: 0.002520, inclination: 96.593991 deg, argument of perigee: 115.322515 deg, right ascension of ascending node: 314.511048 deg, mean anomaly: 63.689870 deg. The above initial state vector is equal to the initial state vector of the reference orbit (RD PSO).

As a measure of the difference between the computed orbit and the reference orbit, the value of RMSD (equation (1)) is adopted. This value is a function of the root mean squares (RMSs) of coordinate differences between the computed orbit and the reference orbit (equation (2)). All geopotential models used in this research, were obtained through International Centre for Global Earth Models (ICGEM).

The RMSD values for a few selected models are showed in Table 1. In the following cases, the satellite motion model is determined by the geopotential and chosen empirical acceleration, which was determined by means of a linear model. It is clear that the addition of the radial acceleration is by far the most effective (RMSD at the level of two hundred meters).

Tables 2a and 2b gives the two variants of RMSD values for fifty selected geopotential models, taking into account the linear model for the empirical accelerations. Here, the computed orbits are based on the satellite motion model containing the given geopotential model and radial acceleration. The values of RMSD for these variants are smaller for the older models than for the newer ones (for example in variant <1> OSU91a – 194.41 m, OSU89a – 199.43 m but EIGEN-51C – 219.31 m, ITG-GRACE2010S - 219.28 m). However, the oldest models, starting from GEMT2, have the highest values of RMSD. When the variant <2> is taken into account, where the geopotential models were truncated at given value degree and order, RMDS values for the older model are still smaller than for the newer ones, but maximal difference between mentioned above values decreased from level of about 25 m (in variant <1>) to the level of near 9 m (variant <2>). In this case the greater decrease of the RMSD value occurred for the newer models. The degree and order of the given model truncation was selected after analysis of RMSD distribution depending on the truncation degree of this model. For all presented here RMSD variants, the two parameters of the radial acceleration model were estimated separately for each geopotential model. These parameters correspond to the values of radial acceleration at the beginning and at the end of orbital arc. Typical values of these parameters were equal to about $-6\cdot10^{-7}$ km/s^2 (directed radially towards the Earth).

Table 3 shows the two variants of RMSD computed using the non-linear model of empirical accelerations. The first variant was obtained for the selected full geopotential models and the radial acceleration, whereas the second one is based on the trucated geopotential models and the radial, along-track, cross-track acclerations and additionally on the acceleration connected with the changes of semi-major axis. In both cases, it is clear visible the decrease of RMSD values for all geopotential models compared to the previous results. For the variant <1>, the value of RMSD decrease varies from about 40 m (OSU86F) to near 64 m (EGM2008 360x360) compared to the analogous case of linear model using. For the truncated geopotential models (variant <2>), the RMSD decrease ranges from 41 m (OSU86F) to about 53 m (JGM3). Similarly to the ”linear model case”, the best results are achieved for truncated OSU91a , JGM3 and EGM96 (149.70 m, 151.29 m, 152.17 m, respectively). For the newer geopotential models, the RMSD values are greater (for example for EIGEN-6S – 153.24 m). It can also be observed, that after application of the non-linear model the maximal difference between the RMSD values is smaller than for ”linear model case”(for example 24.94 m – for the linear model and 5.42 m – for the non-linear model taking into account the full geopotential models and the radial acceleration). The proposed non-linear model contains, among others, numerous periodical components of which the biggest impact have ones with the periods close to 1 day (the length of orbital arc). This feature was noticed for OSU91a 60x60 with the four considered here empirical accelerations.

Summary: The selected geopotential models were compared in terms of the 1-day GOCE satellite orbit computation with respect to the reference orbit - RD PSO orbit. Taking into account the linear and non-linear model of empirical accelerations, it is clear that the older geopotential models work better (especially OSU91a, JGM3, EGM96 with RMSD below 152 m) than the newer ones (for example EIGEN-6S, EIGEN-51C, ITG-GRACE2010S with RMSD values at the level 153 m). This question is not yet well understood. In fact, RMSD value depends on the work of given geopotential model with the empirical accelerations, which absorb to some extent the errors of this model. It seems that the absorption of errors is more efficient in case of the older geopotential models. The newer models are more accurate and less prone (may by more “rigid”) to the error absorption. What's more, the absorption efficiency increases after the given geopotential model truncation at the long-wave length part. However, the efficiency of the error absorpion for the newer geopotential models increased, in comparison to the older models, after application of the non-linear model of empirical accelerations. This is visible in smaller differences between the RMSD values (Table 3). RMSD values for the oldest models are significantly higher (Table 2).

Table 1. *RMSD differences between the 1-day computed orbit and the 1-day GOCE reference orbit (reduced-dynamic Precise Science Orbit (RD PSO)) for the selected geopotential models. The satellite motion model consists of the geopotential and one of the three empirical accelerations. Additionally, it was shown the decrease of the RMSD value after adding the consecutive empirical accelerations for CONS_GCF_2_DIR_R3 model. The linear model of the empirical accelerations was used.*

GEOPOTENTIAL MODEL	RMSD [m] SATELLITE MOTION MODEL: - GEOPOTENTIAL - RADIAL ACCELERATION	RMSD [m] SATELLITE MOTION MODEL: - GEOPOTENTIAL - ALONG-TRACK ACCEL.	RMSD [m] SATELLITE MOTION MODEL: - GEOPOTENTIAL - CROSS-TRACK ACCEL.
CONS_GCF_2_DIR_R3	219.31	5261.63	51363.16
ITG-GRACE2010S	219.28	5261.23	51359.62
OSU91a	194.41	5252.57	51419.92
GEOPOTENTIAL MODEL	**RMSD [m]** SATELLITE MOTION MODEL: - GEOPOTENTIAL - RADIAL ACCELERATION	**RMSD [m]** SATELLITE MOTION MODEL: - GEOPOTENTIAL - RADIAL ACCEL. - ALONG-TRACK ACCEL.	**RMSD [m]** SATELLITE MOTION MODEL: - GEOPOTENTIAL - RADIAL ACCEL. - ALONG-TRACK ACCEL. - CROSS-TRACK ACCEL.
CONS_GCF_2_DIR_R3	219.31	219.10 (-21 cm)	218.79 (-31 cm)

Table 3. *RMSD differences between the 1-day computed orbit and the 1-day GOCE reference orbit (RD PSO orbit) for the selected geopotential models. The satellite motion model includes the geopotential and the linear model of radial empirical acceleration. The geopotential models are truncated at degree and order 14. Such degree and order results from the analysis of the distribution of RMSD differences depending on the model truncation degree. In case of this distribution the satellite motion model includes only geopotential.*

GEOPOTENTIAL MODEL - truncation at 14x14	RMSD [m] SATELLITE MOTION MODEL: - GEOPOTENTIAL - RADIAL ACCELERATION
CONS_GCF_2_DIR_R3	200.70
CONS_GCF_2_TIM_R3	200.72
EIGEN-6C	200.70
ITG-GRACE2010S	200.67
PGM2000a	200.76
OSU91a	201.11
WGS66	270.71

Table 2a and 2b. *Two variants of the RMSD differences between the 1-day computed orbit and the 1-day reference orbit for the selected geopotential models.*
The linear model of radial acceleration was used.

Nr	GEOPOTENTIAL MODEL	RMSD differences between the computed orbit and the reference one	
		<1> [m]	<2> Truncation RMSD [m] / DEGxORD
1	CONS_GCF_2_DIR_R3	219.31	201.59 / 22x22
2	CONS_GCF_2_TIM_R3	219.37	201.54 / 22x22
3	EIGEN-6C	219.34	201.60 / 22x22
4	EIGEN-6S	219.34	201.60 / 22x22
5	GOCO02S	219.33	201.60 / 22x22
6	GO_CONS_GCF_2_DIR_R2	219.37	201.59 / 22x22
7	GO_CONS_GCF_2_TIM_R2	218.86	201.45 / 22x22
8	GO_CONS_GCF_2_SPW_R2	220.60	201.84 / 22x22
9	GO_CONS_GCF_2_DIR_R1	219.30	201.56 / 22x22
10	GO_CONS_GCF_2_TIM_R1	219.11	201.54 / 22x22
11	GO_CONS_GCF_2_SPW_R1	219.41	201.59 / 22x22
12	GOCO01S	219.33	201.60 / 22x22
13	GIF48	219.26	201.57 / 22x22
14	AIUB-GRACE03S	219.31	201.60 / 22x22
15	EIGEN-51C	219.31	201.59 / 22x22
16	AIUB-CHAMP03S	218.99	201.61 / 22x22
17	EIGEN-CHAMP05S	218.50	201.58 / 22x22
18	ITG-GRACE2010S	219.28	201.58 / 22x22
19	AIUB-GRACE02S	219.26	201.60 / 22x22
20	GGM03C	219.23	201.57 / 22x22
21	GGM03S	219.13	201.57 / 22x22
22	AIUB-GRACE01S	219.18	201.60 / 22x22
23	EIGEN-5S	219.18	201.60 / 22x22
24	EIGEN-5C	219.31	201.60 / 22x22
25	EGM2008 1440x1440	219.35	201.59 / 22x22
26	ITG-GRACE03	219.24	201.57 / 22x22
27	AIUB-CHAMP01S	218.29	201.70 / 22x22
28	ITG-GRACE02S	219.26	201.57 / 22x22
29	EIGEN-GL04S1	219.26	201.60 / 22x22
30	EIGEN-GL04C	219.35	201.60 / 22x22
31	EIGEN-CG03C	219.37	201.60 / 22x22
32	EIGEN-CG01C	219.47	201.60 / 22x22
33	EIGEN-CHAMP03S	216.64	201.60 / 22x22
34	EIGEN-GRACE02S	219.06	201.60 / 22x22
35	EIGEN-2	218.30	201.87 / 22x22
36	PGM2000a	212.70	201.56 / 22x22
37	EGM96	212.32	201.57 / 22x22
38	GFZ96	209.95	202.10 / 22x22
39	GRIM4S4	215.20	200.58 / 11x11
40	JGM3	207.35	206.03 / 60x60
41	GFZ93a	211.66	196.98 / 19x19
42	OSU91a	194.41	192.89 / 60x60
43	OSU89a	199.43	197.83 / 60x60
44	GEMT2	209.41	201.04 / 19x19
45	OSU86F	197.17	195.98 / 60x60
46	OSU81	206.74	197.86 / 22x22
47	GEM10b	239.19	199.25 / 19x19
48	OSU68	277.90	277.90 / -
49	WGS66	270.40	245.97 / 9x9
50	SE1	248.08	248.08 / -

<1> - RMSD difference between the computed orbit (with the satllite motion model consisted of the geopotential and the radial empirical acceleration) and the reference orbit – the RD PSO orbit of the GOCE satellite.

<2> - RMSD difference between the computed orbit (obtained from the satellite motion model including the truncated geopotential model and the linear model of radial empirical acceleration) and the reference orbit, i.e. the RD PSO orbit of the GOCE satellite. The degree of the given model truncation was determined taking into account the distribution of the RMSD values depending on the truncation degree of this model. The chosen degree of the truncation corresponds to the minimal value of RMSD. The mentioned distribution was obtained taking into account estimated parameters of the radial acceleration linear model for the entire geopotential model (without truncation).

Table 3. *Two variants of the RMSD differences between the 1-day computed orbit and the 1-day reference orbit for the selected geopotential models.*
The non-linear model of empirical accelerations was applied.

Nr	GEOPOTENTIAL MODEL	RMSD differences between the computed orbit and the reference one	
		<1> [m]	<2> Truncation RMSD [m] / DEGxORD
1	CONS_GCF_2_DIR_R3	155.82	153.24 / 51x51
4	EIGEN-6S	155.52	153.01 / 51x51
7	GO_CONS_GCF_2_TIM_R2	155.47	152.74 / 51x51
15	EIGEN-51C	155.49	152.94 / 51x51
18	ITG-GRACE2010S	155.42	152.90 / 51x51
23	EIGEN-5S	155.58	152.98 / 51x51
25	EGM2008 360x360	155.44	152.93 / 51x51
37	EGM96	154.08	152.17 / 59x59
40	JGM3	153.53	151.29 / 59x59
42	OSU91a	151.26	149.70/ 60x60
45	OSU86F	156.68	153.56 / 51x51

<1> - RMSD difference between the computed orbit (with the satellite motion model consisted of the geopotential and the radial empirical acceleration – non-linear model) and the reference orbit – the RD PSO orbit of the GOCE satellite. The full geopotential models were taken into account. The parameters of the non-linear model were obtained for these geopotential models truncated at 60x60.

<2> - RMSD difference between the computed orbit (obtained from the satellite motion model including the truncated geopotential model and the non-linear models of radial, along-track, cross-track empirical accelerations and of the acceleration causing the changes of semi-major axis) and the reference orbit, i.e. the RD PSO orbit of the GOCE satellite. The degree of the given model truncation was determined taking into account the distribution of the RMSD values depending on the truncation degree of this model. The chosen degree of the truncation corresponds to the minimal value of RMSD. The mentioned distribution was obtained taking into account estimated parameters of the empirical acceleration non-linear model for these geopotential models truncated at 60x60.

Measure of distance between the computed orbit and the reference one

$$\mathrm{RMSD}=\sqrt{(\mathrm{RMS}\,\Delta x)^2+(\mathrm{RMS}\,\Delta y)^2+(\mathrm{RMS}\,\Delta z)^2}, \qquad (1)$$

$$\mathrm{RMS}\Delta x_j=\sqrt{\frac{\sum_{i=1}^{n}\left[(x_j)_{i\,\mathrm{comp}}-(x_j)_{i\,\mathrm{ref}}\right]^2}{n-1}}, \quad (j=1,2,3, x_1=x, x_2=y, x_3=z), \qquad (2)$$

$(x_j)_{i\,\mathrm{comp}}, (x_j)_{i\,\mathrm{ref}}$ *- j-th coordinate of the satellite at the i-th epoch in the computed orbit and in the reference orbit, respectively, n - the total number of epochs taking into account the 1- day orbital arc with the sampling interval of 10 sec.*

General form of the empirical acceleration model used in this work

$$a_i(t_0)=a_i(t_\mathrm{B})|T_1|^{\alpha_{i1}+ms_1(T_1,T_2,q_1,q_2,..,q_7,q_{i8},q_{i9})}+a_i(t_\mathrm{E})|T_2|^{\alpha_{i2}+ms_2(T_1,T_2,r_1,r_2,..,r_7,r_{i8},r_{i9})}+ms_3(p_1,p_2,p_3,..,p_{12},p_{i13},p_{i14}), \qquad (3)$$

where:
for $m=0$, $\alpha_{i1}=1$, $\alpha_{i2}=1$ *– LINEAR MODEL, for* $m=1$, $\alpha_{i1}\neq 1$, $\alpha_{i2}\neq 1$ *– NON – LINEAR MODEL*
$i=1,2,3$ *refers to the radial, along – track, cross – track acceleration, respectively,*
$i=4$ *refers to the acceleration causing the semi – major axis changes,*
$a_i(t_0)$ *– length and direction of the empirical acceleration vector* $\mathbf{a}_i(t_0)$
at the given epoch t_0 *(absolute value of* $a_i(t_0)$ *denotes the length, sign of* $a_i(t_0)$ *determines the direction),*
$a_i(t_\mathrm{B})$ *– length and direction of the empirical acceleration vector* $\mathbf{a}_i(t_\mathrm{B})$
at the beginning of the orbital arc (epoch t_B*),*
$a_i(t_\mathrm{E})$ *– length and direction of the empirical acceleration vector* $\mathbf{a}_i(t_\mathrm{E})$
at the end of the orbital arc (epoch t_E*),*
– in the case of the acceleration causing the semi – major axis changes, the parameters
$a_i(t_0), a_i(t_\mathrm{B}), a_i(t_\mathrm{E})$ *determine the length and direction of the velocity vector*
of semi – major axis changes at the given epochs,

$T_1=(t_\mathrm{E}-t_0)/(t_\mathrm{E}-t_\mathrm{B})$, $T_2=(t_0-t_\mathrm{B})/(t_\mathrm{E}-t_\mathrm{B})$,
$s_1(T_1,T_2,q_1,q_2,...,q_7,q_{i8},q_{i9})=\sum_{k=1}^{20} w_k$, w_k *– "sine / cosine " periodic component,*
$s_2(T_1,T_2,r_1,r_2,...,r_7,r_{i8},r_{i9})=\sum_{n=1}^{20} v_k$, v_k *– "sine / cosine " periodic component,*
$s_3(p_1,p_2,p_3,...,p_{12},p_{i13},p_{i14})=\sum_{j=1}^{16} c_j$, c_j *– "sine / cosine " periodic component,*
$p_1, p_2, p_3,..., p_{12}, q_1, q_2,..., q_7, r_1, r_2,..., r_7$ *– model parameters the same for all accelerations,*
they were estimated once using the OSU91a 60x60 *geopotential model,*
$a_i(t_\mathrm{B}), a_i(t_\mathrm{E}), \alpha_{i1}, \alpha_{i2}, p_{i13}, p_{i14}, q_{i8}, q_{i9}, r_{i8}, r_{i9}$ *– model parameters estimated separately for each acceleration and each geopotential model.*

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