Realized and Planned Improvements in ESA/ESOC Ionosphere Modelling

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Overview

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1. Introduction

- May 1998:
  - *Single layer approaches were widely used.*
  - *1^{d} time resolution of the TEC maps.*
  - *Significant gaps in the IGS ground station network.*
- In the meantime:
  - *+ Time resolution has been enhanced, currently 2^{h}.*
  - *+ Near-real-time processing and real-time processing are considered.*
  - *+ Clear trend to 3-d ionosphere models.*
- At ESOC: Improvement of current single layer modelling:
  - ➢ *New TEC & DCBs estimation scheme to enhance time resolution.*
  - ➢ *Surface spherical harmonics to enhance spatial resolution.*
  - ➢ *Local TEC map service for the ESA sites will commence soon.*
  - ❉ *Activities to develop a 3-d ionosphere model using "classical" TEC data in combination with electron density data.*

2. IONMON Upgrades so far realized

2.1 Enhancement of Time Resolution:

- Normal equations are established with a certain time resolution, e.g. 2^h, for ***i*** time intervals:

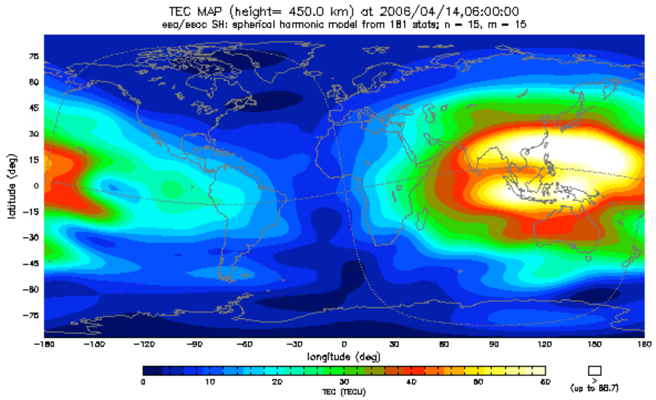
$$\underline{A}_1^T \underline{P}_1 \underline{A}_1 \quad \underline{A}_1^T \underline{P}_1 \underline{l}_1 \quad \underline{l}_1^T \underline{P}_1 \underline{l}_1$$

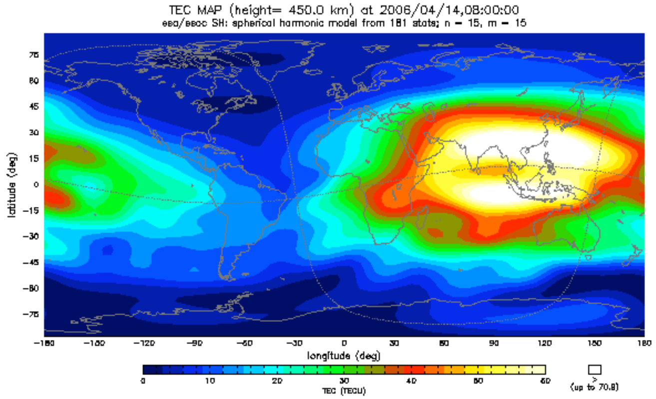
$$\underline{A}_2^T \underline{P}_2 \underline{A}_2 \quad \underline{A}_2^T \underline{P}_2 \underline{l}_2 \quad \underline{l}_2^T \underline{P}_2 \underline{l}_2$$

$$\vdots$$

$$\underline{A}_i^T \underline{P}_i \underline{A}_i \quad \underline{A}_i^T \underline{P}_i \underline{l}_i \quad \underline{l}_i^T \underline{P}_i \underline{l}_i$$

- In this way an archive of normal equations is set up.
- This archive is then used in different ways for TEC estimation and for DCBs estimation.





2. IONMON Upgrades so far realized

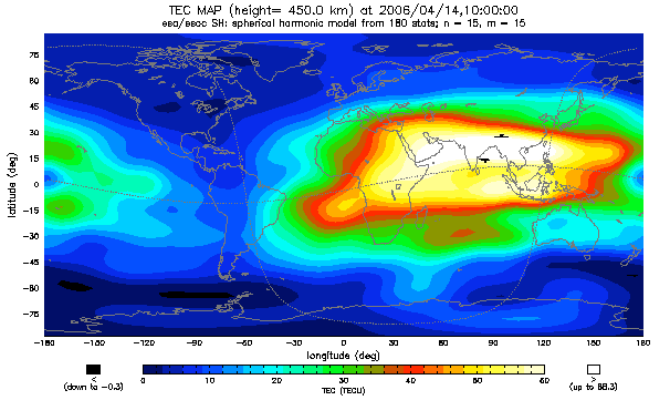
TEC:

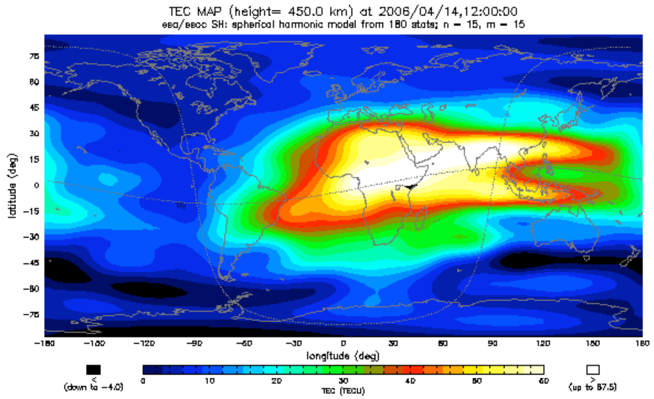
- To make an update for the time interval ***i+1***, pseudo-observation equations are attached to the normal equations

$$\underline{A}_{i+1}^{\,T}\,\underline{P}_{i+1}\,\underline{A}_{i+1} \qquad \underline{A}_{i+1}^{\,T}\,\underline{P}_{i+1}\,\underline{l}_{i+1} \qquad \underline{l}_{i+1}^{\,T}\,\underline{P}_{i+1}\,\underline{l}_{i+1} :$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} \Delta\tilde{x}_1 \\ \Delta\tilde{x}_2 \\ \Delta\tilde{x}_3 \\ \vdots \\ \Delta\tilde{x}_{m-1} \\ \Delta\tilde{x}_m \end{bmatrix} = \begin{bmatrix} x_1 - x_{10} \\ x_2 - x_{20} \\ x_3 - x_{30} \\ \vdots \\ x_{(m-1)} - x_{(m-1)0} \\ x_m - x_{m0} \end{bmatrix}$$

$$\underline{B} \bullet \underline{\Delta\tilde{x}}_{i+1} = \underline{X}_{i+1}$$





2. IONMON Upgrades so far realized

TEC:

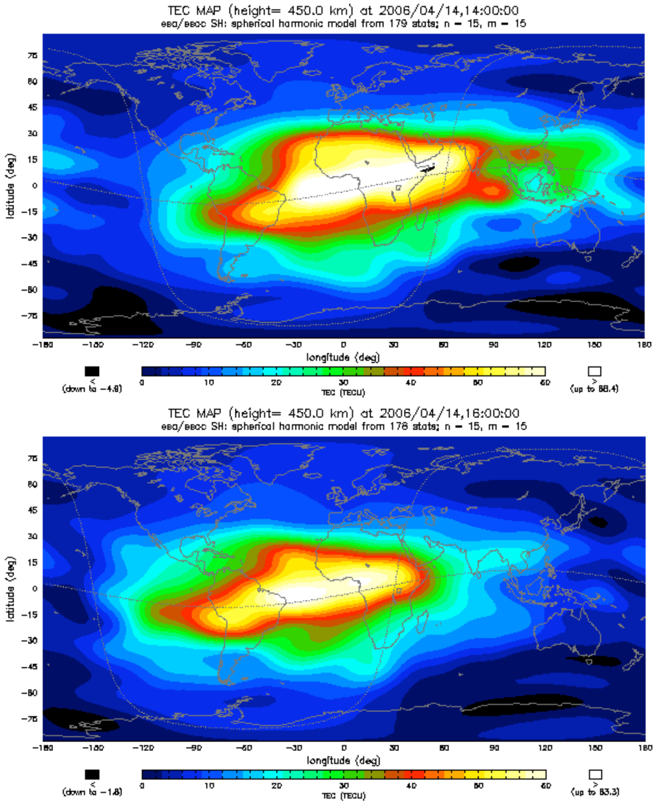
The estimated corrections to the unknowns are then:

$$\underline{\Delta\tilde{x}}_{i+1} = \left(\underline{A}_{i+1}{}^{T}\,\underline{P}_{i+1}\,\underline{A}_{i+1} + \underline{W}_{i+1}\right)^{-1} \bullet \left(\underline{A}_{i+1}{}^{T}\,\underline{P}_{i+1}\,\underline{l}_{i+1} + \underline{W}_{i+1}\,\underline{X}_{i+1}\right)$$

With the weight matrix computed from the previous normal equations:

$$\underline{W}_{i+1} = \frac{F}{rms_i^{\,2}} \bullet \left(f_i \cdot \underline{A}_i{}^{T}\,\underline{P}_i\,\underline{A}_i + f_{i-1} \cdot \underline{A}_{i-1}{}^{T}\,\underline{P}_{i-1}\,\underline{A}_{i-1} + \ldots\right)$$

- The normal equations of update ***i+1*** are then in turn used to establish the normal equations for update ***i+2***, and so on …
- TEC and DCBs are estimated, but only TEC is of relevance here.



2. IONMON Upgrades so far realized

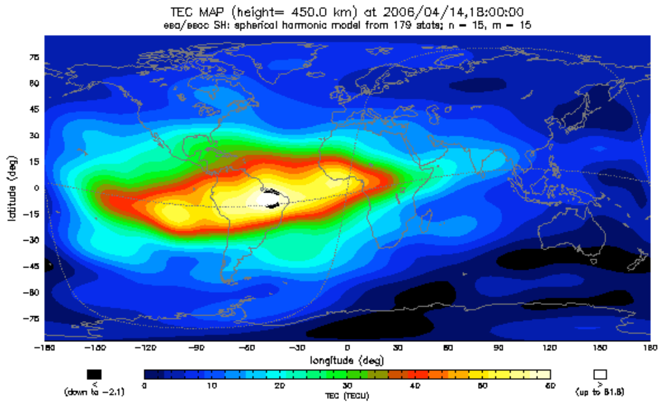
DCBs:

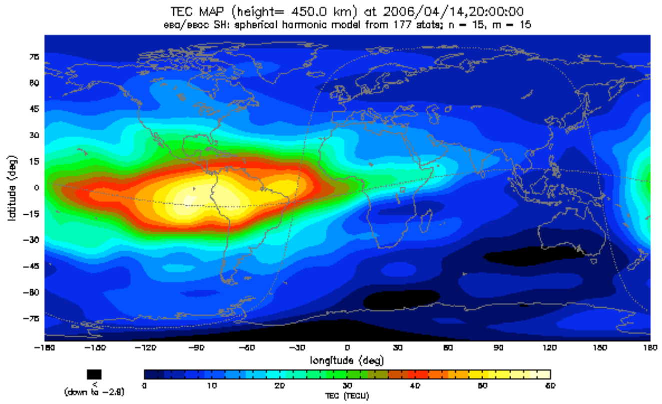
- Satellite and receiver DCB fits are done in daily batch estimates. All ***d*** normal equations of one day are put together as follows from the archive:

$$\left(\underline{A}_1^T \underline{P}_1 \underline{A}_1 + \underline{A}_2^T \underline{P}_2 \underline{A}_2 + \cdots + \underline{A}_d^T \underline{P}_d \underline{A}_d\right) \cdot \underline{\Delta\tilde{x}}$$

$$= \underline{A}_1^T \underline{P}_1 \underline{l}_1 + \underline{A}_2^T \underline{P}_2 \underline{l}_2 + \cdots + \underline{A}_d^T \underline{P}_d \underline{l}_d$$

- DCBs and TEC are estimated, but only DCBs are of relevance here.

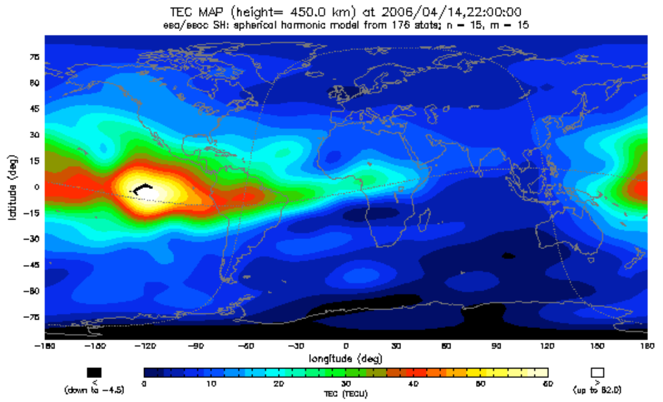


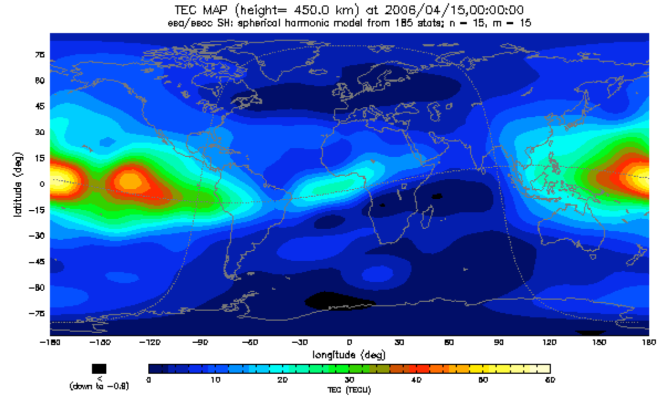


2. IONMON Upgrades so far realized

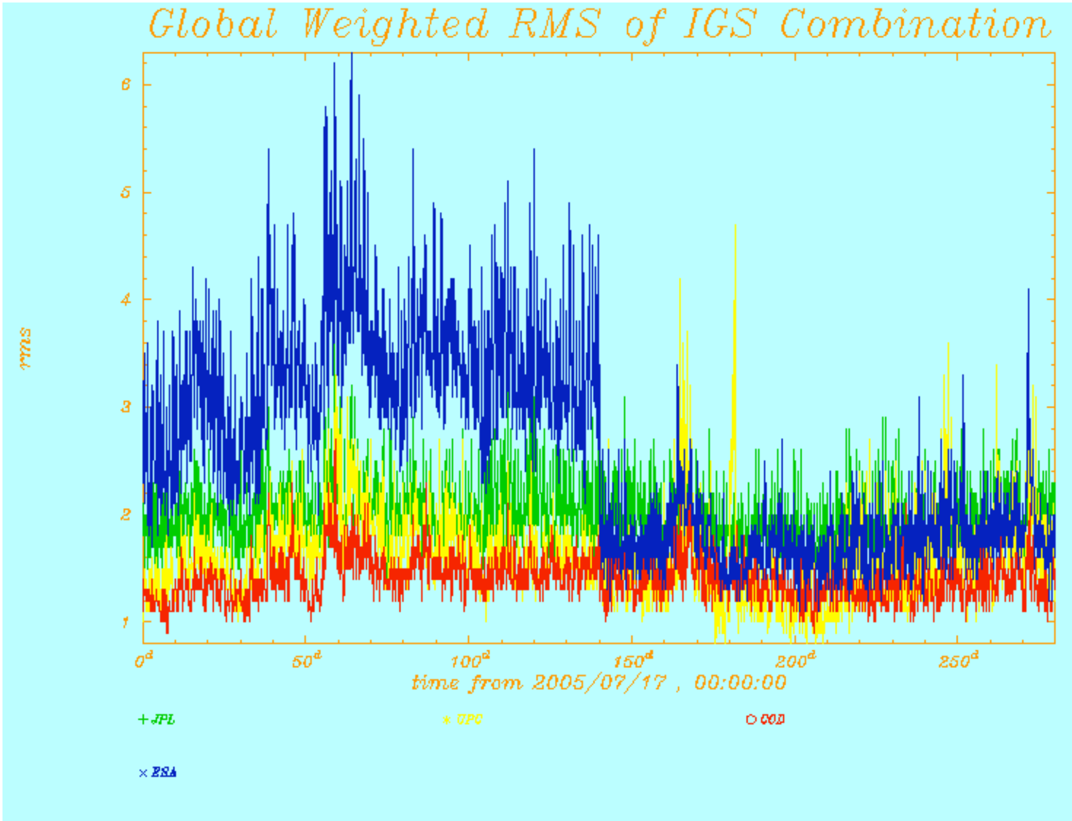
2.2 Additional Improvements:

- Inclusion of RMS maps into the ESA IONEX files.
- Spherical harmonics as surface functions.
- Modified Single Layer Model Mapping Function.
- Number ground stations increased to ≈ 250.
- Inclusion of GLONASS and Galileo TEC data in preparation.
- IONEX file TEC data can be read in as “observables” for tests and validations.





2. IONMON Upgrades so far realized



- The weekly IGS combinations show: The ESA TEC maps are now at the same accuracy level as those of the other IAACs.

3. IONMON Upgrades currently under Work

Establishment of a 3-d ionosphere model.

- In this model the ionosphere can mathematically be set up as superimposition of several layers.
- Exponential correction function for the plasmasphere.
- Alternatively, empirical function for the height dependent scale height.
- Alternatively, several scale height values can be estimated as constants.
- GNSS-derived TEC data will be combined with observed electron density profiles (e.g. from *Champ*, *COSMIC*, *SWARM*, *ionosonde*).
- Occultation data are collected at spacecraft, i.e. also over the oceans.
- The profile parameters themselves will in turn be expressed in terms of global surface functions whose coefficients are then estimated.
- A proper constraining will be necessary to get a stable separation between the different layers.
- By-product: Small program to perform fits of single profile electron density data for preparatory tests - could later be used operationally for single profile analyses.

3. IONMON Upgrades currently under Work

3.1 Profile Functions:

Several profile functions were worked out for 3-d ionosphere modelling.
➡ MacLaurin Series Expansion of the α-Chapman Profile Function:

$$e^{\alpha\cdot\left(1-z-\sec\chi\cdot e^{-z}\right)} = e^{\alpha\cdot(1-z)}\cdot e^{-\alpha\cdot\sec\chi\cdot e^{-z}} \underset{def.}{=} e^{\alpha\cdot(1-z)}\cdot e^{-\gamma\cdot e^{-z}}\ ;\ z = \left(h-h_0\right)/H$$

- Develop the term ***exp{-γ • exp(-z)}*** into a series expansion:

$$f(z) = f(0) + \frac{f'(0)}{1!}\cdot z + \frac{f''(0)}{2!}\cdot z^2 + \frac{f'''(0)}{3!}\cdot z^3 + \ldots$$

- Building the derivatives and evaluating them at ***z = 0***:

$$\begin{aligned}
f(0) &= e^{-\gamma} \\
f'(0) &= (\gamma)\cdot e^{-\gamma} &&= c_1(\gamma)\cdot e^{-\gamma} \\
f''(0) &= -1\cdot\left(\gamma-\gamma^2\right) e^{-\gamma} &&= c_2(\gamma)\cdot e^{-\gamma} \\
f'''(0) &= \left(\gamma-3\gamma^2+\gamma^3\right) e^{-\gamma} &&= c_3(\gamma)\cdot e^{-\gamma} \\
f^{(4)}(0) &= -1\cdot\left(\gamma-7\gamma^2+6\gamma^3-\gamma^4\right) e^{-\gamma} &&= c_4(\gamma)\cdot e^{-\gamma} \\
&\vdots
\end{aligned}$$

3. IONMON Upgrades currently under Work

- Replacing ***exp{-γ • exp(-z)}*** by its series expansion ***R(z)*** in the Chapman Profile Function, one obtains the following profile function and its integral:

$$e^{\alpha\cdot(1-z)}\cdot e^{-\gamma\cdot e^{-z}} \approx e^{\alpha\cdot(1-z)}\cdot R(z) = e^{\alpha\cdot(1-\sec\chi)}\cdot e^{-\alpha z}\cdot\left\{1+c_1\cdot z+\frac{c_2}{2!}\cdot z^2+\frac{c_3}{3!}\cdot z^3+\ldots\right\}$$

$$\int\left(e^{\alpha\cdot(1-z)}\cdot R(z)\right)dz = e^{\alpha\cdot(1-\sec\chi)}\cdot\left\{\int e^{-\alpha z}dz+c_1\cdot\int z\cdot e^{-\alpha z}dz+\frac{c_2}{2!}\cdot\int z^2\cdot e^{-\alpha z}dz+\ldots\right\}$$

- Starting with ***∫exp(-αz) dz = -exp(-αz) / α*** the integral terms can be solved recursively; this scheme can also be applied purely numerically:
- Also the coefficients ***$c_i(\gamma)$*** can be computed with a recursion formula.

$$\int z\cdot e^{-\alpha z}dz = -\frac{1}{\alpha}\cdot z\cdot e^{-\alpha z}+\frac{1}{\alpha}\cdot\int e^{-\alpha z}dz$$

$$\int z^2\cdot e^{-\alpha z}dz = -\frac{1}{\alpha}\cdot z^2\cdot e^{-\alpha z}+\frac{2}{\alpha}\cdot\int z\cdot e^{-\alpha z}dz$$

$$\int z^3\cdot e^{-\alpha z}dz = -\frac{1}{\alpha}\cdot z^3\cdot e^{-\alpha z}+\frac{3}{\alpha}\cdot\int z^2\cdot e^{-\alpha z}dz$$

$$\vdots$$

$$\int z^n\cdot e^{-\alpha z}dz = -\frac{1}{\alpha}\cdot z^n\cdot e^{-\alpha z}+\frac{n}{\alpha}\cdot\int z^{(n-1)}\cdot e^{-\alpha z}dz$$

❈ It is planned to estimate corrections δc_i with observed electron density data.

3. IONMON Upgrades currently under Work

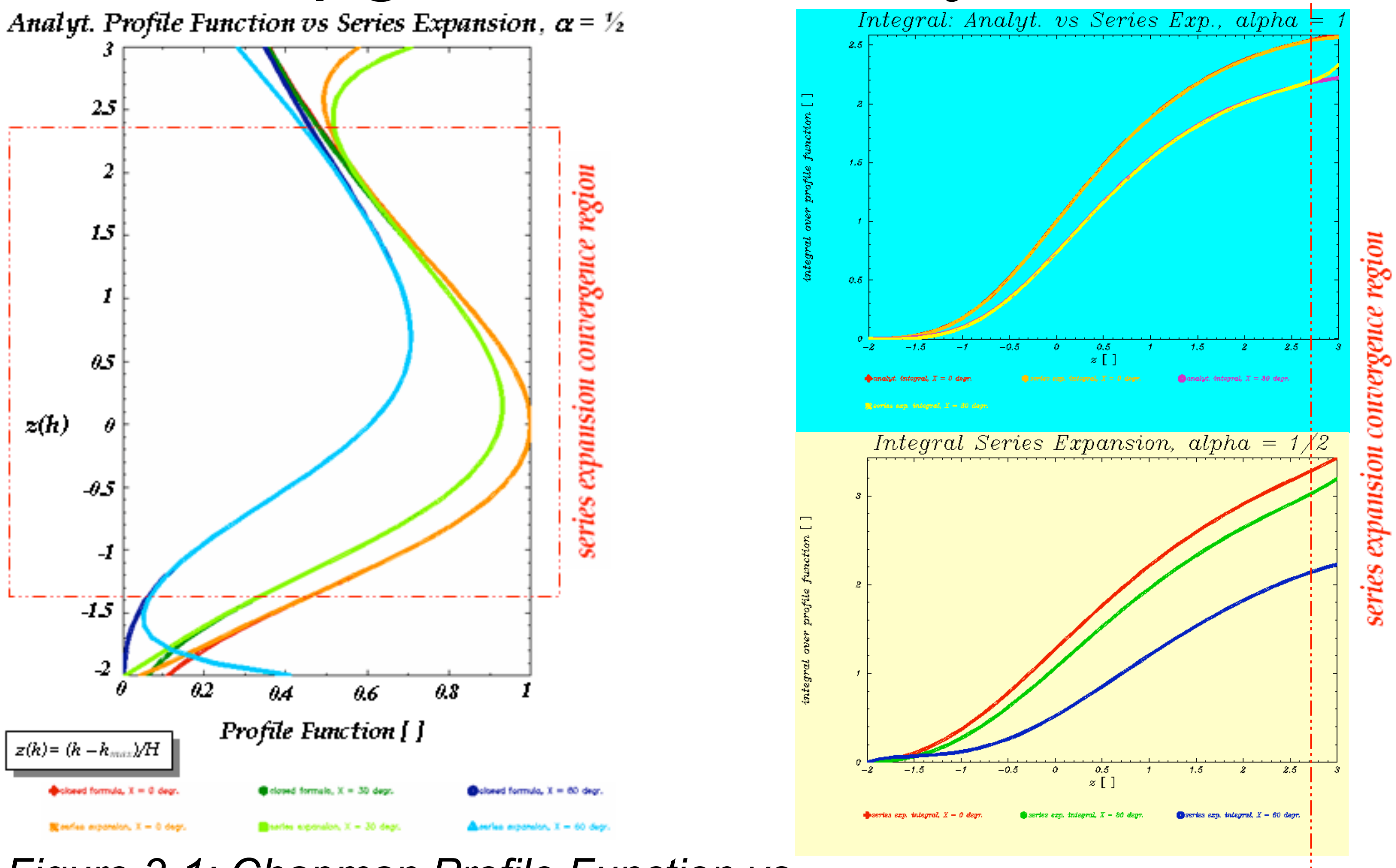


Figure 3-1: Chapman Profile Function vs MacLaurin Series Expansion.

3. IONMON Upgrades currently under Work

3.2 Current Implementation Status:

- The mathematical algorithms for 3-d ionosphere modelling have been worked out, coded and unit tested.
- Prior to the implementation of the new routines into the IONMON software, additional tests and validations will be made, e.g. to find an optimal combination of profile functions and parameters to estimate.
- Currently a small program is prepared as part of these tests to perform fits to single electron density profiles, invoking the new 3-d model routines.
- After the tests are concluded, the new 3-d model routines will be implemented into the IONMON software and final tests will be made.
- Once the new 3-d ionosphere model is ready for operational use, the complete mathematical algorithms will be electronically documented.

4. Future Upgrades

- The inclusion TEC observables derived from GLONASS and Galileo data is under preparation.
- The inclusion of further sources of observed electron density data will be considered.
- The IONMON External User Interface, which allows to access the ionosphere models directly via their estimated coefficients, has to be adapted to the new modelling.
- The currently existing very simple tool to predict the ionosphere's state has significantly to be improved.
- The possibility to establish wavelet-based ionosphere modelling is considered, and also the aspect of the so called "higher order terms".

5. Conclusions

- The ESOC ionosphere modelling has been improved:
  - + *Time resolution was enhanced.*
  - + *Spatial resolution was enhanced.*
  - + *Accuracy of TEC maps is now comparable to those of the other IAACs.*
  - + *Smaller upgrades, e.g. Modified Single Layer Model Mapping Function.*
  - + *Inclusion of data from more ground sites, etc. …*
  - + *Processing of local ionosphere models for the ESA tracking sites shall commence soon.*
- Activities to establish a 3-d ionosphere model at ESOC.
  - + *Takes into account that the ionosphere is composed of several layers.*
  - + *Beyond "classical" TEC data, electron density observation data of different sources will be included into the 3-d processing.*
- Further developments and upgrades are planned.

Thank You for Attention !