Zero Difference Residuals for Multipath Maps and ZTD Quality Indication

Introduction

For the AGRSCLUS network (see Fig. 1) the site–multipath and the quality of the estimated Zenith Total Delays (ZTD) are evaluated using the least–squares residuals. The period, day 121–144 2003, corresponded to the BBC-II campaign. As the output of the Bernese Software are double difference (dd) residuals, first zero difference (zd) residuals have to be computed, see [1], which are necessary for the multipath mapping, variance component estimation and for slant delay retrieval [2], [3].

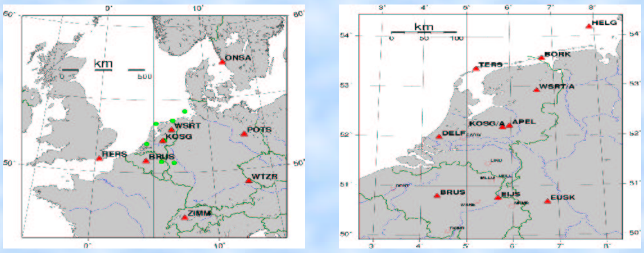

Fig. 1

Zero Difference Residuals

Residuals are defined as the differences between observed and computed ranges using the estimated parameters. The single difference (sd) residuals Δr^i_{AB} between station A and B for satellite i, and the double difference (dd) residuals between the satellites i and j, are related to the zero difference (zd) residuals r^i_A, r^i_B in the line of sight:

$$\Delta r^i_{AB} = r^i_A - r^i_B \quad \text{and} \quad \Delta\Delta r^{ij}_{AB} = \Delta r^i_{AB} - \Delta r^j_{AB} \,.$$

An important property of the zd residuals is that the weighted sum of all zd residuals over all stations to a single satellite, and the weighted sum of sd residuals to all visible satellites for a baseline, are zero. These are the so-called zero mean conditions

$$\sum_{A,B,C,\ldots} w_A \, r^i_A = 0 \quad \text{and} \quad \sum_{i=1,2,3\ldots} w_i \, \Delta r^i_{AB} = 0 \,.$$

This is a consequence of the fact that we have to solve for satellite and receiver clock parameters. This property can be used to derive the inverse transformation, whereby the elevation dependent accuracy of GPS observations should be taken into account by the choice of the weights $w_A, w_B, \ldots$ or $w_1, w_2, \ldots$ in the zero mean conditions.

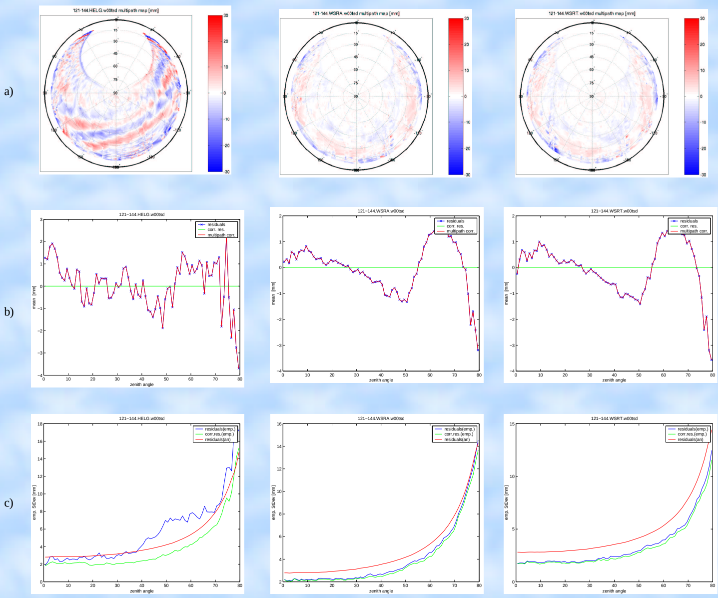

Fig. 2

Multipath Maps

The zd residuals contain noise and unmodeled effects related to atmospheric delay, antenna phase center variations, mapping function errors, multipath, etc.. The contribution of antenna phase center variations and multipath in the zd residuals depends mainly on azimuth and elevation and therefore repeats daily, wheareas the unmodeled atmospheric delay is independent of the receiver-satellite geometry. In order to eliminate the effects of antenna phase center variations and multipath for each station the zd residuals of several days are averaged with respect to azimuth and elevation. The resulting station-dependent "multipath maps" contain only the impact of antenna phase center variations and mutipath, because noise and unmodeled atmospheric delays are assumed to average out. It should be noted that this technique cannot be used to estimate the antenna phase center variations related to $\alpha * sin(e) + \beta / sin(e)$.

Examples of multipath maps are shown in Fig. 2a for stations Helgoland and Westerbork with two receivers. In Fig. 2b only the elevation dependent effects are shown, and in Fig. 2c the rms-error of the residuals before and after correction for the multipath maps of Fig. 2a is shown, together with the formal standard deviation of the residuals. These plots show for Westerbork with the same antenna, but two different receivers, that the noise differs , especially for small elevation angles.

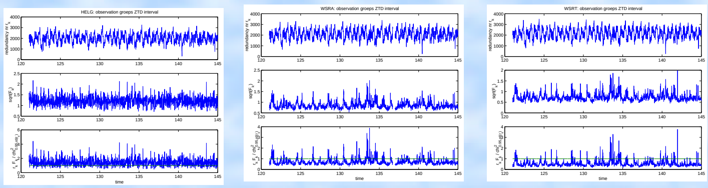

Fig. 3

ZTD quality indicator

The formal standard deviation of the estimated zenith total delay (ZTD) as given by the software is often too optimistic. This can result in problems when ZTD is assimilated into Numerical Weather Prediction models. The actual quality of the observations, systematic errors like multipath, antenna phase center variations, errors in the satellite orbits, do not influence the formal standard deviation of the ZTD. Variance component estimation, using the estimated zd residuals, could solve some of the problems. In the case of uncorrelated observations the Almost Unbiased Variance Component Estimation for a group of observations reads

$$F_k = \frac{\widehat{\sigma^2_k}}{\sigma^2_0} = \frac{1}{r_k} \sum_{i \in S_k} \hat{e}_i / \sigma^2_{y_i} \quad \text{with} \quad r_k = \sum_{i \in S_k} \sigma^2_{\hat{e}_i} / \sigma^2_{y_i} \,, \quad \hat{e}_i \text{ zd residual} \,.$$

The ratio $F_k = \frac{\widehat{\sigma^2_k}}{\sigma^2_0}$ is expected to be one, if σ_0 approximates σ_k and systematic errors are to be neglected. The redundancy numbers r_k describe the contribution of the observation group $\boldsymbol{y}_k$ to the complete redundancy. Results for the ZTD estimation are given in Fig. 3 for station Helgoland and twice for Westerbork, showing the redundancy number r_k, $\sqrt{F_k}$ and results of hypothesis tests with a 95% confidence level under the assumption of the Fisher distribution $F_k \sim F(round(r_k), \infty)$. For station Helgoland with strong multipath effects the estimated F_k are larger than one and most of the hypothesis tests are rejected. Station Westerbork shows less multipath and smaller standard deviations. The estimated F_k are smaller and hypothesis tests are more often accepted. Nevertheless, a number of peaks mark high F_k or clearly rejected hypothesis and give hints for a bad quality of the corresponding ZTD.

Outlook

The further aim is to retrieve slant delays from the multipath corrected residuals and mapped zenith delays. In this approach, an important issue is to find the optimal sampling rates for slant delays and for ZTD estimation.

References

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Acknowledgements

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