Combined System Integrity Performance Analysis of Multi-constellation Navigation

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Abstract

Integrity is the capability of sending the alarm to the users timely when the navigation system is out of use and it is a significant performance parameter of satellite navigation system. The GPS, Galileo and COMPASS may be applied in combination in future, which would improve and enhance the positioning accuracy and availability. Therefore, users could take full advantage of three constellations of integrity information. As different navigation systems calculate the integrity in different ways, it is important to give an appropriate combined integrity strategy of the three systems. Since GPS employs the protection level concept, while Galileo prefers to compute the integrity risk at the alarm limit, also the COMPASS uses the similar Integrity concept with GPS/SBAS system, this paper examines GPS and Galileo integrity concept separately first. Using both "GPS-Based Integrity" strategy and "Galileo-Based Integrity" strategy, corresponding different integrated integrity equations for combined integrity approaches using data from GPS and Galileo and COMPASS are constructed, and their results are analyzed for comparison. Accordingly the improvement of combined positioning and integrity performance is analyzed theoretically. The Combined integrity analysis with comparison of different strategies of GPS, Galileo and COMPASS would provide support for the future choice of combining integrity for multiple navigation constellations.

1 Introduction

The GPS, Galileo and COMPASS may be used in combination (hereinafter referred to as GPS + Galileo + COMPASS) in future, which would improve and enhance the positioning accuracy and availability. Therefore, it is necessary that users take full advantage of the integrity information of the three constellations. Combined strategy of multiple systems need to address the following key issues:

1) From the viewpoint of user, how to use the integrity information of different systems equivalently.
2) Based on 1), how to build the integrity equation to make full use of the integrity information of combined navigation system.

3. Positioning accuracy and integrity performance of combined constellations

As for navigation positioning of two constellation, assume that the number of visible satellite is n1 for the first constellation 1, the number of visible satellite is n2 for the second constellation. The bias parameter error covariance matrix of user positioning and time delay can be recorded as: G=(HTH)－1, where H means the observation matrix:

$$H=\begin{bmatrix} a_{x1,1} & a_{y1,1} & a_{z1,1} & 1 & 0\\ a_{x2,1} & a_{y2,1} & a_{z2,1} & 1 & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ a_{xn1,1} & a_{yn1,1} & a_{zn1,1} & 1 & 0\\ a_{x1,2} & a_{y1,2} & a_{z1,2} & 0 & 1\\ a_{x2,2} & a_{y2,2} & a_{z2,2} & 0 & 1\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ a_{xn2,2} & a_{yn2,2} & a_{zn2,2} & 0 & 1 \end{bmatrix}$$

$$GDOP=\sqrt{trace(H^TH)^{-1}} =\sqrt{trace\left(diag\left(\frac{1}{\lambda_1},\frac{1}{\lambda_2},\frac{1}{\lambda_3},\frac{1}{\lambda_4},\frac{1}{\lambda_5}\right)\right)}=\sqrt{\sum_{i=1}^{5}\frac{1}{\lambda_i}}$$

$$H^TH=\begin{bmatrix} \sum_{i=1}^{n1}a_{xi,1}^2+\sum_{i=1}^{n2}a_{xi,2}^2 & \sum_{i=1}^{n1}a_{xi,1}a_{yi,1}+\sum_{i=1}^{n2}a_{xi,2}a_{yi,2} & \sum_{i=1}^{n1}a_{xi,1}a_{zi,1}+\sum_{i=1}^{n2}a_{xi,2}a_{zi,2} & \sum_{i=1}^{n1}a_{xi,1} & \sum_{i=1}^{n2}a_{xi,2}\\ \sum_{i=1}^{n1}a_{xi,1}a_{yi,1}+\sum_{i=1}^{n2}a_{xi,2}a_{yi,2} & \sum_{i=1}^{n1}a_{zi,1}^2+\sum_{i=1}^{n2}a_{zi,2}^2 & \sum_{i=1}^{n1}a_{yi,1}a_{zi,1}+\sum_{i=1}^{n2}a_{yi,2}a_{zi,2} & \sum_{i=1}^{n1}a_{yi,1} & \sum_{i=1}^{n2}a_{yi,2}\\ \sum_{i=1}^{n1}a_{xi,1}a_{zi,1}+\sum_{i=1}^{n2}a_{xi,2}a_{zi,2} & \sum_{i=1}^{n1}a_{yi,1}a_{zi,1}+\sum_{i=1}^{n2}a_{yi,2}a_{zi,2} & \sum_{i=1}^{n1}a_{zi,1}^2+\sum_{i=1}^{n2}a_{zi,2}^2 & \sum_{i=1}^{n1}a_{zi,1} & \sum_{i=1}^{n2}a_{zi,2}\\ \sum_{i=1}^{n1}a_{xi,1} & \sum_{i=1}^{n1}a_{yi,1} & \sum_{i=1}^{n1}a_{zi,1} & n1 & 0\\ \sum_{i=1}^{n2}a_{xi,2} & \sum_{i=1}^{n2}a_{yi,2} & \sum_{i=1}^{n2}a_{zi,2} & 0 & n2 \end{bmatrix}$$

In theory, for single constellation: $GDOP_1 \geq \sqrt{\frac{10}{n_1}}$

as for two constellations: $GDOP_{1,2} \geq \sqrt{\frac{9}{(n_1+n_2)}+\frac{1}{n_1}+\frac{1}{n_2}}$

As for integrity performance, we consider the trace:

$$M_{TOPO}\cdot P^{-1}\cdot M_{TOPO}^T = N_{TOPO}^T\cdot(H^TPH)^{-1}\cdot N_{TOPO}$$
$$tr(N_{TOPO}^T(H^TPH)^{-1}N_{TOPO})=tr(N_{TOPO}^TN_{TOPO}(H^TPH)^{-1})=tr((H^TPH)^{-1})$$

By standardizing the weight matrix, we consider

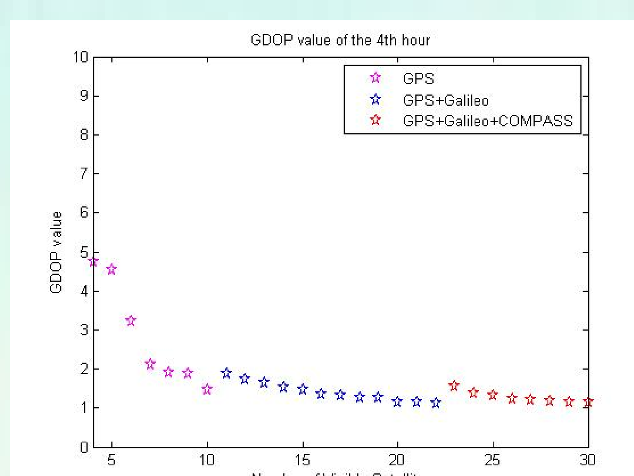
$$H^TH=\begin{bmatrix} H_1{}^TH_1+B_{21}{}^TB_{21} & H_1{}^TB_{12}+B_{21}{}^TB_{22}\\ B_{12}{}^TH_1+B_{22}{}^TB_{21} & B_{12}{}^TB_{12}+B_{22}{}^TB_{22} \end{bmatrix}\hat{=}\begin{bmatrix} V_{11} & V_{12}\\ V_{21} & V_{22}\end{bmatrix} \quad \tilde{V}_{22}=V_{22}-V_{21}V_{11}{}^{-1}V_{12}$$

The difference between $(H_1{}^TP_1H_1)^{-1}$ and $(H^TPH)^{-1}$?

If it holds that

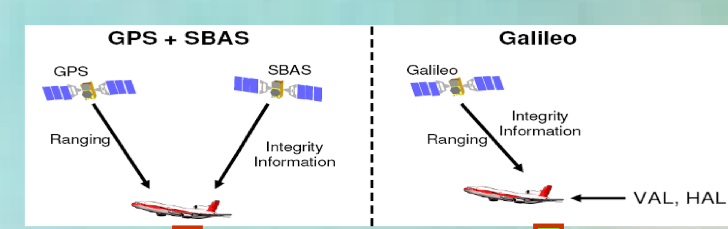
$$\begin{aligned} tr(H^TH)^{-1} &= tr(V_{11}{}^{-1}+V_{11}{}^{-1}V_{12}\tilde{V}_{22}{}^{-1}V_{21}V_{11}{}^{-1})\\ &= tr[(H_1{}^TH_1+B_{21}{}^TB_{21})^{-1}+V_{11}{}^{-1}V_{12}\tilde{V}_{22}{}^{-1}V_{21}V_{11}{}^{-1}]\\ &< tr[(H_1{}^TH_1)^{-1}] \end{aligned}$$

the positioning accuracy and integrity would be improved as adding one constellation.



During one day in an area the number of visible satellites are almost 10、21 and 31 for single GPS system, GPS +Galileo system and GPS +Galileo +COMPASS(14 satellites: 5GEO+5IGSO+4MEO) combined system. It can be seen that, the GDOP reduced as adding the visible Satellite.

2. Integrity Concept of GPS+SBAS/Galileo



HPL/VPL **Integrity Risk**

GPS+SBAS: HPL/VPL

Several input parameters for GPS+SBAS integrity algorithms:
Geometry between user and visible GPS satellites;
Variance of residual clock and ephemeris errors in corrected range to the satellite;
Variance of residual ionospheric errors in corrected range to the satellite;
Variance of residual tropospheric error in corrected range to the satellite;
Variance of the local receiver noise and multipath.

$$(G^TWG)^{-1}=\begin{bmatrix} d_{east}^2 & d_{EN} & d_{EU} & d_{ET}\\ d_{EN} & d_{north}^2 & d_{NU} & d_{NT}\\ d_{EU} & d_{NU} & d_U^2 & d_{UT}\\ d_{ET} & d_{NT} & d_{UT} & d_T^2 \end{bmatrix}$$

$$\sigma_i^2=\sigma_{i,flt}^2+\sigma_{i,UIRE}^2+\sigma_{i,air}^2+\sigma_{i,tropo}^2$$

$$d_{major}=\sqrt{\frac{d_{east}^2+d_{north}^2}{2}+\sqrt{\left(\frac{d_{east}^2-d_{north}^2}{2}\right)+d_{EN}^2}}$$

$$HPL=\begin{cases} K_{H,NPA}\cdot d_{major} & \text{en route inclusive non-precision approach mode}\\ K_{H,PA}\cdot d_{major} & \text{precision approach mode}\end{cases}$$

$$VPL=K_{V,PA}\cdot d_U \quad \text{precision approach mode}$$

The assessment of the integrity is achieved by comparing PL(protection limit) with AL(alert limit): if PL> AL, the integrity of the alarm mechanism is triggered.

Galileo: User Integrity Equation

$$\sigma_{u.V.FF}^2=\sum_{i=1}^{n}M_{TOPO}^2[3][i]\cdot(SISA_i^i+\sigma_{u.L.i}^2)$$

$$M_{TOPO}=\tilde{N}_{TOPO}^T(\tilde{H}^T P\tilde{H}^T)^{-1}\cdot(\tilde{H}^TP)$$

$$P_{HMI}(HAL,VAL)=P_{HMI}(HAL)+P_{HMI}(VAL)=1-erf\left(\frac{VAL}{\sqrt{2}\cdot\sigma_{u.V.FF}}\right)+e^{\left(-\frac{HAL^2}{2\cdot\xi_{FF}^2}\right)}$$
$$+\frac{1}{2}\sum_{j=1}^{N}P_{fail,sat_j}\left(\left(1-erf\left(\frac{VAL+\mu_{u.v.j}}{\sqrt{2}\cdot\sigma_{u.V.j.FM}}\right)\right)+\left(1-erf\left(\frac{VAL-\mu_{u.v.j}}{\sqrt{2}\cdot\sigma_{u.V.j.FM}}\right)\right)\right)$$
$$+\sum_{j=1}^{N}P_{fail,sat_j}\cdot\left(1-\chi^2_{2,\delta_{u.H.j}}cdf\left(\frac{HAL^2}{\xi_{FM.j}^2}\right)\right)$$

SISE: the maximum error of the SIS in the range domain caused by the SV, the SV payload, and the navigation message.

SISA: a prediction of the minimum standard deviation of a Gaussian distribution that overbounds the SISE distribution for fault-free SIS.

SISMA: The difference between SISE and estimated SISE (SISEest) has a distribution. This distribution shall be overbounded by a Gaussian distribution with a standard deviation called SISMA.

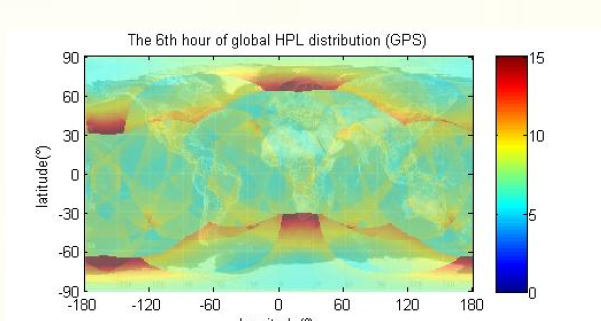
4. Combined integrity analysis of GPS, Galileo and COMPASS

Two ways of comparison:

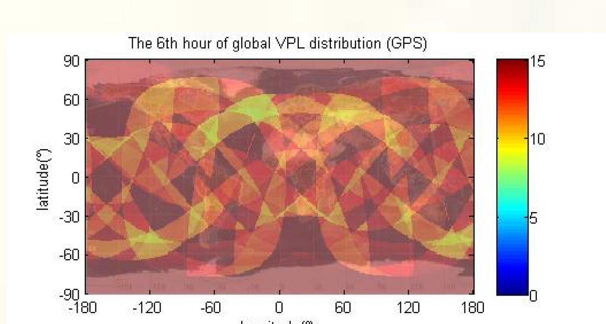
(1) GPS baseline integrity concept.
- GPS
- GPS system in cooperation with Galileo system
- Combined GPS+ Galileo+COMPASS system

(2) Galileo baseline integrity concept
- Galileo
- Galileo in collaboration with GPS system
- Combined Galileo+GPS+COMPASS system

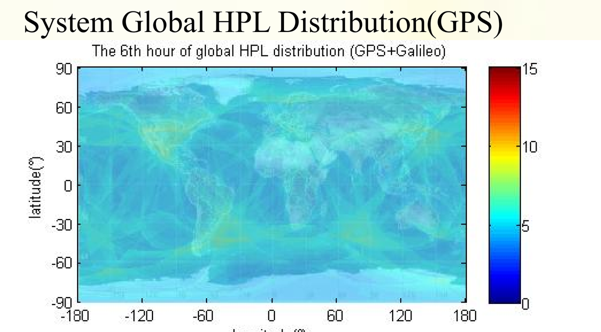
"GPS baseline Integrity" combined strategy



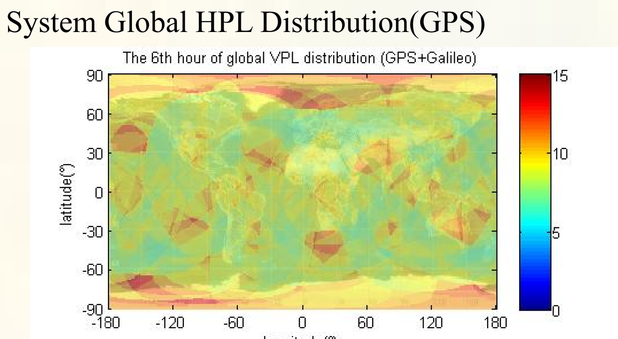
System Global HPL Distribution(GPS)



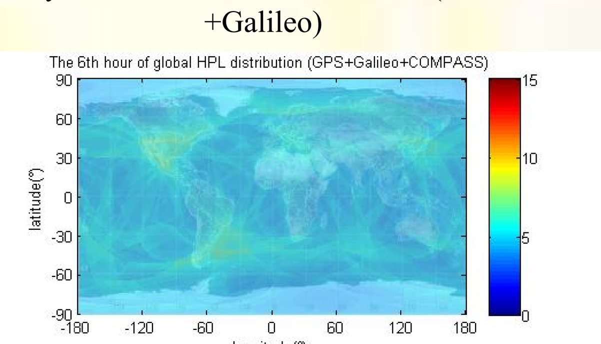
System Global HPL Distribution(GPS)



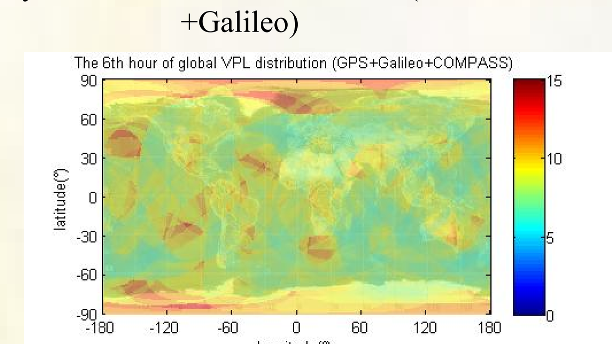
System Global HPL Distribution(GPS +Galileo)



System Global HPL Distribution(GPS +Galileo)



System Global HPL Distribution(GPS +Galileo +COMPASS)



System Global HPL Distribution(GPS +Galileo +COMPASS)

"Galileo baseline Integrity" combined strategy

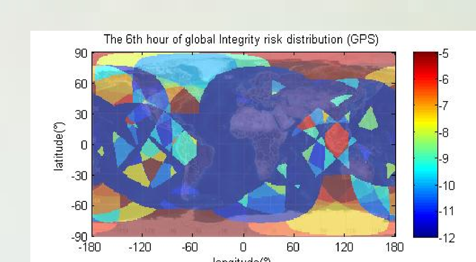
$$\tilde{P}_{HMI}(VAL,HAL)=\tilde{P}_{HMI}(VAL)+\tilde{P}_{HMI}(HAL)=1-erf\left(\frac{VAL}{\sqrt{2}\sigma_{u,V,FF}}\right)+e^{-\frac{HAL^2}{2\xi_{FF}^2}}+$$
$$+\frac{1}{2}\sum_{j=1}^{N}P_{fail,sat_j}\left(\left(1-erf\left(\frac{VAL+\mu_{u,V}}{\sqrt{2}\sigma_{u,V,FM}}\right)\right)+\left(1-erf\left(\frac{VAL-\mu_{u,V}}{\sqrt{2}\sigma_{u,V,FM}}\right)\right)\right)+$$
$$+\sum_{j=1}^{N}P_{fail,sat}\left(1-\chi^2_{2,\delta_u,H}cdf\left(\frac{HAL^2}{\xi_{FM}^2}\right)\right)$$

The fault-free mode integrity risk of all the visible satellites for GPS+ Galileo+COMPASS

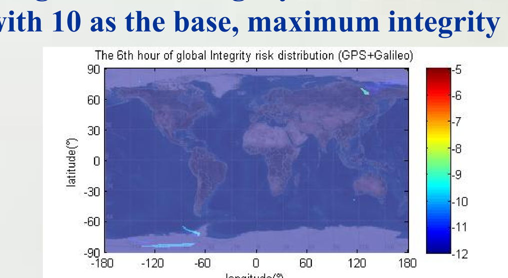
The latter two terms represent the fault mode of the integrity of "available" GPS, Galileo and COMPASS satellites, where *N* is the number of all satellites marked as "available"

$$P_{fail,sat_j}=10^{-6}$$

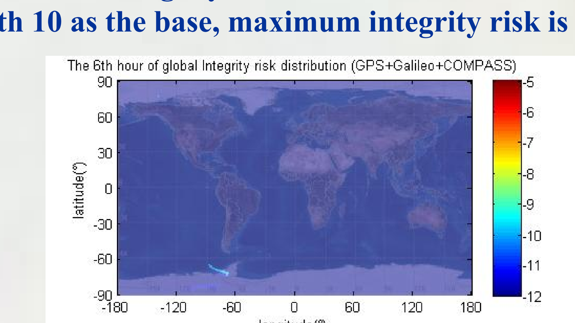
GPS : UERE = 1.4m; and the false alert probability is 10^{-3}.
Galileo: SISA = 0.85m, SISMA = 1.2m, UERE = 1.8m;
COMPASS: Uses GPS-like integrity mechanism,
UERE = 1.8m; (for IGSO,MEO)
UERE = 3.6m; (for GEO)



Logarithm of integrity risk distribution（GPS）(with 10 as the base, maximum integrity risk is ~10^{-3})



Logarithm of integrity risk distribution（GPS+Galileo）(with 10 as the base, maximum integrity risk is ~10^{-6})



Logarithm of integrity risk distribution (GPS +Galileo +COMPASS) (with 10 as the base, maximum integrity risk is ~10^{-7})

Results:

(1) The integrity performance of collaborated system is better than that of no collaboration, no matter based on GPS baseline integrity concept or Galileo baseline integrity concept.

(2) Using the integrity concept of GPS+SBAS, HPL and VPL's global distribution of GPS +Galileo combined system reduced significantly on the basis of GPS system. However, the improvement of HPL and VPL distribution not very significant adding COMPASS, in China area it has a more obvious improvement due to its regional constellation deployment.

(3)Using the integrity concept of Galileo integrity risk, the similar conclusion could be deduced that the integrity performance of combined constellation is better than a single constellation.

Conclusions

We try to find an appropriate way to utilize all integrity information from different satellite navigation systems, such as GPS, Galileo, COMPASS, simultaneously. Although the protection levels(GPS baseline integrity concept) and integrity risks at the alert limit(Galileo baseline integrity concept) are mathematically an inversion of the same concept, they could not use other information directly.

From two aspects of integrity: HPL/VPL and integrity risk, this paper compares different combined integrity strategies. Based on GPS+SBAS integrity concept and Galileo integrity concept, an integrated equation is constructed to realize the integrity risk assessment for combined constellation of GPS+Galileo+COMPASS system. Accordingly the improvement of combined positioning and integrity performance is analyzed theoretically.

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