# Seasonal Height Errors and the TRF 

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## Introduction

Seasonal errors from many effects have magnitudes of only a few millimeters. With vertical repeatability on a global scale of 5-10 millimeters for both VLBI and GPS, errors of only a few millimeters are often overlooked. However, when averaged over decades, as is important for the determination of site velocities and positions for the terrestrial reference frame, such effects are significant. This has long been recognized, and it is common practice to estimate annual and semi-annual terms of site coordinates along with the linear terms to remove unmodeled effects. While it is clear that it is preferable to remove time-variable effects, it is important to also remove errors that appear to first order to be time-invariant. This will reduce the sensitivity to unmodeled or unknown effects that would not affect the measurements of a perfect system. A simple example is the dependence on minimum elevation angle of most GPS antennas which does not give an apparent height change if the az-el distribution remains the same, but will cause a seasonal error if the lowest elevation observations are blocked by leaves during the summer.

In this poster I would like to bring attention to five sources of uncorrected height errors and possible model improvements that must be implemented in order to achieve height accuracy of a few millimeters.

The five model improvements are:

1) atmosphere delay mapping functions (models better than NMF are available)
2) thermal deformation, e.g. building or monument expansion (model or measure)
3) local hydrologic effects, e.g. groundwater withdrawal (some measurements are available, e.g. for the Tsukuba GPS and VLBI antennas (Munakane et al))
4) radome (can be measured; minimum-elevation dependent; therefore sensitive to seasonal variation in satellite visibility)
5) site specific antenna/monument phase error (estimated from post-fit residuals (e.g. Hurst et al))

## Height error due to NMFh mapping function

The simple seasonal model of the NMF hydrostatic mapping function (Niell 1996) leaves fictitious position height errors at annual and semi-annual periods in addition to a random error (Figure 1). These errors are obtained by using the actual az/el distribution of a 24 -hour GPS session for latitude $42^{\circ}$ and treating the difference in mapping function between NMFh and IMFh (Niell 2000) as the error in NMFh. IMFh is based on a Numerical Weather Model and is significantly more accurate than NMFh, especially for seasonal terms.


Figure 1. Height error in a solution made with NMFh compared to using IMFh for minimum elevation of $5^{\circ}$. The solid line is the sum of the annual and semi-annual terms in the fourier transform of a twenty-two year record of the calculated height error.


Figure 2. Amplitude of annual and semi-annual error in height that would be incurred using NMFh (compared to IMFh) with a minimum elevation of $5^{\circ}$. The dashed lines are least-squares fits of the form $a^{*} \cos (2 *$ latitude $) .+b$.

Such errors in both the geodetic estimates and in atmosphere delay estimates are largest at high latitudes and are smallest in the equatorial region (Figure 2). At mid-latitudes the annual term of the height error has an amplitude of approximately 5 mm for minimum elevation of $5^{\circ}$, and the semi-annual error is about half that. The errors are smaller by a factor of about two for $7.5^{\circ}$ minimum and by about six for a $15^{\circ}$ minimum (Figure 3).


Figure 3. Error in height that would be incurred using NMFh (compared to IMFh) for three different minimum elevations. The dashed lines are least-squares fits of the form $\mathrm{a} * \cos (2 *$ latitude $) .+\mathrm{b}$.

The sense of the error is generally to depress the height in northern hemisphere winter (Figure 1). Thus a more correct model of the atmosphere delay will reduce the apparent annual change in height that might otherwise be interpreted to hydrologic loading due, for example, to snow.

These errors can be significantly reduced by using atmosphere delay mapping functions based on a Numerical Weather Model, such as IMF or VMF (Boehm and Schuh 2004).

Using the more accurate mapping function will allow lower elevation data to be included, thus improving the intrinsic geometric precision of the daily position estimate (figure 4).

|  | available | a priori hydrostatic gradient | height std dev (mm) ${ }^{1}$ |
| :---: | :---: | :---: | :---: |
| IMFh | globally | globally | 4 |
| VMFh | specified sites | specified sites | 1 |

[^0]

Figure 4. Height uncertainty for twenty-four hours for point position solution for latitude $42^{\circ}$ (solid black line), and additional uncertainty due to atmosphere hydrostatic mapping function error, averaged over a year: NMF (blue x), IMF (red + ).

## Thermal deformation:

Many GPS antennas are mounted on pillars, towers, or buildings that are on the order of 10 m high. At high latitudes the winter to summer temperature variation may be up to 50 C . For example, the IGS site at Algonquin Park, Canada, fits these conditions. The expected expansion of concrete or steel for these circumstances is about 2 mm , or 1 mm amplitude. The phase is such that this mimics the height change expected for snow loading, and is of the order discussed for degree-1 loading.

Measurements of height variations of VLBI antennas at Onsala, Sweden, Wettzell, Germany, and Westford, USA, have demonstrated that a simple model of deformation based on ambient temperature can reproduce the height variations to a few times 0.1 mm .

## Local hydrologic changes:

Munekane et al (GSI) reported at the 2003 Dec AGU that the height changes of the TSKB GPS site are correlated with vertical displacements measured by subsidence meters in a nearby well. The amplitude of the height change is about 4 mm . These changes are attributed to withdrawal and replenishment of water related to the growing of rice nearby. The height of the Tsukuba 32 m VLBI antenna relative to TSKB was measured using a GPS antenna at the vertex of the antenna. By comparison of annual changes with temperature and with well depth they demonstrate that the GPS antenna sees changes in both the deep and shallow aquifers, while the VLBI vertex is affected by the deep aquifers and thermal expansion. Munakane et al conclude that with the hydrologic and temperature measurements the heights of both the VLBI and GPS can be modeled to approximately one millimeter.

## Radome:

Several authors have demonstrated the effect of radomes on the measured GPS heights, including the seasonal effect of snow accumulation. While these changes are somewhat difficult to model, the static effect of the radome can be measured, for example by differential comparison with a nearby reference antenna (Figure 5). These phase differences (radome on - radome off) could be applied as a correction as is done for phase center variation. Changes in the sky coverage, due for example to seasonal foliage change or to sky blockage due to new construction,
would then be accommodated at the observable level.


Figure 5. Minimum elevation dependence of height error due to Ashtech choke-ring radome.

## Site specific phase errors:

In a manner similar to the radome effect, the removal of site specific phase errors determined from the post-fit phase residuals (as described by Hurst et al) would reduce the sensitivity to seasonal changes in sky coverage. From data over several years it might be possible to derive a seasonal model of such SSPCs. The dependence of height on minimum elevation due to site specific phase error is shown in the Figure 6 for the site SANT (Santiago, Chile).


Figure 6. Dependence of height on minimum elevation for the site SANT. The antenna is a Dorne-Margolin choke ring. Note the change of 6 cm with only a $4^{\circ}$ change in minimum elevation.

## Implementation issues

An important issue for the implementation of time variable phenomena, such as atmospheric pressure loading or thermal deformation of the antenna mount, is the definition of the reference "point", which must include both an epoch and a value (or set of values if horizontal variability is also to be allowed). Furthermore, the reference point must be consistent among the techniques. It is likely that perceived differences between GPS and VLBI site coordinates are due to failure to account for thermal deformation in both the VLBI antenna structures and GPS supporting structures.

## Where in analysis should a correction be made?

Careful consideration should be given to how corrections are made. In principle the correction should be made as close to the basic observable as possible. For GPS this would be at L1/L2 or at LC (currently). For example, although the radome correction shown above could be made as a height correction after the position is determined, this requires knowledge of the minimum elevation included. Furthermore, the effective minimum elevation depends on the actual distribution in elevation of the observations and whether elevation-dependent weighting was used. Correction of the phases, which might best be done as LC corrections, would eliminate the error due to the uncertain distribution of observations. This becomes more important if there are azimuthal variations as well.

## References

Boehm, J. and H. Schuh: Vienna Mapping Functions in VLBI Analyses, Geophys. Res. Letters, 2004.

Hurst, K., et al, in preparation; AGU, 1996.
Munekane, H, M. Tobita, K. Takashima, Groundwater-induced vertical movements in Tsukuba, Japan, 2004 (Draft report).

Niell, A.E.: Improved atmospheric mapping functions for VLBI and GPS, Earth, Planets, and Space, 52, 699-702, 2000.


[^0]:    ${ }^{1} \quad 5^{\circ}$ minimum elevation, mid-latitude.

