IGS Rapid Orbits: Systematic Error at Day Boundaries

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Abstract

When one fits a GPS spacecraft trajectory through several days of orbit positions from IGS Rapid orbit SP3 files, the orbit position residuals show discontinuities at the day boundaries between SP3 files. The discontinuities can be of order 10 cm, especially the component in the along-track direction. The discontinuity component values for a specific spacecraft usually have the same sign for several months, that is, the variation from day to day is not random. SVN 44 (PRN 28) during the year 2005 showed an along-track discontinuity that slowly varied over the range +2 to +13 cm. IGS Final orbits show similar discontinuities at each 00 hr GPS. The biased residual discontinuities reflect a discontinuity in Rapid orbit systematic position error across day boundaries; this error is much larger than the 2 cm RMS difference between orbits from different Analysis Centers. We show that some reasonable orbit modelling errors could produce these discontinuities.

To indicate the magnitude of systematic orbit error, the IGS should include day boundary discontinuity values in the Rapid orbit combination reports.

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INTRODUCTION

One way to estimate a lower bound on the orbit determination accuracy of a system is to take determined orbits for successive time intervals and look at the difference between orbits where they overlap. If there is a difference D between computed positions at the overlap, at least half of this difference (D/2) must be due to errors in one of the orbits.

This paper looks at the difference between the determined orbit positions in successive IGS Rapid orbit SP3 files. Each file gives Earth-referenced GPS orbit positions at 15 minute intervals from 00^h00^m GPS through 23^h45^m GPS for one day. Since there is no overlap in the reported satellite positions at the 00^h00^m GPS day boundary between files, it is necessary to extrapolate. In principle, one can

1) use a suitable set of Earth Rotation Parameters (erp's) to transform the satellite positions from both files to inertial axes,
2) fit an orbit to one file's positions,
3) extrapolate this orbit across the day boundary to the nearest time of a position in the other file, and
4) use this extrapolated position to evaluate the difference between file orbits at the day (file) boundary.

This study uses a slight modification to this procedure. We take IGS Rapid orbit data spanning 4 to 6 days, transform this data to inertial axes, and fit a single spacecraft trajectory to the entire data span. We use the "Trajedy" program in JPL's GIPSY-OASIS II software package. The program adjusts 6 orbital elements (the starting position and velocity vector) plus 6 corrections to the solar radiation force model to obtain a least-squares trajectory fit to the orbit positions. The software has been modified to use my modifications to the CODE radiation force model. No ad hoc (stochastic) velocity impulses are applied to the trajectory in the fit.

The computer program computes the position residuals for the fit with the sense

(Observed position) minus (Computed position vector)
vector from the file) (from the trajectory)

and finds the residual components along three orthogonal local directions:

- the radial direction (away from the geocenter),
- the transverse direction (normal to radial direction, positive near the direction of inertial motion ["along-track"]), and
- the orbit normal direction ($\vec{r} \times \vec{v}$ direction, "cross-track").

Figure 1 illustrates the transverse residuals from a 2 day fit. In our residual plots, the horizontal time axis lists the time in hours since the first point; numerical values are given at 24 hour intervals to indicate the day boundary positions. The time label gives the Gregorian calendar date (GPS) for the first point.

Successive residual points (at 15 minute intervals) in Fig. 1 show a gradual variation with time due to modelling errors and possible systematic errors in the Rapid orbit positions. At the day boundary, where the residual change between successive data points is marked by a vertical line segment, there is a 90 cm step change in the residual level. This step change (the "day boundary discontinuity") represents the difference in orbit position systematic error between two adjacent-time files.

DISCUSSION OF DAY-BOUNDARY DISCONTINUITIES

Figure 1 shows a particularly bad case. It is probably atypical since it is for a GPS satellite one week after it was launched. One could object that it shows exceptional errors
a) because its surfaces are still outgassing in the vacuum of space, with a resulting unmodelled recoil acceleration of the spacecraft, and
b) because its signal usability has been marked "unhealthy" so few GPS stations would be reporting data for this spacecraft. The result is a less accurate determined orbit.

Figures 2, 3, and 4 show that relatively large (12 cm level) discontinuities also occur for fully operational GPS satellites. These figures display data from different satellites and times to show that several large discontinuities can occur on successive days for the along-track, cross-track, and radial directions, respectively. These cases were selected from recent data, IGS Rapid orbits produced during 2005.

The along-track residuals in Fig. 2 show a peculiar behavior near the day boundary in that the residuals show a marked departure from zero on only one side of each day boundary (large positive values on the right side of each boundary in Fig. 2). In addition, the extreme residual value occurs very near the day boundary and the residual values on that side tend monotonicaly toward zero over a 4 hour interval as one moves away from the boundary. These "tails" are fairly common when large, along-track discontinuities occur.

In contrast, the cross-track residuals in Fig. 3 show day-boundary discontinuities that are more centered about zero. The discontinuities lie within the range of residual oscillation away from the day boundaries.

Finally, we note that the radial-direction residuals in Fig. 4 show a runoff to large negative values during the 4 hours just before the day boundary.

The question arises of whether large discontinuities are something inherent only in the Rapid orbit determination process, a result of processing with a short deadline. Fig. 5 shows a fit to IGS Final orbit data for 6 days near the end of November 2004, a time when IGS orbits show particularly large discontinuities. The figure shows that day-boundary discontinuities occur for the IGS Final orbit SP3 files also; the transverse discontinuities of order +10 cm found here for SVN 44 Final orbits are comparable to the Fig. 2 transverse discontinuities from Rapid orbits for the same spacecraft. This result indicates that the discontinuities are inherent in the present GPS orbit determination process.

The question also arises of whether these large discontinuities are just values in the tails of a normal distribution of errors - that our figures only show 3-sigma cases. That question can be answered by looking at discontinuity values as a function of time through the year 2005. Figs. 6, 7, and 8 show the daily discontinuity values for the along-track, cross-track, and radial directions, respectively, for the same spacecraft whose orbits show large discontinuities in Figs. 2, 3, and 4.

The discontinuity values on successive days do not vary randomly about zero. Instead, over any ~1 week interval they show a ±2 cm oscillation about a bias value. The ±2 cm oscillation arises from random error in the determined orbits. The bias value over the week arises from a difference in systematic error across the day boundary, a difference that persists month after month with only slow changes, oftentimes showing an irregular oscillation with a period of order 3 or 4 months. There is no apparent correlation with eclipse seasons; eclipse season midpoints occur at ~6 month intervals when the satellite orbit passes through Earth's shadow. These considerations show that large discontinuities do not reflect the tails of a normal distribution of random errors; they reflect large systematic errors in the determined orbits.

Fig. 6 shows the along-track discontinuity values for SVN 44 (PRN28), the worst case GPS satellite. Its bias slowly varies between +2 and +13 cm; the discontinuity value is always positive for this satellite and shows a mean value during 2005 of +9 cm. Since at least half of the discontinuity bias must be due to systematic error on one side of the day boundary, the 13 cm discontinuity region indicates that systematic errors >6 cm occur in the GPS determined orbits.

And finally, the question arises of whether these discontinuities are an artifact of the JPL-GIPSY software used at USNO, or perhaps occur because we do not properly set up the GIPSY computer runs. These objections are answered by Fig. 9 which shows the residuals for a run made at the University of Bern using the Bernese software. This figure shows that the Bernese software also finds sizeable day boundary discontinuities. The along-track discontinuities for SVN 44 here have the opposite sign from what is shown in Figs. 2, 5, and 6 because the Bernese software defines the residual to have the opposite sign to what is used in our other graphs.

Note that two sources of erp data were used in Fig. 9; the bottom run used Bulletin A values (that is, erp values that are continuous across day boundaries) while the top run used values for each day from the IGS .erp files that go with each SP3 file. Either choice results in similar day-boundary discontinuities.

POSSIBLE SOURCES OF SYSTEMATIC ERROR

We now discuss some possible causes of the systematic along-track discontinuities.

Radial Bias

Consider a GPS spacecraft whose mass center S describes a circular orbit of radius a about the geocenter. Consider the effect of an error in the radial position of the spacecraft antenna's phase center relative to S. Computer processing of the GPS signals then locates the spacecraft in a circular orbit with a slightly different geocentric radius

$$a_0 = a - \Delta a \qquad (1)$$

where Δa = resulting error in orbit radius.

Figure 10 illustrates the geometry. During day 1 (a time interval T=1 day), the satellite moves along its true orbit from A to E through a geocentric angle nT, where n is the angular velocity along its circular orbit from Kepler's Third Law [Eq.(A-3) in Appendix A with α_r=0].

Because the computed satellite position puts it in a slightly higher orbit (as assumed for

Fig. 10), the satellite moves along its computed orbit from B to C through a slightly smaller geocentric angle n_0T, where n_0 is the angular velocity from Eq.(A-4) for an orbit of radius a_0.

In order to minimize the least-squares error in the analysis of orbit data extending over day 1, computer processing will tend to center the computed orbit arc BC over the true orbit arc AE as shown in the figure. Point B will thus be located at a geocentric angle

$$\begin{aligned}\gamma &= (nT - n_0 T)/2 \\ &= (\Delta n)\,T/2 \end{aligned} \tag{2}$$

ahead of point A (with Δn from Eq.(A-6)) while point C will be located at a geocentric angle γ behind point E.

During the following day (day 2), the satellite moves along its true orbit from E to H through a similar geocentric angle nT. Least-squares processing of orbit data from day 2 gives a computed orbit arc, FG in the figure, which has the same relation to the true orbit arc as occurred for day 1. The day 2 computed arc is again centered over the true orbit arc EH, with point F located at a geocentric angle γ ahead of point E. There is a discontinuity CF between the day 2 00^h points C and F of the two computed orbit arcs. CF spans a geocentric arc 2γ, so the transverse linear discontinuity displacement is

$$\begin{aligned}D_s &= 2\gamma a_0 \\ &= a_0 T \Delta n \quad . \end{aligned} \tag{3}$$

Since the nominal orbit period is 0.5 sidereal day $\simeq$ T/2 here, $n_0 \simeq 4\pi/T$ by Eq.(A-1). Eq.(A-7) with $\alpha_r=0$ allows us to eliminate Δn from the last equation and obtain

$$\begin{aligned}D_s &= -\frac{3 n_0 T}{2} \Delta a \\ &= -6\pi \Delta a \quad . \end{aligned} \tag{4}$$

For SVN 44 discussed previously, $D_s \sim 9$ cm during 2005 so the last equation gives $\Delta a=-0.5$ cm for the simple error model just discussed; by Eq.(1) the computed orbit is 0.5 cm above the true orbit. This example shows that even a small error in measured radial distance (as from a phase center location error) can give rise to a significant day boundary discontinuity.

Unmodelled Radial Force

Next consider the case where the computed circular orbit places the GPS satellite at the correct geocentric distance a, so $a_0=a$ and $\Delta a=0$, but the computation omits a small, constant radial acceleration α_r resulting from an unmodelled radial force (or from the radial error in an incorrectly modelled force). We apply the Appendix A theory to this case. The geometry of Fig. 10 applies as the special case $\Delta a=0$, so Eq.(3) holds here. The computed orbit in this case has the nominal angular velocity n_0 computed using Eq.(A-4), but satellite motion along the real orbit has a different angular velocity ($n_0 + \Delta n$) with Δn given by Eq.(A-7) with $\Delta a=0$. Using this Δn value in Eq.(3) then gives

$$D_s = \frac{-\alpha_r T}{2\, n_0} \tag{5}$$

which when evaluated with T=86,400 s and n_0=1.458×10^{-4} rad/s for GPS orbits gives

$$\alpha_r = -3.38\ nm/s^2\ (D_s/m) \tag{6}$$

for D_s in meters, where nm=10^{-9}m.

Applying this error model to SVN 44 with D_s=0.09 m gives α_r=-0.30 nm/s, that is, an extra geocentric radial acceleration with this magnitude could produce the observed along-track discontinuity. Solar radiation force models could have a radial acceleration error of the required magnitude and thus be responsible for some of the discontinuities.

Gravitational Field Errors

An error in the Earth gravitational field model could provide the unmodelled radial acceleration suggested in the previous section. We will only consider a possible error in the main, inverse-square term, as used in Appendix A. The computed satellite motion could use a value $(Gm_E)_{comp}$ while the real satellite motion experiences an acceleration based on a slightly different value $(Gm_E)_{true}$, so the extra radial acceleration is

$$\alpha_r = -\frac{\Delta(Gm_E)}{a^2} \tag{7}$$

where

$$\Delta(Gm_E) = (Gm_E)_{true} - (Gm_E)_{comp} \quad . \tag{8}$$

Using the example α_r value from the last section in Eq.(7) with the GPS orbit value a=2.656×10^7 m gives

$$\begin{aligned} \Delta(Gm_E) &= -\alpha_r a^2 \\ &= (0.30 \times 10^{-9}\ m/s^2)(2.656 \times 10^7\ m)^2 \\ &= 2.1 \times 10^5\ m^3/s^2 \quad . \end{aligned} \tag{9}$$

This hypothetical correction is much less than the formal error ±8×10^5 m^3/s^2 in the IERS value for (Gm_E) [IERS Conventions (2003), p.12, Table 1.1]. This correction thus indicates that possible errors in the gravitational field model may give noticeable contributions to day-boundary discontinuities. But this exercise does not indicate any particular correction to (Gm_E); different GPS satellites show along-track discontinuities with different signs which would require corrections to (Gm_E) with both algebraic signs, a contradictory result.

SUMMARY AND RECOMMENDATION

We have shown that sizeable discontinuities occur at the day boundaries between successive IGS Rapid (and Final) SP3 orbit position files. These discontinuities are inherent in the file data and are not an artifact of the processing software. The discontinuity values on successive day boundaries do not result only from random error in the determined orbits; since they cluster near a non-zero bias value, they indicate systematic errors in the determined orbits. For SVN 44 (PRN28), the systematic error in the along-track direction can be >6 cm.

In order to indicate the magnitude of systematic error in the IGS determined orbits, the IGS should include a day boundary discontinuity value for each satellite in the Rapid IGS Orbit Combination reports.

Appendix A - Circular Orbital Motion with an Additional Constant Radial Force

Consider motion along a circular orbit with the satellite subject to
(1) the inverse square gravitational force, $-Gm_E m_s/a^2$, and
(2) an additional constant radial force expressed as $m_s \alpha_r$
where

α_r = the additional constant, radial acceleration that results from this force.

Let

a = geocentric radius of orbit;
m_s = satellite mass;
n = angular velocity about geocenter,
= (2π rad)/(orbit period); and (A-1)
Gm_E = Earth gravitational parameter,
= 398,600.44 km^3/s^2.

Taking forces and accelerations as positive in the radial direction away from the geocenter, we note that the radial (centripedal) acceleration is $-na^2$. Applying Newton's Second Law to the satellite gives

$$-m_s(n^2 a) = -\frac{Gm_E m_s}{a^2} + m_s \alpha_r \tag{A-2}$$

which may be rearranged to give a modified Kepler's Third Law,

$$n^2 a^3 = Gm_E - a^2 \alpha_r \quad . \tag{A-3}$$

Let a_0 and n_0 be the radius and angular velocity for a nominal orbit with no additional acceleration (α_r=0), so

$$n_0^2 a_0^3 = Gm_E \quad , \tag{A-4}$$

and consider the orbit with small departures Δa and Δn from the nominal values when α_r is non-zero.

$$a = a_0 + \Delta a \quad . \tag{A-5}$$

$$n = n_0 + \Delta n \quad . \tag{A-6}$$

Substituting these values in Eq.(A-3) and expanding, keeping terms to the first order of smallness, and using Eq.(A-4) gives finally

$$\alpha_r \simeq -2 n_0 a_0 \Delta n - 3 n_0^2 \Delta a \quad . \tag{A-7}$$

Figure 1. SVN 53 (PRN 17) ALONG-TRACK DISCONTINUITY

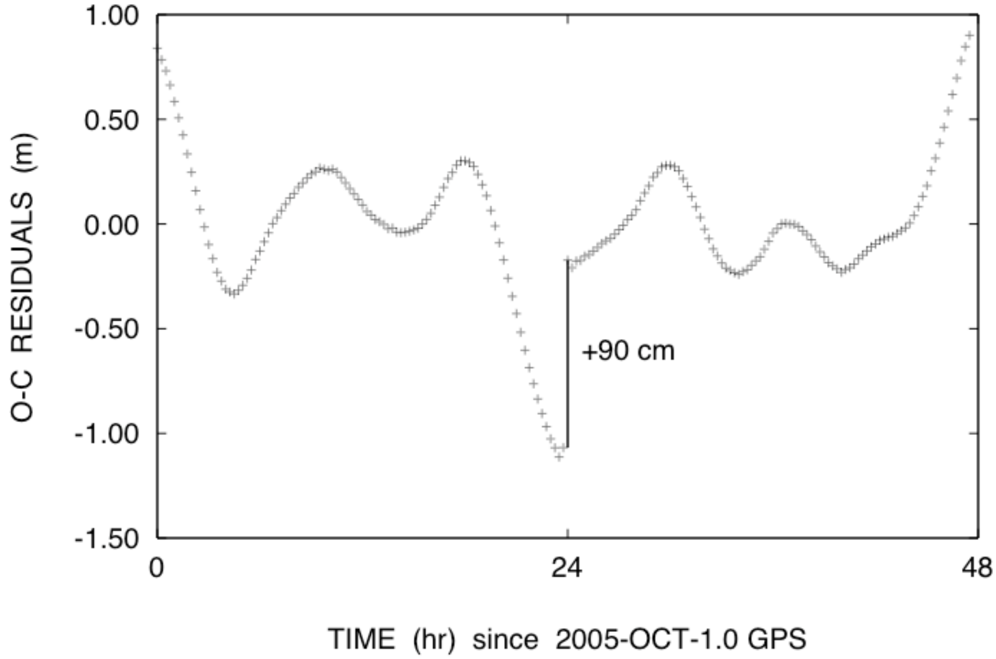

Figure 2. SVN 44 (PRN 28) ALONG-TRACK DISCONTINUITY

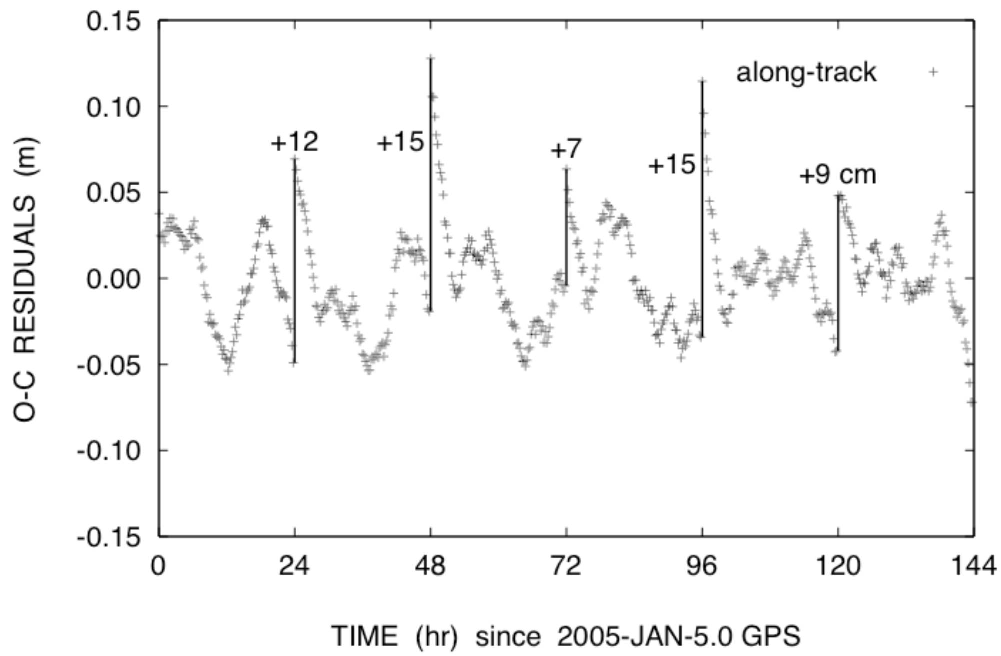

Figure 3. SVN 44 (PRN 28) CROSS-TRACK DISCONTINUITY

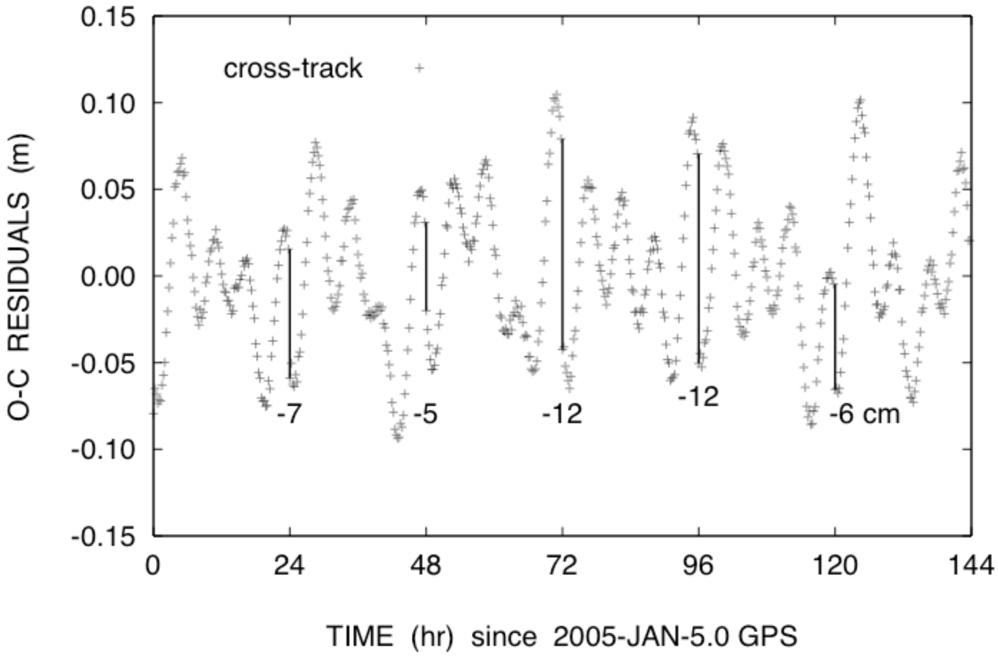

Figure 4. SVN 24 (PRN 24) RADIAL DISCONTINUITY

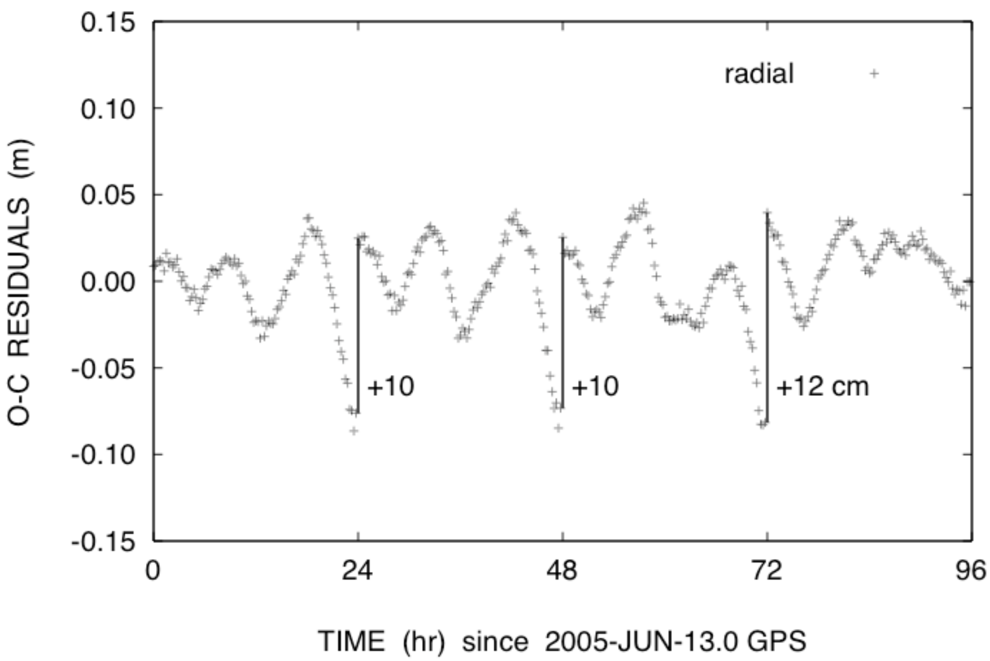

Figure 5. SVN 44 (PRN 28) ALONG-TRACK DISCONTINUITY

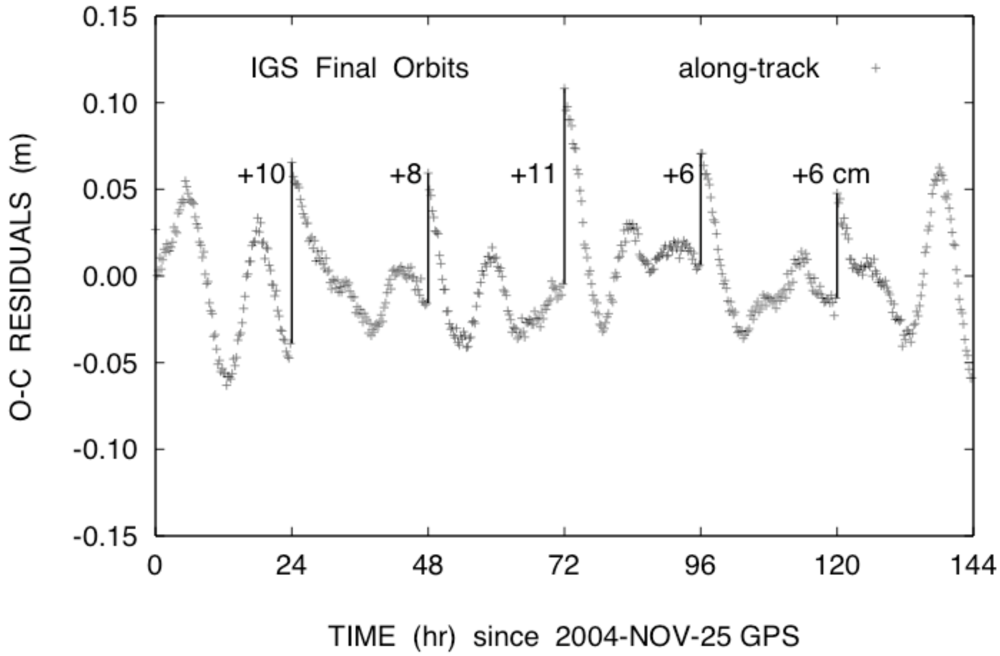

Figure 6. SVN 44 (PRN 28) ALONG-TRACK DISCONTINUITY during 2005

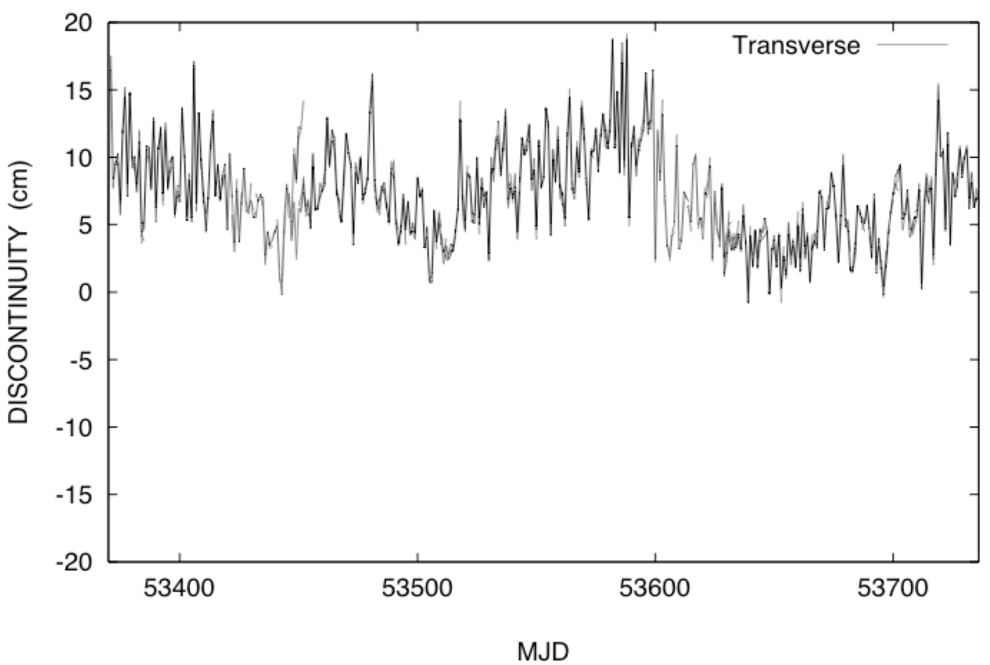

Figure 7.   SVN 44 (PRN 28) CROSS-TRACK DISCONTINUITY during 2005

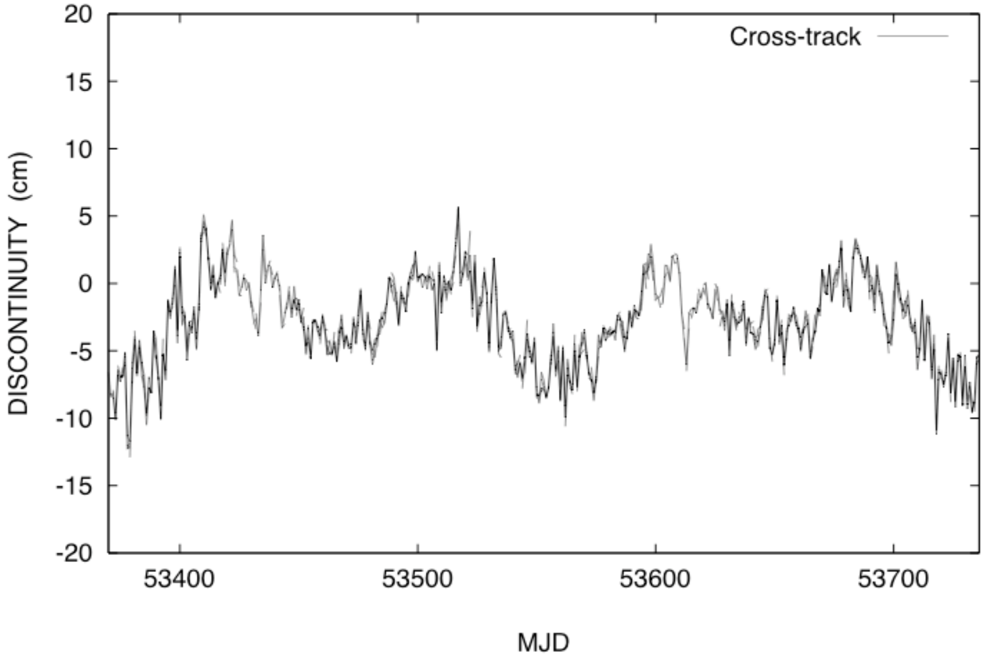

Figure 8.   SVN 24 (PRN 24) RADIAL DISCONTINUITY during 2005

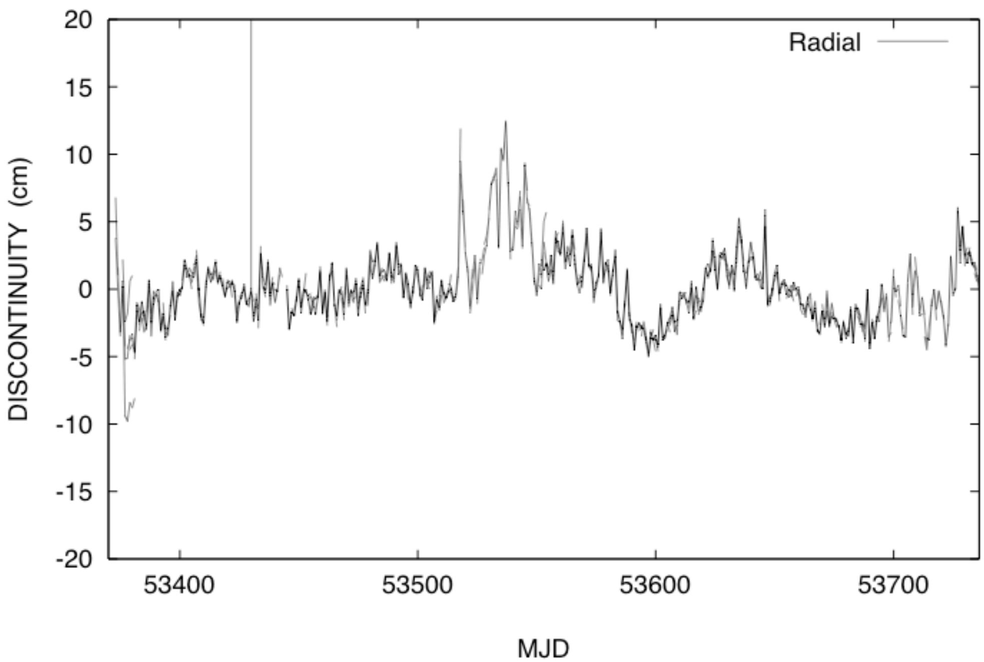

Figure 9. SVN 44 (PRN 28) ALONG-TRACK DISCONTINUITY
using Bernese orbit software

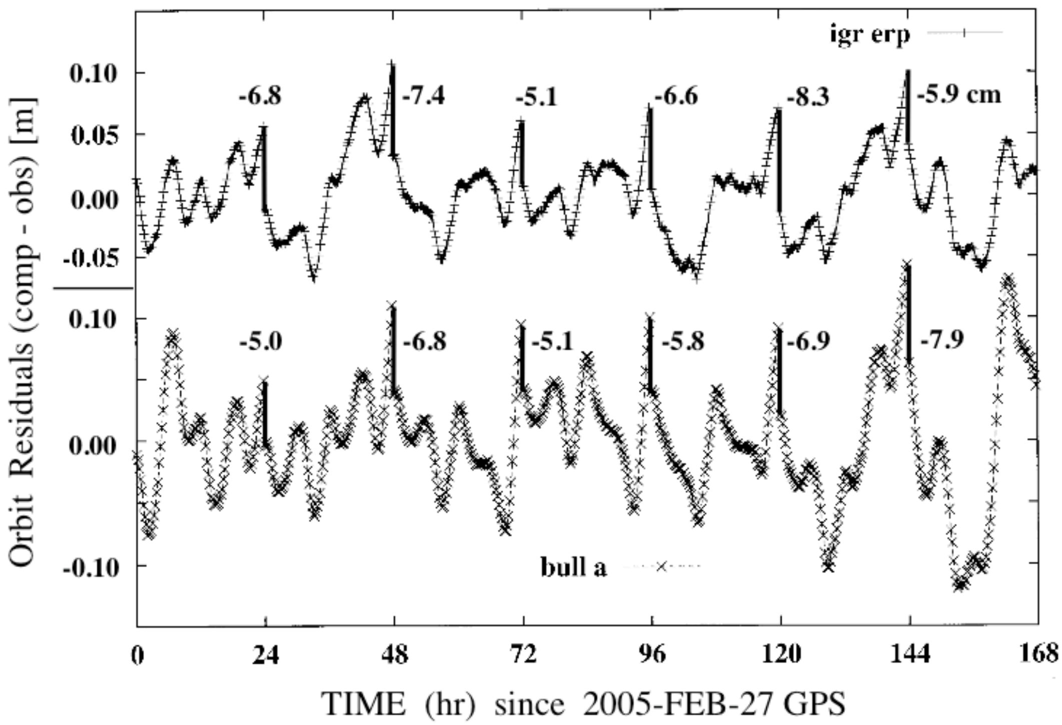

Figure 10. ALONG-TRACK DISCONTINUITY
AT DAY BOUNDARY