A High Precision Analytical Surface Force Model For GPS Block IIR Satellites

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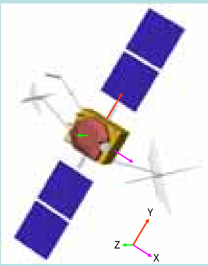
Abstract

This poster discusses the development and testing of an analytical high precision non-conservative force model for the GPS Block IIR spacecraft, carried out at University College London. The computation uses a suite of newly developed techniques that model precisely the dynamic effects on the spacecraft due to solar radiation pressure (SRP), thermal re-radiation, radiation both emitted and reflected by the Earth and the transmission of microwave signals by the satellite antenna array. In the modelling process the complexity of the spacecraft structure is retained – that is, no structural simplifications are made. The environment of the satellite as modelled is based upon in-situ measurements of the respective radiation fluxes by space-based probes. Additional precision has been gained from modelling eclipse transition times based on an oblate Earth and from introducing a reduced solar flux model during passage through the penumbral regions. Model testing is based on numerical integration of the spacecraft model using an 8th order embedded Runge-Kutta integrator with adaptive step size control and using GRACE gravity field coefficients to 15 x 15.

Weighted mean RMS orbit prediction errors (when comparing the integrated trajectory with the precise orbits) for the month of March 2001 using all available GPS IIR satellites over 12 hour arcs were 0.14 m in height, 0.07 m across track and 0.51 m along track. These results have been obtained purely from numerical integration of the differential equations of motion derived from analytical models alone – no empirical estimation has been applied.

Introduction

Real-time GNSS applications rely upon a prediction of the satellite motion. This orbit prediction relies fundamentally upon the knowledge of the forces acting on the spacecraft. Increasingly system users expect more accurate absolute positioning with shorter observation periods. This in turn drives system developers to re-examine the models that support the applications. Well understood forces that drive the shape of GPS satellite trajectories are: Earth's gravity field; the gravitational forces due to the Moon and Sun; tidal Earth gravity field effects and relativistic force effects.



Another group of forces which change the energy state of GNSS spacecraft, are known as non-conservative forces. Of these the largest is solar radiation pressure (SRP) - the force caused by the impact of photons on the satellite surface. Others include thermal re-radiation (TRR), the force due to anisotropic emission of radiation, and the force due to radiation emitted and reflected by the Earth.

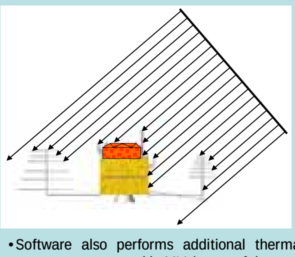
Non-conservative forces are problematic to characterize as they depend instantaneously on the satellite's attitude, surface geometry and optical properties, as well as the incident radiation flux.

This poster outlines a novel approach to modelling these non-conservative forces which act on a GPS Block IIR satellite using the complex geometry of the real spacecraft.

SRP and TRR Modelling Method

Due to the attitude constraints of GPS satellites, in the body-fixed (BFS) coordinate frame the Earth is continually on the Z-axis, and the Sun appears to rise and set in the X-Z plane, never falling below the Y-Z plane. As the Sun rises and sets the solar panels rotate to track the Sun. This relatively simple attitude behaviour is exploited by the modelling method (Ziebart, 2004).

- Photon flux simulated using a pixel array (2mm resolution)
- Spacecraft structure built from 182 geometric primitives, with no simplification of the surface geometry (see image above)
- Pixels projected onto structure - accounts for shadowing effects
- Accelerations determined at discrete points (every 2° in XZ plane)
- Three Fourier series (one each for the satellite X,Y and Z axes) to the data points, means Y-bias is modelled *a priori*
- Software can be used to assess the response of the satellite to the effects of non-nominal attitude



The pixel array projected onto the spacecraft structure

- Software also performs additional thermal force calculations for components covered in MLI (most of the spacecraft bus).
- To determine this force, the temperature of the MLI is required.
- This is evaluated using an energy balance calculation which yields the following equation :

$$T_{MLI}^4 = \frac{\alpha W \cos\theta + \varepsilon_{eff}\,\sigma T_{sc}^4}{\sigma(\varepsilon_{MLI} + \varepsilon_{eff})}$$

Effective emmissivity (ε_{eff}) describes the amount of heat conducted and radiated into the spacecraft.

α=absorptivity, W=solar irradiance, θ=angle of attack, σ=Stephan Boltzmann constant, ε_{MLI}=emmissivity of the MLI, T_{sc} = temp of spacecraft walls

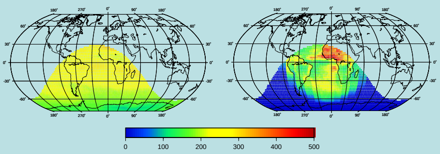
- An analytical function has been developed for TRR due to the solar panels
- Including a thermal model improves orbit prediction at the metre level

Antenna thrust

Transmission of the navigation signals result in a reactive force acting on the satellite. The force is given by the power divided by the speed of light. In the study, a value for the signal power of 515 Watts (supplied by Lockheed Martin) has been used. This is equivalent to a force of 1.7 x 10-6 N, which acts constantly in the satellite radial direction. Adding this effect reduced the RMS along track residuals by approximately ~0.4m.

Earthshine and Albedo

Accelerations due to radiation from the Earth are derived from monthly averaged longwave (LW) and shortwave (SW) data from CERES (Clouds and the Earth's Radiant Energy System) and ERBE (Earth Radiation Budget Experiment).

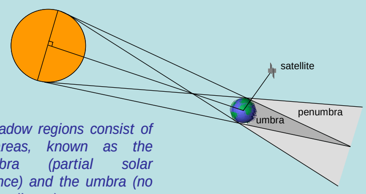


Left: TOA LW Radiation (W/m^2) *Right: TOA SW Radiation (W/m^2)*

- Together, ERBE and CERES provide global 2.5 degree gridded monthly estimates of LW and SW earth radiation fluxes at the top of the atmosphere (TOA = 30km above WGS84 Ellipsoid) from 1984.
- For a particular epoch, TOA radiation values from each grid (that is visible to the satellite) are mapped and scaled onto the satellite position assuming Lambert scattering.
- The effect upon a box and wing model is summed and final ECI accelerations derived. A Fourier series is then fitted to a time series of Earth radiation accelerations.
- Eventually, these forces will scale an analytical Earth radiation force model computed in much the same manner as the current SRP model, thus removing the approximation enforced by the current box and wing implementation.

Eclipse and Penumbral Flux Determination

GPS Block IIR satellites experience eclipse seasons every six months, which last about 4-6 weeks and the time spent in eclipse in one pass can be up to an hour.

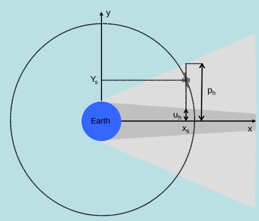


The shadow regions consist of two areas, known as the penumbra (partial solar irradiance) and the umbra (no solar irradiance)

Mistiming entry into or exit from these shadow regions impacts upon numerical integration of orbit trajectories. Often, a spherical Earth model is used as this allows simple geometric arguments to be developed for the calculation of crossing times. Oblate Earth models traditionally require the use of either a series of rotations or the solution of a quartic equation and are computationally intensive. A new treatment of the problem has been developed during the course of this research which still accounts for the Earth's polar flattening but uses a very computationally straightforward method. The mathematical formalism of this method is described in Adhya et al., and it has been validated using photometry for LEO satellites.

In the penumbra, the solar flux gradually diminishes, reaching zero at the umbra. A GPS satellite can spend over 14 minutes in the penumbra, and the resultant fall off in solar flux can be modelled. A very accurate solution again depends on the solution of quartic equations at each epoch. A simpler way to model the flux in the penumbra region is to linearly reduce it from the value it has when it enters the penumbra to zero at the umbra, depending on its depth into the penumbra. The fractional depth into the penumbra is:

$$1 - \frac{y_s - u_h}{p_h - u_h}$$



Schematic showing how penumbral depth is calculated.

Multiplying the right-hand term by the flux present just before entering eclipse gives the flux parameter.

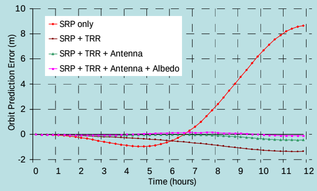
This model is based on a spherical earth, and not on the oblate earth. Although this introduces a discrepancy, the benefits in terms of accuracy of the computed orbits gained by incorporating the above model with an oblate earth do not outweigh the computational costs. Including this model reduces RMS orbit prediction errors at the level of 0.1m.

Testing Methodology

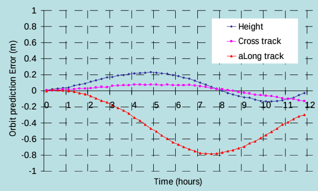
The model tests performed in this paper attempt to predict the motion of the satellite over a twelve hour period using initial conditions taken from the JPL precise ephemerides. The orbit prediction uses numerical integration of the spacecraft dynamic model. This employs: an 8th order embedded Runge-Kutta integrator with adaptive step-size control; high order (15x15) GRACE gravity field coefficients; periodic variations to the gravity field coefficients using the pole and solid Earth tides; third body accelerations for the Sun, Venus, the Moon, Mars and Jupiter with positions generated using the JPL DE405 ephemerides; general relativistic forces and International Earth Rotation Service conventions to compute exact frame transformations.

Results and Discussion

The figures below show differences between integrated trajectories and JPL precise ephemerides:



Along-track orbit prediction errors over 12 hours for one GPS satellite with different non-conservative force models



HCL orbit prediction errors over 12 hours for one GPS satellite with the full non-conservative force model

SVN	H	C	L
41	0.1223	0.071	0.4301
43	0.1763	0.0853	0.648
44	0.122	0.0763	0.445
46	0.185	0.0533	0.590
51	0.183	0.0783	0.780
W Mean	**0.143**	**0.070**	**0.513**

Weighted mean RMS residuals for GPS-IIR satellites over the whole of March 2001

- As each extra factor is modelled clear improvements in the orbit prediction errors are seen
- Remaining errors are small, and appear to have once per rev signatures
- All of the satellites, including the eclipsing SVN46, perform well
- Further improvements could be gained by using more accurate mass histories than are available, or by modelling yaw manoeuvres more closely
- The remaining discrepancies are now approaching the precision of the precise orbits

Conclusion and Outlook

- With SRP, TRR, antenna thrust, penumbra flux and albedo models, weighted mean RMS orbit prediction errors (integrated compared to precise orbits) for the month of March 2001 using all available GPS IIR satellites over 12 hour arcs were 0.14 m in height, 0.07 m across track and 0.51 m along track
- No empirical estimation was used. The models are based upon an understanding of the physical principles governing the dynamics of the satellite
- The method deals with the modelling complexity of real satellites.
- Enhancing the *a priori* accuracy of the satellite dynamic model reduces the likelihood of retaining systematic biases in the post-processed orbit, with implications for calibration, global network scale and determination of satellite antenna phase centre. The models produced by the group at UCL are freely available to any IGS centre, along with code to support implementation.

Selected Papers and Contributors

- Adhya, S., Sibthorpe, A., Ziebart, M., and Cross, P., (2004), "An Oblate Earth Eclipse state Algorithm for Low Earth Orbiting Satellites", *Journal of Spacecraft and Rockets*, vol.41 (1), pp. 157-159
- Ziebart, M.,Adhya, S., Sibthorpe, A., and Cross, P., (2003a), "Residual Systematic Biases in Spacecraft Orbits", Proceedings of Workshop on the State of GPS Vertical Positioning Precision Separation of Earth Processes by Space Geodesy, European Centre for Geodynamics and Seismology, Luxembourg, April 2003, in press
- Ziebart,M., Adhya, S., Sibthorpe, A., and Cross, P., (2003b), "Taking the Long View: The Impact of Spacecraft Structural Design and High Precision Force Modelling on Long Term Orbit Evolution", Proceedings of ION GPS/GNSS 2003, Portland, Oregon, USA, pp. 1002-1008
- Ziebart, M., Adhya, S., Sibthorpe, A., and Cross, P., (2003c), "GPS Block IIR Non-conservative Force Modelling: Computations and Implications", Proceedings of ION GPS/GNSS 2003, Portland, Oregon, USA, pp. 2671-2678. *Winner of the Best Paper in Session award.*
- Ziebart,M., (2004), "Generalised Analytical Solar Radiation Pressure Modelling Algorithm for Spacecraft of Complex Shape", *Journal of Spacecraft and Rockets*, in press

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